THIRTY-SEVENTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 1993

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Trevecca Nazarene College
Coordinated by Larry Buess

Scoring formula: 4R - W + 40

Edited by: Larry Bouldin, Roane State
Community College, Harriman, TN

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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NOTE: 1994 CONTEST DATE--APRIL 12

ALGEBRA II

1. Find the difference between two positive numbers whose sum is 100 and whose product is as large as possible.

   a. 25 b. 21 c. 20 d. 50 e. 0

2. What is the equation of the function through (-2,3) perpendicular to $f(x) = -\frac{3}{2}x - 8$?

   a. $2x - 3y = -13$ b. $2x - 3y = -7$ c. $3x - 2y = 5$

   d. $3x - 2y = -12$ e. $2x + 3y = -7$

3. What is the smallest value of A such that $|2x + 3| \leq A$ for all x in the interval (-1,5)?

   a. 1 b. -1 c. 5 d. 13 e. -13

4. Simplify the following. Assume all variables used as exponents are integers and that all other variables are non zero real numbers.

$$\frac{z^{-3q}\left(z^{q-5}\right)^{q}}{\left(z^{q+1}\right)^{-q}}$$

   a. z^{6-q} b. z^{2q^2-3q-6} c. z^{2q^2-8q-1} d. z^{2q^2-1}

   e. z^{2q^2-7q}

5. In any triangle, the sum of the lengths of the 2 shorter sides must be greater than the length of the longest side. Find all possible values of x if a triangle has sides of lengths

$$\frac{1}{x+2}, \quad \frac{1}{x+1}, \quad \text{and} \quad \frac{1}{x}.$$

   a. $0 < x < \sqrt{2}$

   b. $-2 - \sqrt{2} \leq x < -2$ or $-1 < x < -2 + \sqrt{2}$

   c. $\sqrt{2} < x < \infty$

   d. $-2 < x < -\sqrt{2}$ or $-1 < x < 0$ or $\sqrt{2} < x < \infty$

   e. $0 \leq x < \infty$

6. The contrapositive of "A graph which is not symmetric about the origin is not symmetric about both the x-axis and the y-axis" is

   a. A graph which is symmetric about the origin is symmetric about both the x-axis and the y-axis.

   b. A graph which is symmetric about the x-axis or is symmetric about the y-axis is symmetric about the origin.

   c. A graph which is symmetric about both the x-axis and the y-axis is symmetric about the origin.

   d. A graph which is not symmetric about both the x-axis and the y-axis is not symmetric about the origin.

   e. A graph which is symmetric about the y-axis and the origin is symmetric about the x-axis.

7. The sum of the coordinates of the center of the circle $x^2 - 4x + y^2 - 10y + 4 = 0$ is

   a. 3 b. -3 c. -7 d. $2\sqrt{3}$ e. 7

8. The coefficient of the 5th term of the expansion of $(2a - b)^7$ is

   a. $\frac{7!}{5!2!}$ b. $\frac{7!}{5!}2^5$ c. $5!\cdot 2^5\cdot 1^2$ d. $\frac{7!}{4!3!}2^3$ e. $\frac{7!}{3!4!}2^4$

9. Which of the following could not be a candidate for a rational zero of the equation $8x^3 - 25x^2 + 4x - 3$?

   a. $-\frac{4}{3}$ b. $\frac{3}{8}$ c. 3 d. $-\frac{1}{4}$ e. 1

10. The sum of the roots of $x^4 - 5x^2 + 4 = 0$ is

    a. 3 b. -2 c. 0 d. 5 e. -4

11. $8(5 - 3i)(-2 - 4i) =$

    a. $40 - 60i$ b. $-80 + 96i$ c. $-1408 - 896i$

    d. $-176 - 14i$ e. $-176 - 112i$

12. $(54b^6)^{\frac{2}{3}}$ in simplest radical form is

a. $9b^4\sqrt[3]{4}$ b. $36b^4$ c. $9b^6\sqrt[3]{b^2}$ d. $36b^6\sqrt[3]{b^2}$

e. $36b^4\sqrt{4}$

13. The combined ages of a man and his sister is 55 years. He is three times as old as she was when he was as old as she is now. What is the product of their ages?

a. 567.1875 b. 726 c. 484 d. 750 e. 574

14. The nth term of the sequence $1, -\frac{2}{3}, \frac{4}{9}, \ldots$ is

a. $\frac{3}{5}$ b. $(-\frac{4}{3})^{n-1}$ c. $\frac{1 - (-\frac{2}{3})^n}{1 + \frac{2}{3}}$ d. $(-\frac{2}{3})^{n-1}$

e. $(\frac{3}{2})^{n+1}$

15. A company has 80 computers which it rents. When the rent for each unit is \$250 per month, all computers are rented. However, for each \$10 increase in monthly rent per unit, one of the units is not rented. Each unrented unit costs the company \$15 per month for depreciation, and each rented unit costs the company company \$65 per month for insurance, repair, and deprecia tion. What rent should be charged for maximum profit?

a. \$550 b. \$250 c. \$600 d. \$1000 e. \$180

16. The standard deviation of 6, 24, 37, 49 and 64 is

a. 58 b. 36 c. 399.6 d. 19.99 e. 44.70

17. $\frac{3 - 2i}{3 + i} + \frac{4i}{3 - 7i} =$

a. $\frac{1}{2} - \frac{1}{3}i$ b. $\frac{63}{290} - \frac{201}{290}i$ c. $\frac{11}{130} + \frac{3}{130}i$ d. $-\frac{8}{13} + \frac{12}{13}i$

e. $-\frac{8}{13} - \frac{12}{130}i$

18. $i^{2007} =$

a. i b. -1 c. $-i$ d. $1 + i$ e. $1 - i$

19. The interval for which $\frac{(x + 1)(x - 4)}{x - 2} \leq 0$ is

a. $[-\infty, 2) \cup [4, \infty)$ b. $(-1, 2) \cup (4, \infty)$ c. $(-\infty, -1) \cup [2, 4]$

d. $[-1, 2] \cup [4, \infty)$ e. $(-\infty, -1] \cup (2, 4]$

20. Given $y = ax^2 + bx + 3$. Which of the following ordered pairs (a,b) could cause the function to have a maximum value at $x = 1$?

a. $(-1, 2)$ b. $(-2, 5)$ c. $(-1, -1)$ d. $(-1, -2)$

e. $(-2, 1)$

21. $f(x) = 2x^2 + 3$ and $g(x) = \sqrt{x + 2}$. $f \circ g =$

a. $2\sqrt{x + 2} + 3$ b. $2x + 7$ c. $2x + 5$ d. $\sqrt{2x^2 + 5}$

e. $(2x^3 + 3)(\sqrt{x + 2})$

22. The augmented matrix for the system $\begin{matrix} x + 6y = 7 \\ y = 11x = 10 \end{matrix}$ is

a. $\begin{bmatrix} 1 & 6 & 7 \\ -1 & 11 & 10 \end{bmatrix}$ b. $\begin{bmatrix} 1 & 6 & 7 \\ 1 & -11 & 10 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 6 & 7 \\ 11 & -1 & -10 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 6 \\ -11 & 1 \end{bmatrix}$ e. $\begin{bmatrix} 7 & 6 \\ 10 & -1 \end{bmatrix}$

23. The interval for which $x^2 \leq 2x + 2$ is

a. $[\, 2 - \sqrt{3},\ 2 + \sqrt{3}\,]$ b. $[\, 1 - \sqrt{3},\ 1 + \sqrt{3}\,]$

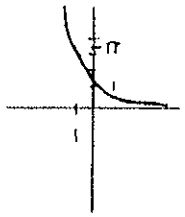
c. $[1 - 2\sqrt{3}, 1 + 2\sqrt{3}\,]$ d. $[\, 1 + \sqrt{3},\ \infty\,)$

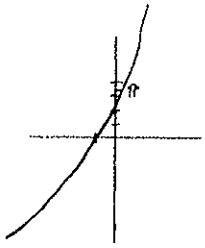
e. $(-\infty,\ 2 - \sqrt{3}\,]$

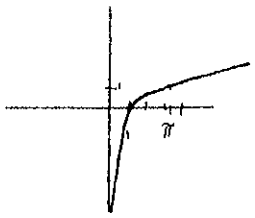
24. $(\sqrt[3]{x} + 3)^2 - 8\sqrt[3]{x} = 12$. The solution set is

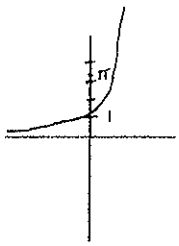
a. $\{-1\}$ b. $\{-3, -1\}$ c. $\{-1, 27\}$ d. $\{27\}$ e. $\{1, -3\}$

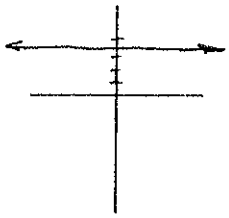
25. Sketch the graph of π^x

a. 

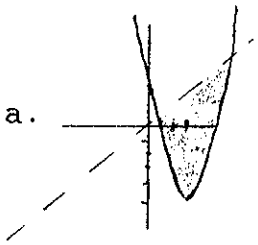
b. 

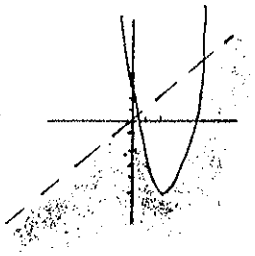
c. 

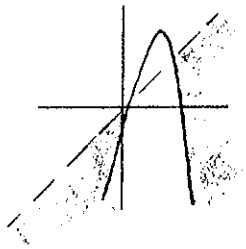
d. 

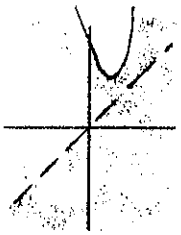
e. 

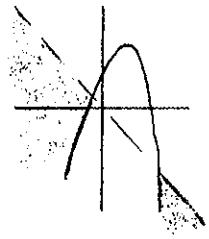
26. The graph of $y > x^2 - 6x + 4$ and $y < x$ looks like

a. 

b. 

c. 

d. 

e. 

27. $\dfrac{5 + i}{4 - 5i} =$

a. $\frac{5}{4} - \frac{1}{5}i$ b. $-\frac{15}{9}$ c. $\frac{15}{31}$ d. $\frac{55}{41}$ e. $\frac{15}{41} + \frac{29}{41}i$

28. The number of possible negative roots of $y = 2x^6 - x^4 + 3x + 13$ according to Descartes' rule of signs is

a. 0, 2, or 4 b. 1 or 3 c. 0 d. 1, 3, or 5 e. 0 or 2

29. $x + 2$ is a factor for which of the following polynomials?

a. $x^3 + 3x - 3$ b. $5x^5 - 8x^4 + 2x^3 + 7x^2 - 6x + 15$

c. $x^2 + 4$ d. $x^3 - 8$ e. $4x^4 + 13x^3 - 13x^2 - 40x + 12$

30. Suppose that y varies directly as x^2 and inversely as z. If $y = 32$ when $x = 8$ and $z = 10$, find y when $x = 12$ and $z = 15$.

a. 5 b. 4 c. 48 d. 64 e. 22

31. The remainder when $x^5 - 2x^4 + x^3 - 3x^2 + 8$ is divided by $x - 2$ is

a. 8 b. 4 c. $x + 2$ d. $x - 4$ e. 6

32. A restaurant developed a new kind of hamburger recently. They tried 9 different buns, 40 sauces, 3 types of lettuce, and 2 sizes of tomatoes. How many different kinds of hamburgers could they have tested?

a. 4920 b. 54 c. 360 d. 2160 e. 256

33. $|7x - 3| = -1$, the solution set is

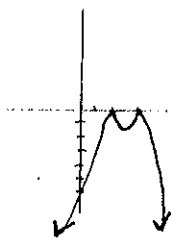
a. {2/7,4/7} b. {0} c. {1} d. {4/7} e. $\emptyset$

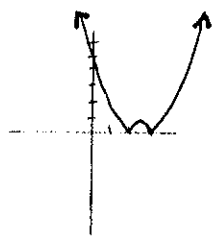
34. $9x^2 + 42x + 49 = 0$ has

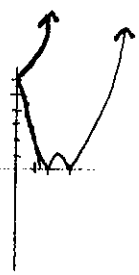
a. 2 unique real roots b. one root c. no real roots

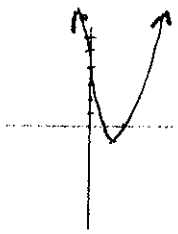
d. one complex root and one real root e. 2 complex roots

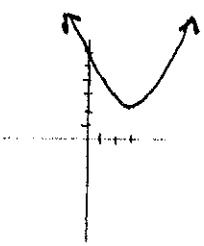
35. The graph of $y = |x^2 - 5x + 6|$ looks like

a. 

b. 

c. 

d. 

e. 

36. $\sqrt{10 - x} - 8 = 2x$, the solution set is

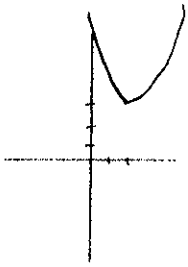
a. {-9/4} b. {-6} c. {-9/4,-6} d. Ø e. {2/3}

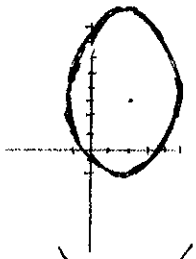
37. Sand is poured on the ground and forms a right circular cone whose height is one-third the diameter of the base. What is the radius of the base when the cone has a volume of 192 m^3?

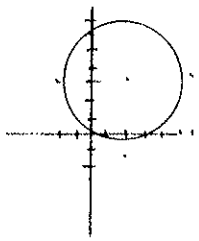
a. $\frac{6\sqrt{2\pi}}{\pi}$ m. b. $\frac{6\sqrt[3]{4\pi^2}}{\pi}$ m. c. $8\sqrt{3}$ m. d. $\frac{12\sqrt{2\pi}}{\pi^2}$ m.

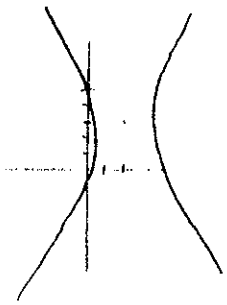
e. $\frac{3\sqrt[3]{4\pi^2}}{\pi}$ m.

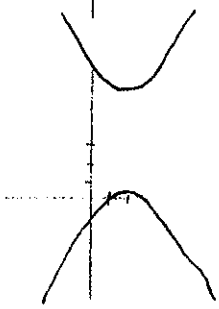
38. The graph of $25x^2 + 9y^2 - 100x - 54y = 44$ looks like

a. 

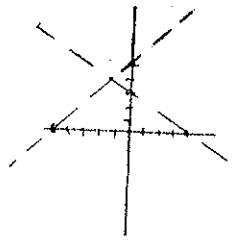
b. 

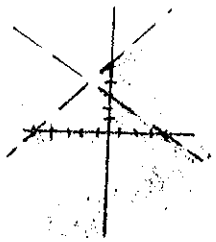
c. 

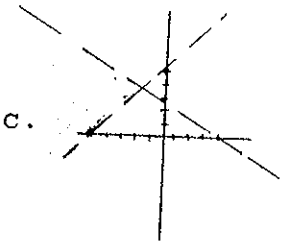
d. 

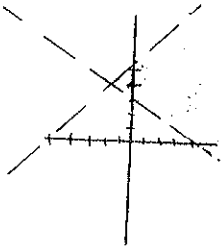
e. 

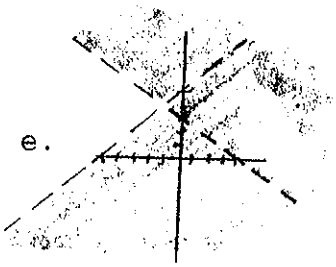
39. The graph of $x > -5 + y$ or $3x + 4y > 12$ looks like

a. 

b. 

c. 

d. 

e. 

40. An evening class has 20 students in it. Eight of them are single men and twelve are single women. Three of the men and eight of the women have full-time jobs. All of the others are full-time college students. If one is chosen at random, find the probability that the chosen student will be a woman who is a full-time student.

a. 1/5 b. 3/5 c. 3/11 d. 4/9 e. 9/20