THIRTY-NINTH ANNUAL MATHEMATICS CONTEST

sponsored by

THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 1995

Prepared by:	Reviewed By:
TMTA Examinations Director	Mathematics Faculty Roane State CC Harriman, TN Oak Ridge, TN

Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State CC, Gallatin, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
TRW Commercial Steering Division, Lebanon, Tennessee
Wright Industries, Inc., Nashville, Tennessee

Algebra II

1. Coordinates of the vertex for the parabola $y = -2x^2 + 12x - 3$ are:

   (a) (3,15) (b) (3,21) (c) (3,12) (d) (-3,-15) (e) (-3,-57)

2. Find the solution to:
$$\begin{aligned} 3x - 2y &= 4 \\ 5x + 3y &= 13 \\ x + y &= 4 \end{aligned}$$

   (a) (2,1) (b) (1,2) (c) (12/5,8/5) (d) (1/2,7/2) (e) $\varnothing$

3. Which of the following is not a function of x?

   (i) $y = |x|$ (ii) $x + 2y = y$ (iii) $y = 1/x$ (iv) $x^2 + y^2 = 4$

   (a) i and iv (b) ii and iii (c) iii and iv (d) iv (e) iii

4. If the expression $(x^2 + 1)^3(x^4 + 3)(2x - 1)$ is multiplied out, the highest power of x would be:

   (a) 10 (b) 11 (c) 13 (d) 7 (e) 8

5. The solution of $\frac{3}{x-2} + \frac{3}{x+2} = \frac{5x-4}{x^2-2x}$ is:

   (a) {4,2} (b) {4} (c) {0,-1} (d) {-4} (e) {-4,0}

6. If $(x + 3)$ is a factor of $x^3 - 2kx + k^2$, then k is:

   (a) 3 (b) -3 or 9 (c) 3 or -9 (d) -3 (e) -9

7. A polynomial with real coefficients and zeros 3, -1, and $1 + 2i$ is:

   (a) $x^4 - 2x^2 + 16x - 15$ (b) $x^4 - 4x^3 + 6x^2 - 4x - 15$

   (c) $x^4 - 6x^2 - 8x - 3$ (d) $x^4 - 2x^3 + x^2 - 8x - 12$ (e) $x^3 - 7x - 6$

8. What is the smallest value of A such that $|2x + 3| \leq A$ for all x in the interval (-1,5)?

   (a) 1 (b) -1 (c) 5 (d) 13 (e) -13

9. The sum of the roots of $x^4 - 5x^2 + 4 = 0$ is:

   (a) 3 (b) -2 (c) 0 (d) 5 (e) -4

10. The combined ages of a man and his sister is 55 years. He is 3 times as old as she was when he was as old as she is now. What is the product of their ages?

    (a) 567 (b) 726 (c) 484 (d) 750 (e) 574

11. Solve the inequality: $x^2 - 3x - 7 \leq x - 2$

(a) $-5 \leq x \leq 1$ (b) $x \leq 5$ (c) $x \leq -1$ or $5 \leq x$

(d) $-1 \leq x \leq 5$ (e) $x \geq 5$

12. What is the value of i^{2003}?

(a) i (b) -1 (c) $-i$ (d) 1 (e) 0

13. If $f(x) = 2x^2 + 3$ and $g(x) = \sqrt{x+2}$, find $f \circ g = ?$

(a) $2\sqrt{x+2} + 3$ (b) $2x + 7$ (c) $2x + 5$

(d) $\sqrt{2x^2 + 5}$ (e) $(2x^3 + 3)(\sqrt{x+2})$

14. The augmented matrix for the system $\begin{matrix} x + 6y = 7 \\ y = 11x + 10 \end{matrix}$ is:

(a) $\begin{bmatrix} 1 & 6 & 7 \\ -1 & 11 & 10 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 6 & 7 \\ 1 & -11 & 10 \end{bmatrix}$ (c) $\begin{bmatrix} 1 & 6 & 7 \\ 11 & -1 & -10 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 6 \\ -11 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 7 & 6 \\ 10 & -1 \end{bmatrix}$

15. $\dfrac{5 + i}{4 - 5i} = ?$

(a) $\frac{5}{4} - \frac{1}{5}i$ (b) $-\frac{15}{9}$ (c) $\frac{15}{31}$ (d) $\frac{55}{41}$ (e) $\frac{15}{41} + \frac{29}{41}i$

16. Suppose that y varies directly as x^2 and inversely as z. If $y = 32$ when $x = 8$ and $z = 10$, find y when $x = 12$ and $z = 15$.

(a) 5 (b) 4 (c) 48 (d) 64 (e) 22

17. $9x^2 + 42x + 49 = 0$ has

(a) two unique real roots (b) one double root (c) no real roots

(d) one complex and one real root (e) two complex roots

18. Suppose the function $f(x) = 2x^3 + 4x^2 - 3x - 5$ is divided by $x + 2$. The remainder is:

(a) 31 (b) -11 (c) f(2) (d) -31 (e) f(-2)

19. The equation of the circle with end points of the diameter at (-3,-2) and (5,4) is:

(a) $x^2 + y^2 - 2x - 2y + 27 = 0$ (b) $x^2 + y^2 - 2x - 2y - 23 = 0$

(c) $x^2 + y^2 - 2x - 2y - 27 = 0$ (d) $x^2 + y^2 + 2x + 2y + 23 = 0$

(e) $x^2 + y^2 + 2x + 2y - 23 = 0$

20. What is the coefficient of the x^4y^5 in the expansion of $(2x-y)^9$?

(a) -16 (b) 126 (c) -126(16) (d) -126 (e) 16

21. If $3^{x+2} = 2^{2x-1}$ then x is:

(a) $\frac{\log 18}{\log \frac{4}{3}}$ (b) $\frac{\log 12}{\log \frac{4}{3}}$ (c) $\frac{\log 27}{\log 2}$

(d) $\frac{\log 3}{\log 2}$ (e) $\frac{\log \frac{4}{3}}{\log 18}$

22. $\begin{bmatrix} \cos A - i \sin A & \sin A \\ 2 \sin A & \cos A + i \sin A \end{bmatrix} = ?$

(a) sin 2A (b) cos 2A - i sin 2A (c) cos 2A + i sin 2A

(d) 1 (e) cos 2A

23. If sin A = $\frac{4}{5}$ for some angle A in quadrant I and cos B = $-\frac{12}{13}$ for some angle B in quadrant II, then the value of tan(A + B) is:

(a) $\frac{33}{56}$ (b) $-\frac{33}{56}$ (c) $\frac{63}{56}$ (d) $-\frac{63}{56}$ (e) $\frac{33}{16}$

24. If Jach can do a job in 104 hours and it takes Jach and Jank working together 40 hours to do the same job, how many hours will Jank take to do the job alone?

(a) 64 (b) 80 (c) 28 (d) 72 (e) 65

25. Which of the following does <u>not</u> represent a function:

(i) $\{(x,y) \mid y = 2x + 1\}$ (ii) $\{(x,y) \mid x^2 + y^2 = 10,\ y \geq 0\}$
(iii) $\{(3,1),(4,1),(5,2),(6,2),(7,3)\}$ (iv) $\{(x,y) \mid y = 2^x\}$
(v) $\{(x,y) \mid x^2 - y^2 = 2\}$

(a) iv (b) v (c) iii (d) ii (e) i

26. Simplify: $(3 + i)^3$

(a) 18 (b) 11 + 7i (c) 18 + 26i (d) 27 - i (e) 27

27. The solution of $|7x - 3| = -1$ is:

(a) {2/7,4/7} (b) {0} (c) {1} (d) {4/7} (e) $\varnothing$

28. How many committees consisting of 4 men and 2 women can be chosen from 10 men and 7 women?

(a) 4410 (b) $\frac{10!}{6!} \cdot \frac{7!}{5!}$ (c) 2205

(d) (10!) $\frac{10!}{6!} \cdot \frac{7!}{5!}$ (4!) (e) 560

29. Simplify: $-\sqrt{-121}$

(a) $-11i$ (b) $11i$ (c) 11 (d) -11 (e) $i\sqrt{121}$

30. The height h in feet above the ground at time t in seconds of an object launched vertically is given by:

$$h(t) = -16t^2 + 16t + 32$$

In how many seconds will it hit the ground?

(a) 3 (b) 1 (c) 4 (d) 2 (e) 1/2

31. A race car averages 30 mph for one lap of a one-mile oval track. In what time would it have to complete a second lap in order to average 60 mph for both laps?

(a) 1 minute (b) 30 seconds (c) 1.5 minutes

(d) zero minutes (e) .1 hours

32. In any triangle, the sum of the lengths of the 2 shorter sides must be greater than the length of the longest side. Find all possible values of x if a triangle has sides of lengths

$$\frac{1}{x+2}, \quad \frac{1}{x+1}, \quad \frac{1}{x}$$

(a) $0 < x < \sqrt{2}$ (b) $-2 - \sqrt{2} \le x < -2$ or $-1 < x < -2 + \sqrt{2}$

(c) $\sqrt{2} < x < \infty$ (d) $-2 < x < -\sqrt{2}$ or $-1 < x < 0$ or $\sqrt{2} < x < \infty$

(d) $0 \le x < \infty$

33. The standard deviation of 6, 24, 37, 49, and 64 is:

(a) 58 (b) 36 (c) 399.6 (d) 19.99 (e) 44.7

34. The measure of central tendency most affected by extreme values is:

(a) mean (b) median (c) mode (d) range (e) midrange

35. A pizza parlor offers 8 different toppings and 4 different cheeses. A deluxe pizza contains 5 different toppings and 2 different cheeses. How many different deluxe pizzas are possible?

(a) 10 (b) 48 (c) 336 (d) 80,640 (e) $(8^5)(4^2)$

36. A palindrome is a numeral within which the digits read the same from left to right or right to left. How many different palindromes exist for a three digit numeral if zero is not used?

(a) 144 (b) 81 (c) 99 (d) 18 (e) 49

37. Using the same conditions indicated in the previous problem, how many more palindromes will be possible with a four digit numeral?

(a) 9 (b) 99 (c) 10 (d) 5 (e) 0

38. After a genetics experiment, the number of pea plants having certain characteristics was tallied with the following results:

22 were tall; 25 had green peas; 39 had smooth peas;
9 were tall and had green peas; 17 were tall and had smooth peas;
20 had both green and smooth peas; 6 had all three characteristics;
4 had none of the characteristics

Find the total number of plants counted.

(a) 75 (b) 40 (c) 65 (d) 50 (e) 35

39. Simplify by rationalizing the denominator. Assume all radicands represent positive real numbers.

$$\frac{\sqrt{x} + \sqrt{x+1}}{\sqrt{x} - \sqrt{x+1}}$$

(a) $-2x - 2\sqrt{x(x+1)} - 1$ (b) $2x - 2\sqrt{x(x+1)} - 1$

(c) $-2x + 2\sqrt{x(x+1)} - 1$ (d) $2x + 2\sqrt{x(x+1)} + 1$

(e) $-2x - 2\sqrt{x(x+1)} + 1$

40. Simplify using positive exponents. Assume that the variable represents positive real numbers.

$$\frac{8y^{\frac{2}{3}}y^{-1}}{2^{-1}y^{\frac{3}{4}}y^{-\frac{1}{6}}}$$

(a) $\frac{61}{y^{\frac{11}{12}}}$ (b) $\frac{16}{y^{\frac{7}{12}}}$ (c) $\frac{16}{y^{\frac{11}{12}}}$

(d) $\frac{12}{y^{\frac{7}{15}}}$ (e) $\frac{4}{y^{\frac{11}{12}}}$