THIRTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 1994

Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes. The use of calculators is prohibited.

NOTE: 1995 Contest date: April4

Contributors to TMTA for Annual Mathematics Contest:

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1. Which of the following is not a solution of the absolute value inequality, $|3x + 5| \geq 4$.

a) -5 b) -2 c) 0

d) 4 e) 287

2. Write the equation whose graph is the line through the points (3,-4) and (-2,3).

a) $5x + 7y = 0$ b) $y = 5x - 7$ c) $y = x + 5$

d) $5x - 7y - 31 = 0$ e) $7x + 5y - 1 = 0$

3. Simplify $\frac{xy - xw}{-x}$.

a) -y - w b) $x^2y - x^2w$ c) w - y

d) $x^2(w-y)$ e) $(y - w)^{-1}$

4. Determine the zeros of the polynomial $P(x) = x^3 - 7x - 6$.

a) -1,1,6 b) 1,-1,-6 c) 0,0,0

d) -2,-1,3 e) -3,1,2

5. Find the coordinates of the vertex of the parabola $y = -2x^2 + 12x - 3$.

a) (3,15) b) (3,21) c) (3,12)

d) (-3,-15) e) (-3,-57)

6. Ginny leaves school at 1:00 pm in her car traveling north at a rate of 24 mph. At the same time, Penny leaves school traveling west on her bicycle at a rate of 10 mph. At 2:00 pm, what is the distance between Ginny and Penny ?

a) 26 miles b) $\sqrt{34}$ miles c) 14 miles

d) 34 miles e) $\sqrt{476}$ miles

7. Alice has \$2.60 in nickels and dimes. The number of dimes is two less than three times the number of nickels. How many of each coin does Alice have?

a) 8 nickels, 22 dimes b) 7 nickels, 19 dimes c) 12 nickels, 20 dimes

d) 2 nickels, 25 dimes e) 32 nickels, 10 dimes

8. The equation $\frac{x-3}{2} = \frac{1}{x-4}$ has two solutions. Their <u>product</u> is

a) 6 b) 7 c) 10

d) 12 e) 14

9. When the polynomial $x^4 - 3x^3 - 7x^2 + 7x + 2$ is divided by $x + 2$ the quotient is $x^3 + Bx^2 + cx + 1$. The value of $B + C$ is

a) -8 b) -2 c) 0

d) 2 e) 10

10. Find the solution to the system of equations:

$$\begin{aligned} 3x - 2y &= 4 \\ 5x + 3y &= 13 \\ x + y &= 4 \end{aligned}$$

a) (2,1) b) (1,2) c) $(\frac{12}{5}, \frac{8}{5})$

d) $(\frac{1}{2}, \frac{7}{2})$ e) there is no solution to the system

11. The graph of the equation $x^2 - 2x + y^2 - 4y + 5 = 0$ is

a) a circle b) a hyperbola c) two intersecting lines

d) a single point e) a parabola

12. Suppose $y = \frac{3}{(x^2+1)^2}$. Choose functions f(x) and g(x) so that $y = f(g(x))$.

a) $f(x) = \frac{3}{x}$ $g(x) = (x^2 + 1)^2$ b) $f(x) = 3$ $g(x) = \frac{1}{(x^2+1)^2}$

c) $f(x) = \frac{3}{x^2}$ $g(x) = x^2 + 1$ d) $f(x) = \frac{3}{x^2+1}$ $g(x) = \frac{1}{x^2}$

e) Both a) and c) are correct

13. Solve the equation $\log_{10} 10^x = x^2 - 6$.

a) $\pm\sqrt{6}$ b) 3, -2 c) 4, -4

d) 0, 6 e) 1, -5

14. Which of the following equations is <u>not</u> a function of x.

(i) $y = |x|$ (ii) $x + 2y = y$ (iii) $y = 1/x$ (iv) $x^2 + y^2 = 4$ (v) $x^2 + y = 4$

a) i, iv and v b) i, ii and iii c) iv and v

d) iv e) i

15. Solve for x: $\frac{1}{1 - \frac{1}{1-x}} = 2$

a) -1 b) 1 c) -1, 1

d) 0 e) 0, 1

16. Multiply the two complex numbers: $(1 + i\sqrt{2})(1 - i\sqrt{2})$

a) 1 - 2i b) 1 + 2i c) -3

d) 3 e) 5

17. Simplify the expression $\dfrac{x^3-1}{x+1} \bullet \dfrac{x^2+1}{x^2-1}$.

a) $\dfrac{(x^2+x+1)(x^2+1)}{(x+1)^2}$ b) $x^2 + x + 1$ c) $x^2 + 1$

d) $\dfrac{(x^2-x+1)(x^2+1)}{(x+1)^2}$ e) $x^2 - x + 1$

18. A pizza restaurant offers 8 different toppings and 4 different cheeses. A deluxe pizza contains your choice of 5 different toppings and your choice of 2 different cheeses. How many different deluxe pizzas are possible?

a) 10 b) 48 c) 336

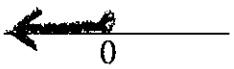
d) 80,640 e) $8^5 \cdot 4^2$

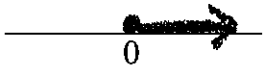
19. Solve the equation $2x^2 + x - 1 = 0$.

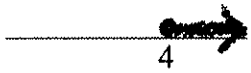
a) 1/2, -1 b) -1/2, 1 c) $\dfrac{-1 \pm i\sqrt{7}}{4}$

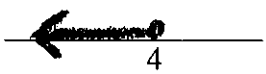
d) $\dfrac{1 \pm i\sqrt{7}}{4}$ e) $\dfrac{-1 \pm i\sqrt{7}}{2}$

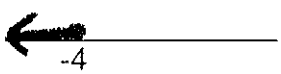
20. Determine the solution to the inequality $4 - 2x \leq -4$.

a) 

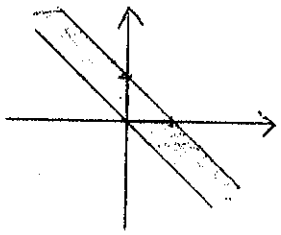
b) 

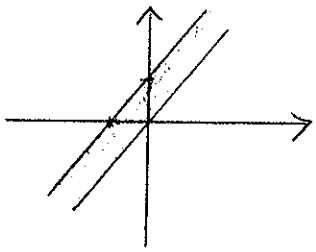
c) 

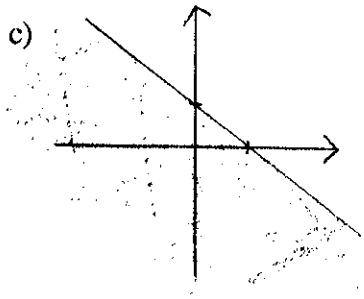
d) 

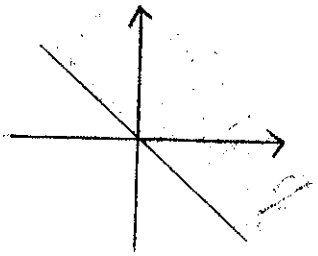
e) 

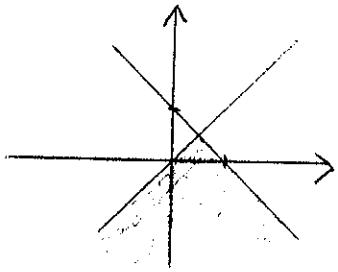
21. Determine the solution to the system of inequalities $x + y \leq 1$
$x + y \geq 0$

a) 

b) 

c) 

d) 

e) 

22. If the expression $(x^2 + 1)^3(x^4 + 3)(2x - 1)$ was multiplied out, the highest power on x would be

a) 10 b) 11 c) 13

d) 7 e) 8

23. Determine the relationship between the lines whose equations are $x + 2y = 3$ and $y = 3x - 2$.

a) parallel b) they are the same line

c) perpendicular d) intersect at the point (1,1), but are not perpendicular

e) intersect at the point (3,-2), but are not perpendicular

24. Which pair of points are the greatest distance apart ?

a) (0,0) and (6,8) b) (0,0) and (0,12) c) (1,1) and (10,1)

d) (-7,2) and (7,2) e) (0,0) and (3,4)

25. Evaluate the expression $4^{3/2} + 9^{1/2} + 1^{2/3}$.

a) 12 b) $11\frac{1}{2}$ c) 10

d) $13\frac{1}{2}$ e) $12\frac{2}{3}$

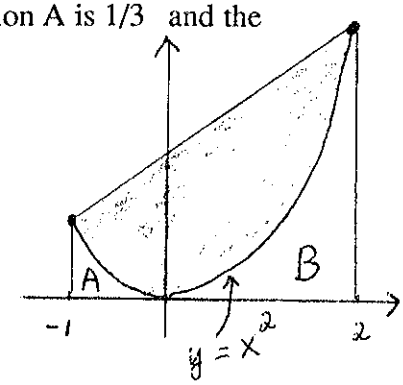
26. Which of the following are TRUE statements for all real numbers A and B ?

(i) $\sqrt{A}\,\sqrt{B} = \sqrt{AB}$ (ii) $\sqrt{A^2 + B^2} = A + B$ (iii) $\sqrt{A^2 - B^2} = A - B$ (iv) $\sqrt{A^2} = A$

a) all are true b) i and iv c) i

d) i, ii and iii e) i, ii and iv

27. The graph of $y = x^2$ from $x = -1$ to $x = 2$ is sketched below. The area of region A is 1/3 and the area of region B is 8/3. Determine the area of the shaded region.



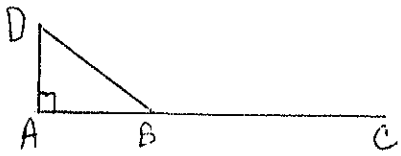
a) 3 b) 7/3 c) 15/2

d) 21/2 e) 9/2

28. In the diagram below, assume that the distance from A to C is 20, the distance from A to D is 4, and the distance from A to B is x. Determine the sum of the distances from D to B and B to C.

a) 24 b) $x + 4$ c) $\sqrt{16 + x^2} + 20 - x$

d) $\sqrt{16 - x^2} + 20 - x$ e) $\sqrt{16 - x^2} + 20$



29. If $\sqrt{x - 5} + 7 = x$, then $x =$ _____.

a) 6, 9 b) $\frac{1 + \sqrt{89}}{2}$ c) 6

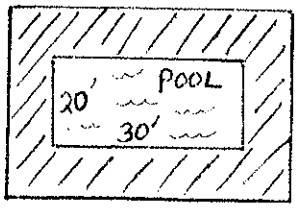
d) 9 e) 14

30. The solution of the inequality $\frac{1}{x} \leq \frac{x}{4x - 4}$ is

a) $(-\infty, 0) \cup (1, \infty)$ b) $(0, 1)$ c) $(-1, 0)$

d) $(-\infty, \infty)$ e) $(0, \infty)$

31. A pool that is 30 feet long and 20 feet wide is to have a concrete path of uniform width and 4 inches deep constructed around it (see drawing). Each cubic yard of concrete will pour 81 square feet of 4 inch deep concrete. The contractor has budgeted enough money to purchase 6 cubic yards of concrete. Which number is closest to the largest width possible for the path.



a) 3 feet b) 8 feet c) 5 feet

d) 6 feet e) 4 feet

32. If $\frac{3}{x-2}+\frac{3}{x+2}=\frac{5x-4}{x^2-2x}$, then $x =$ _____.

a) 4, 2 b) -4 c) 0, -1

d) 4 e) -4, 0

33. The domain of $f(x) = \sqrt{3x-2x^2}$ is _____.

a) All real numbers b) [0, 3/2] c) [0, 1]

d) (0, ∞) e) (- ∞, 0] ∪ [2, ∞)

34. If $f(x) = \frac{x}{x-2}$, then the inverse of $f(x)$ is $f^{-1}(x) =$ _____ .

a) $\frac{x-2}{x}$ b) $\frac{x}{x-2}$ c) $\frac{2x}{x-1}$

d) does not exist e) $\frac{x}{x-1}$

35. If $x + 3$ is a factor of $x^3 - 2kx + k^2$, then $k =$ _____.

a) 3 b) -3, 9 c) 3, -9

d) -3 e) -9

36. A polynomial with real coefficients and zeros 3, -1, and 1 + 2i is _____.

a) $x^4 - 2x^2 + 16x - 15$ b) $x^4 - 4x^3 + 6x^2 - 4x - 15$ c) $x^3 - 7x - 6$

d) $x^4 - 6x^2 - 8x - 3$ e) $x^4 - 2x^3 + x^2 - 8x - 12$

37. If $x = \log_4 (1/16)$, then $x =$ _____.

a) -1/4 b) -4 c) 1/2

d) 2 e) -2

38. The solution, (x, y, z), to the system $\begin{array}{l} x + y + z = 2 \\ 2x + 3y - z = 3 \\ 3x + 5y + z = 8 \end{array}$ is _____.

a) (3, 0, -1) b) (1, 2, -1) c) (3, -1, 0)

d) (3, -1, 4) e) (-1, 2, 1)

39. The solution to the system $\begin{array}{l} x^2 + y^2 = 25 \\ x - y = 1 \end{array}$ is _____

a) $(\pm 4, 3)$ b) (4, 3), (-3, -4) c) $(\pm 4, 3)$, $(\pm 3, -4)$

d) $(\pm 3, -4)$ e) (3, 4), (-4, -3)

40. The equation of the circle with center at (4, -1) and tangent to $3x - 4y = -9$ is _____.

a) $x^2 + y^2 - 8x + 2y = 25$ b) $x^2 + y^2 + 8x - 2y = 8$ c) $x^2 + y^2 + 8x - 2y = -9$

d) $x^2 + y^2 - 8x + 2y = 8$ e) $x^2 + y^2 - 8x + 2y = -9$