FORTIETH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 1996

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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1. Which of the following equations describes a line that passes through the point (6, -2) and has slope $\frac{1}{3}$?

(a) $x - 3y = 12$ (b) $x - 3y = 0$ (c) $3x - y = 20$ (d) $3x + y = 16$

(e) $x + 3y = -12$

2. The distance from the point (3, 10) to the line $y = 2x + 3$ is:

(a) $\sqrt{5}$ (b) $2\sqrt{5}$ (c) $-\sqrt{5}$ (d) $\frac{1}{\sqrt{5}}$ (e) 0

3. The value of the expression $(x - 1)^{\frac{2}{3}} + 2x^0$ when $x = 9$ is:

(a) 6 (b) 8 (c) 10 (d) 4 (e) 22

4. The equation $x^2 + y^2 - 6y = 16$ describes a circle with what radius?

(a) 3 (b) 4 (c) 5 (d) 16 (e) 25

5. If the three points (-2, -1), $(x, 2)$, and (8, 14) are collinear, then the value of x is:

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

6. Which of the following rational numbers has the decimal representation $0.\overline{132}$?

(a) $\frac{33}{250}$ (b) $\frac{33}{100}$ (c) $\frac{17}{111}$ (d) $\frac{119}{900}$ (e) $\frac{44}{333}$

7. The complete factorization of the polynomial $x^4 + 3x^3 - 13x^2 - 27x + 36$ is:

(a) $(x + 3)(x + 3)(x + 4)(x + 1)$ (b) $(x + 3)(x - 3)(x + 4)(x - 1)$

(c) $(x - 3)(x - 3)(x + 4)(x - 1)$ (d) $(x + 3)(x - 3)(x - 4)(x + 1)$

(e) $(x - 3)(x - 3)(x - 4)(x - 1)$

8. If an automobile costs a dealer \$16,240, what is the least sticker price that will allow the dealer to give a 20% discount on the sticker price and still make 30% profit on the cost of the car?

(a) \$26,390 (b) \$30,100 (c) \$30,205 (d) \$25,000 (e) \$24,360

9. In the complex number system, the product $\sqrt{-4}\sqrt{-9}$ simplifies to:

(a) -6 (b) 6i (c) -6i (d) -5 (e) 6

10. If (a, b, c) denotes the unique solution of the system of linear equations shown below, what is the value of $a + b + c$?

$$\begin{aligned} x + 2y + 3z &= 3 \\ 2x + 3y + z &= 2 \\ 3x + y + 2z &= 1 \end{aligned}$$

(a) -1 (b) $-\frac{1}{3}$ (c) 0 (d) 1 (e) $\frac{1}{3}$

11. The graph of the equation $9x^2 + y^2 - 54x + 6y + 81 = 0$ is:

(a) a point (b) an ellipse (c) a hyperbola (d) a circle (e) a parabola

12. In descending powers of c, the third term in the expansion of $(c - 2)^5$ is:

(a) $-107c^2$ (b) $60c^3$ (c) $40c^3$ (d) $20c$ (e) $80c^3$

13. The largest possible number of positive real roots for $P(x) = 2x^6 - x^4 - x^3 + 3x^2 - 7x + 13$ is:

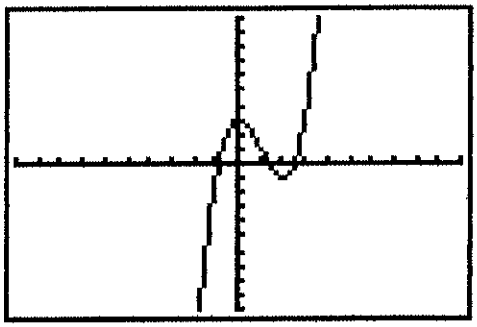
(a) 6 (b) 5 (c) 4 (d) 3 (e) 0

14. The amount of silt carried by a stream is directly proportional to the sixth power of its velocity. A certain stream normally carries 150 tons of silt per day. However, during the summer's flood the velocity was doubled. How much silt did the stream carry per day during the flood?

(a) 75 tons (b) 300 tons (c) 9600 tons (d) 4800 tons

(e) 1800 tons

15. Which of the following equations has the graph shown below?



(a) $y = x^3 + 3x^2 + 3$

(b) $y = x^3 - 3x^2 - 3$

(c) $y = -x^3 + 3x^2 + 3$

(d) $y = -x^3 - 3x^2 + 3$

(e) $y = x^3 - 3x^2 + 3$

16. The product of $\sqrt[3]{4}$ and $\sqrt[4]{8}$ equals:

(a) $2\sqrt[7]{12}$ (b) $2\sqrt[12]{32}$ (c) $\sqrt[7]{32}$ (d) $\sqrt[12]{32}$ (e) $\sqrt[7]{12}$

17. Which of the following are solutions of the equation $\ln e^{x^2} = x + 6$?

(a) 4, -4 (b) 3, -2 (c) $\pm\sqrt{6}$ (d) 0, 6 (e) 2, -3

18. The inverse function, $f^{-1}(x)$, of $f(x) = \dfrac{-5x - 1}{-3x - 2}$ is:

(a) $\dfrac{-5 - x}{-3 - 2x}$ (b) $f^{-1}(x)$ does not exist (c) $\dfrac{-3x - 2}{-5x - 1}$ (d) $\dfrac{-2x + 1}{-3 - 2x}$

(e) $\dfrac{2x - 1}{-3x + 5}$

19. Which of the following equations describes a parabola with vertex (-3, 2) and focus (-3, -10)?

(a) $x^2 + 6x - 5y + 59 = 0$ (b) $x^2 + 6x - 48y + 105 = 0$ (c) $x^2 + 6x + 5y - 41 = 0$

(d) $x^2 + 6x + 48y - 87 = 0$ (e) $x^2 - 6x + 3y - 33 = 0$

20. After t seconds, the height h in feet above the ground of an object launched vertically is given by $h(t) = -16t^2 + 16t + 32$. The maximum height of the object is:

(a) 32 feet (b) 64 feet (c) 36 feet (d) 48 feet (e) 40 feet

21. If $f(t) = 3t + 2$ and $g(t) = 2t + 3$, then the sum of all real solutions of $f(x) = g(x^2)$ is:

(a) $\frac{4}{3}$ (b) 4 (c) $\frac{3}{2}$ (d) 1 (e) 0

22. Given $\log 2 = 0.301$, $\log 3 = 0.477$, and $\log 5 = 0.699$, then the value of $\log\left(\frac{2}{25}\right)$ is:

(a) -10.970 (b) -2.097 (c) 0.903 (d) -1.097 (e) 0.0120

23. In the diagram below, the distance from A to C is 50, the distance from B to D is 15, and the distance from A to B is x. The algebraic expression for the sum of the distances from A to D and B to C is:

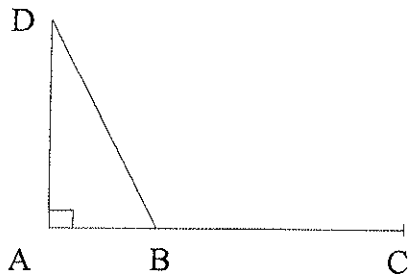
(a) $\sqrt{225 + x^2} - 50 + x$

(b) $65 - 2x$

(c) $\sqrt{225 + x^2} + 50 - x$

(d) $275 - 2x$

(e) $\sqrt{225 - x^2} + 50 - x$



24. If $f(x) = \frac{1}{2}x$, then for $h \neq 0$, $\frac{f(x+h) - f(x)}{h}$ equals:

(a) 2 (b) 1 (c) $\frac{1}{2}$ (d) $\frac{x + \frac{1}{2}h}{h}$ (e) 0

25. If $xy = a$ and $x + y = b$, then the expression $x^2 + y^2$ equals:

(a) b^2 (b) $a^2 + 2b$ (c) $a^2 - 2b$ (d) $b^2 + 2a$

(e) $b^2 - 2a$

26. Which of the following is the equation of the line passing through the point (3, 2) and perpendicular to the line $2x - 3y + 4 = 0$?

(a) $3x + 2y - 13 = 0$ (b) $3x - 2y - 5 = 0$ (c) $3x - 2y - 13 = 0$

(d) $2x+3y-12=0$ (e) $2x-3y+13=0$

27. The expression $\left(\frac{x^{-3}y^2}{z}\right)^{-4}$, written without negative exponents, simplifies to:

(a) $\frac{z^4}{x^7y^6}$ (b) $\frac{y^2z^4}{x^7}$ (c) $\frac{x^{12}y^8}{z^4}$ (d) $\frac{z^4}{x^{12}y^8}$ (e) $\frac{x^{12}z^4}{y^8}$

28. Which of the following are vertical asymptotes of the graph of $y = \frac{x^3 - 3x^2 - 4x + 12}{x^2 - 9}$?

(a) x = -2, x = 2 (b) $x = 3$ (c) x = -3, x = 3 (d) x = -3

(e) x = -3, x = 3, x = -2, x = 2

29. The solution of the equation $3^{x+4} = 4^{2x+3}$ in terms of natural logarithms is:

(a) $x = \ln 4 - \ln 3$ (b) $x = \frac{3\ln 4 - 4\ln 3}{\ln 3 - 2\ln 4}$ (c) $x = \ln\left(\frac{4^3}{3^4} - \frac{3}{4^2}\right)$

(d) $x = 5\ln 4 - 5\ln 3$ (e) $x = 3\ln 4 - 3\ln 3$

30. The sum of the convergent geometric series $\sum_{k=1}^{\infty} 3\left(-\frac{2}{3}\right)^{k-1}$ is:

(a) 12 (b) $\frac{9}{5}$ (c) $\frac{6}{5}$ (d) 9 (e) $\frac{5}{9}$

31. Which of the following are solutions of the equation $x(2x + 1) = 1$?

(a) $-\frac{1}{2}, 1$ (b) $-\frac{1}{2}, -1$ (c) $\frac{1}{2}, 1$ (d) $\frac{1}{2}, -1$

(e) $\frac{-1 \pm i\sqrt{7}}{2}$

32. The equation of the axis of symmetry for the graph of $x = 4y^2 + 2y + 1$ is:

(a) $x = \frac{1}{4}$ (b) $x = -\frac{1}{4}$ (c) $y = -\frac{1}{4}$ (d) $x = -2$

(e) $y = -\frac{1}{4}x + 2$

33. If $f(x) = \frac{3}{x}$, then $f(f(x))$ equals:

(a) $\frac{1}{x}$ (b) $\frac{3}{x}$ (c) $\frac{9}{x^2}$ (d) x (e) 9

34. The solution set for the equation $\frac{1}{\frac{4}{x-2}+2} = \frac{1}{2}$ is:

(a) {2} (b) {4} (c) the empty set (d) {0} (e) {0.25}

35. Which of the following is an equation of the hyperbola with center (0, 0), vertex at (4, 0) and an asymptote of $y = \frac{3}{2}x$:

(a) $\frac{x^2}{16} - \frac{y^2}{36} = 1$ (b) $\frac{x^2}{16} - \frac{y^2}{24} = 1$ (c) $\frac{x^2}{4} - \frac{y^2}{9} = 1$ (d) $-\frac{x^2}{16} + \frac{y^2}{36} = 1$

(e) $-\frac{x^2}{4} + \frac{y^2}{9} = 1$

36. The domain of the function $f(x) = \sqrt{\frac{x}{x^2+4}}$ is:

(a) $(-\infty, \infty)$ (b) $[0, \infty)$ (c) $(0, \infty)$

(d) $(-\infty, 0) \cup (0, \infty)$ (e) $(-\infty, -2) \cup (-2, 0) \cup (0, 2) \cup (2, \infty)$

37. A survey is made of household pets in a certain town by randomly selecting 70 houses. The number of houses and pets owned is as follows:

cat	19 houses	cat and bird only	2 houses
dog	38 houses	cat and dog only	3 houses
bird	10 houses	dog and bird only	1 house
		cat, dog, and bird	2 houses

If this survey is representative of the whole town, the probability that the next house has no pets is:

(a) 1 (b) 0 (c) $\frac{13}{70}$ (d) $\frac{2}{7}$ (e) $\frac{8}{70}$

38. If $f(x) = (x - 2)^3$, then an expression for the inverse function, $f^{-1}(x)$, is:

(a) $\sqrt[3]{x - 2}$ (b) $\sqrt[3]{x} + 2$ (c) $\sqrt[3]{x + 2}$ (d) $\sqrt[3]{x} - 2$

(e) $(x + 2)^3$

39. The solution set of the inequality $x^2(x + 1)(x - 2)(x^2 - x + 1) < 0$ is:

(a) $[-1, 2]$ (b) $(-1, 2)$ (c) $[-1, 0) \cup (0, \infty)$

(d) $(-1, 0) \cup (0, 2)$ (e) $(-1, 0) \cup (2, \infty)$

40. The graph of $y = x^2 + 3$ from $x = -1$ to $x = 2$ is sketched below. If the area of region A is $3\frac{1}{3}$ and the area of region B is $8\frac{2}{3}$, then the area of the shaded region is:

(a) $\frac{16}{3}$ (b) 12 (c) $\frac{33}{2}$

(d) $\frac{9}{2}$ (e) $\frac{7}{2}$