THIRTY-SIXTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 1992

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Coordinated by Johnny Jackson

Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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NOTE: 1993 CONTEST DATE--APRIL 6

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1. Write the complex number $\frac{3 + 2i}{-2 + 5i}$ in standard form:

a) $\frac{4}{29} + \frac{19i}{29}$ b) $\frac{4}{21} - \frac{19i}{21}$ c) $\frac{-4}{21} + \frac{19i}{21}$

d) $\frac{4}{29} - \frac{11i}{29}$ e) $\frac{4}{29} - \frac{19i}{29}$

2. The equation for the line through (1/2, -1/3) with slope -2 is:

a) $3y + 6x - 2 = 0$ b) $y + 2x + 1 = 0$ c) $3y - 6x + 2 = 0$

d) $3y + 6x + 2 = 0$ e) $3y + 6x - 4 = 0$

3. If $f(x) = x^2 + 5$ and $g(x) = -3x + 4$ then $f[g(x)]$ equals:

a) $9x^2 - 24x + 21$ b) $-3x^2 - 11$ c) $9x^2 + 24x + 21$

d) $3x^2 + 19$ e) $9x^2 - 24x - 11$

4. The solutions of $|2x + 3| + 3 = 7$ are:

a) $x = -5$ or $x = 2$ b) $x = \frac{-13}{2}$ or $x = \frac{7}{2}$

c) $x = 5$ or $x = -2$ d) $x = \frac{-7}{2}$ or $x = \frac{1}{2}$

e) $x = \frac{13}{2}$ or $x = \frac{-7}{2}$

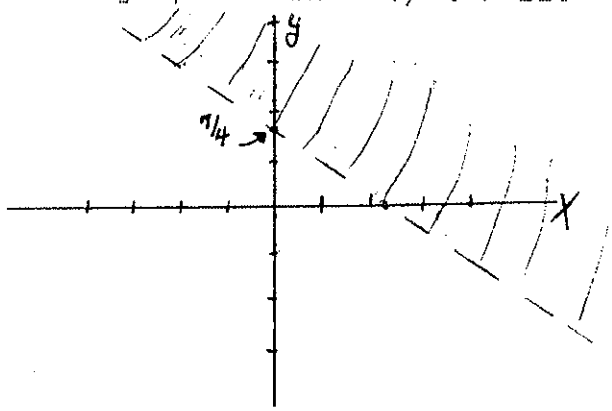
5. If $f(x) = x - x^2$ then $\frac{f(x + h) - f(x)}{h}$ for $h \neq 0$ is:

a) $1 + 2x + h$ b) $\frac{2x^2 + 2x(1 + h) - h^2}{h}$

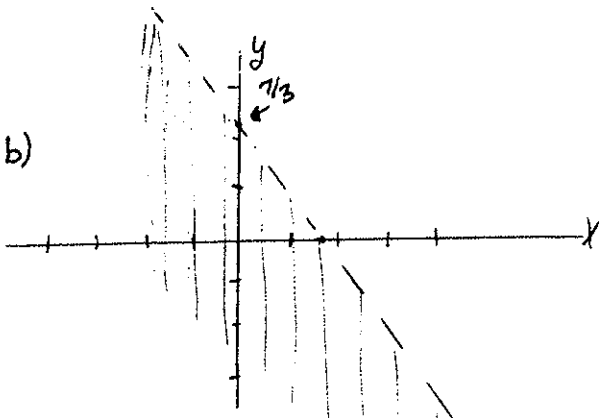
c) $1 - 2x + h$ d) $1 + 2x - h$ e) $1 - 2x - h$

6. The graph of $3x + 4y < 7$ is:

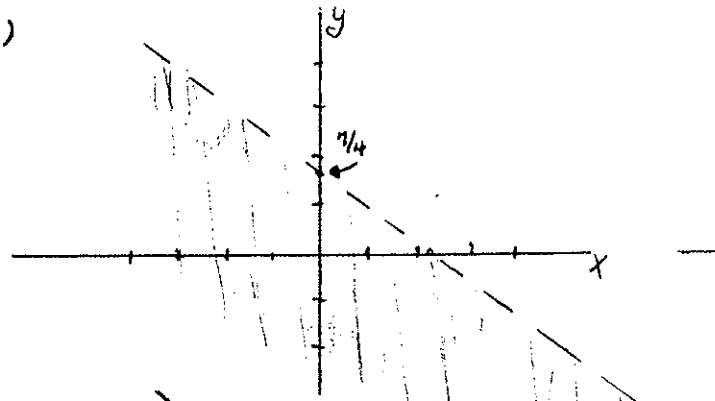
a)



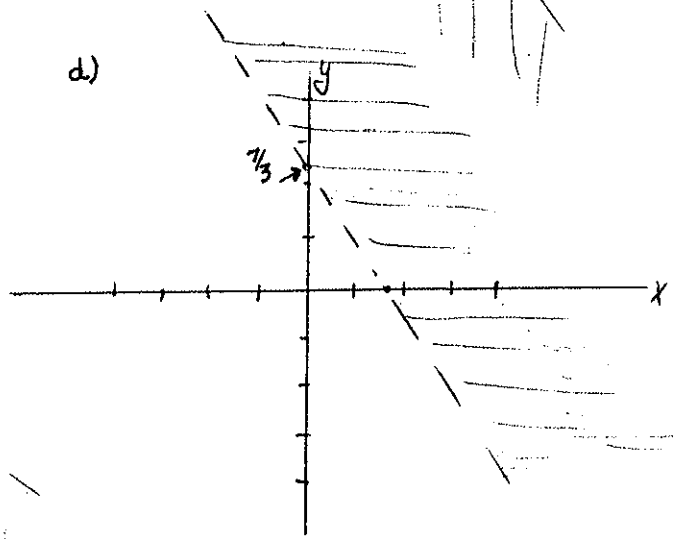
b)



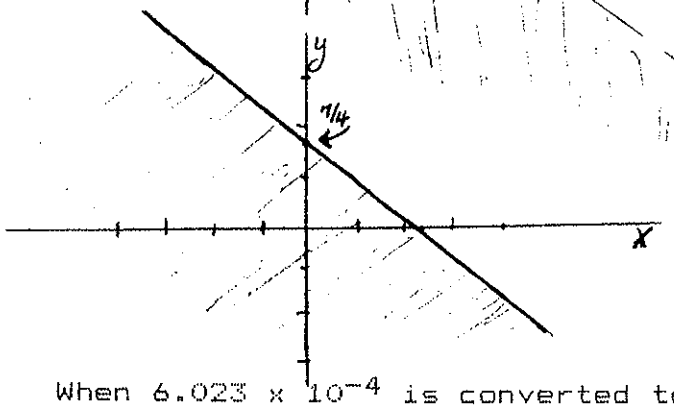
c)



d)



e)



7. When 6.023×10^{-4} is converted to decimal form it is:

a) 60230 b)6023 c) .006023 d) .0006023 e) .00006023

8. The solution to the equation $\sqrt{3x + 4} = 1 + \sqrt{x + 5}$ is:

a) $x = 1$ b) $x = -1$ c) $x = -1$ and $x = 4$ d) $x = 4$

e) $x = 1$ or $x = -4$

9. In simplest form $\frac{x^{-2}y^{2}z}{x^{-1}y^{-3}z^{5}}$ =

a) $\frac{y^5}{xz^2}$ b) $\frac{y^5}{x^3z^4}$ c) $\frac{y}{x^3z^4}$ d) $\frac{y}{xz^4}$ e) $\frac{y}{x^3z^2}$

10. Simplify $\frac{\left(a^{\frac{1}{3}}b^{\frac{3}{5}}c^{\frac{2}{3}}\right)^2}{\left(a^{\frac{2}{3}}b^{\frac{4}{5}}c\right)^3}$

a) $a^{\frac{4}{3}}b^{\frac{6}{5}}c^{\frac{5}{3}}$ b) $a^{-\frac{8}{3}}b^{-\frac{18}{5}}c^{-\frac{13}{3}}$ c) $a^{-\frac{4}{3}}b^{-\frac{6}{5}}c^{-\frac{5}{3}}$

d) $a^{-\frac{4}{3}}b^{-2}c^{-\frac{5}{3}}$ e) $a^{\frac{8}{3}}b^{\frac{18}{5}}c^{\frac{13}{3}}$

11. The 6th term of the recursively defined sequence with $a_1 = 11$ and $a_n = \frac{a_{n-1}}{n}$ is:

a) $\frac{11}{6}$ b) $11 \cdot 6$ c) $11 \cdot (6!)$ d) $\frac{1}{6!}$ e) $\frac{11}{6!}$

12. In interval notation the solution to $|2x + 3| \geq 5$ is:

a) $(-\infty, 4) \cup (1, \infty)$ b) $(-\infty, -4] \cup (1, \infty)$

c) $(-\infty, -4) \cup [1, \infty)$ d) $(-\infty, -4] \cup [1, \infty)$

e) $[-\infty, -4] \cup [1, \infty]$

13. The least root of $x^4 - 6x^3 + 3x^2 + 26x - 24 = 0$ is:

a) -1 b) $+1$ c) -3 d) -2 e) -4

14. The equation for the circle which has endpoints of the diameter at $(-3,-2)$ and $(5,4)$ is:

a) $x^2 + y^2 - 2x - 2y + 27 = 0$ b) $x^2 + y^2 - 2x - 2y - 23 = 0$

c) $x^2 + y^2 - 2x - 2y - 27 = 0$ d) $x^2 + y^2 + 2x + 2y + 23 = 0$

e) $x^2 + y^2 + 2x + 2y - 23 = 0$

15. If y varies jointly as x and the square of z and inversely as the cube of w, what is the resulting change in y when z is doubled and w is divided by 3 if x remains the same?
a) y is increased by a factor of 4/27
b) y is increased by a factor of 27/4
c) y is increased by a factor of 1/108
d) y is increased by a factor of 108
e) y is increased by a factor of 36

16. If $3^{x+2} = 2^{2x-1}$ then x is

a) $\dfrac{\log 18}{\log \frac{4}{3}}$ b) $\dfrac{\log 12}{\log \frac{4}{3}}$ c) $\dfrac{\log 27}{\log 2}$

d) $\dfrac{\log 3}{\log 2}$ e) $\dfrac{\log \frac{4}{3}}{\log 18}$

17. $\begin{vmatrix} \cos A - i \sin A & \sin A \\ 2 \sin A & \cos A + i \sin A \end{vmatrix} =$

a) sin 2A b) cos 2A − i sin 2A c) cos 2A + i sin 2A

d) 1 e) cos 2A

18. A man riding a bicycle at 10 mph leaves town A to ride to town B which is 55 miles away. How long after the cyclist leaves can a man driving a car at 35 mph leave town A to arrive in town B before the cyclist?

a) $< \frac{99}{14}$ hrs. b) $< \frac{14}{99}$ hrs. c) $< \frac{14}{55}$ hrs. d) $< \frac{55}{14}$ hrs.
e) $< \frac{55}{7}$ hrs.

19. The third term in the expansion of $(3x + 2y^3)^5$ is:

a) $720x^2y^9$ b) $360x^3y^6$ c) $1080x^3y^6$ d) $1080x^2y^9$
e) $720x^3y^6$

20. If $\sin A = \frac{4}{5}$ for some angle A in quadrant I, and $\cos B = -\frac{12}{13}$ for some angle B in quadrant II, then the value of tan(A + B) is:

a) $\frac{33}{56}$ b) $-\frac{33}{56}$ c) $\frac{63}{56}$ d) $-\frac{63}{56}$ e) $\frac{33}{16}$

21. The solution for $\log_2(5x + 6) - \log_2(2x - 2) = \log_2 3 - \log_2 4$ is:

A) $-\frac{15}{7}$ B) $\frac{15}{7}$ C) $\frac{15}{13}$ D) $\left|-\frac{15}{7}\right|$ E) ϕ

22. The factored form of $x^3 + x^2 - 2x - 8$ is:

A) $(x + 2)(x^2 + 3x + 4)$ B) $(x - 2)(x^2 + 3x + 4)$

C) $(X - 2)(x + 4)(x - 1)$ D) $(x - 2)(x^2 - 3x + 4)$

E) $x^2(x + 1) - 2(x + 4)$

23. The simplified form of

$$\frac{3x^2 - 14x - 5}{2x^2 - x - 3} \cdot \frac{3x^2 - 23x - 8}{2x^2 - 11x + 12} \cdot \frac{3x^2 - x - 4}{x^2 - 9x + 20}$$

is:

A) $\frac{(3x + 1)^2(x - 5)(3x - 4)}{(2x - 3)^2(x - 4)^2}$ B) $\frac{(2x + 3)(x + 4)(3x - 4)}{(2x - 3)(x - 8)(x - 4)}$

C) $\frac{3x - 4}{x - 8}$ D) $\frac{3x + 4}{x + 8}$ E) $\frac{(3x - 4)(x - 4)}{(x - 8)(x + 4)}$

24. The solution of $\frac{x}{x - 3} - \frac{2x^2 + 9}{2x^2 - 3x - 9} = \frac{1}{2x + 3}$ is:

A) 3 B) -3 C) ϕ D) -6 E) 6

25. If A can do a job in 104 hours and it takes A and B working together 40 hours to do the same job, how long will it take B working alone to do the same job?

A) 64 hrs B) 80 hrs C) 28 8/9 hrs D) 72 hrs
E) 65 hrs

26. The solution to $|x + 6| = |2x - 3|$ is:

A) ϕ B) $\{9\}$ C) $\{9, -1\}$ D) $\{3\}$ E) $\{-9, 1\}$

27. The standard form of $i^6 - i^{-10} - i^{15}$ is:

A) i^6 B) $0 + i$ C) $-i$ D) $-1 + i$ E) $1 - i$

28. For what values of K does the equation $Kx^2 + 5\sqrt{2}x - 3 = 0$ have two imaginary roots?

A) $K < -\frac{25}{6}$ B) $K < \frac{25}{6}$ C) $K \leq -\frac{25}{6}$ D) $K > \frac{25}{6}$

E) $K \geq \frac{25}{6}$

29. Which of the following expressions does NOT represent a function?
A) $\{(x,y) \mid y = 2x + 1\}$
B) $\{(x,y) \mid x^2 + y^2 = 10,\ y \geq 0\}$
C) $\{(3,1)(4,1)(5,2)(6,2)(7,3)\}$
D) $\{(x,y) \mid y = 2^x\}$
E) $\{(x,y) \mid x^2 - y^2 = 2\}$

A) D B) E C) C D) B and C E) C and E

30. If $f(x) = 7 - 3x$ then the inverse $f^{-1}(x) =$

A) $\frac{7 + x}{3}$ B) $\frac{-x - 7}{3}$ C) $7x - 3$ D) Does not exist

E) $\frac{7 - x}{3}$

31. Given $\log_a 2 = .301$, $\log_a 3 = .477$ and $\log_a 5 = .699$ then $\log_a \left(\frac{25}{8}\right) =$

A) $-.495$ B) $.386$ C) $.541$ D) $-.541$ E) $.495$

32. The center and vertices of the ellipse $4x^2 + 9y^2 - 16x - 54y + 61 = 0$ are:

A) $C(-2,3)\ V\ (-2 \pm 3, 3)$ B) $C(2,3)\ V\ (2 \pm 3, 3)$

C) $C(-2,3)\ V\ (-2 \pm 3, -3)$ D) $C(2,3)\ V\ (2 \pm 3, 0)$

E) $C(0,0)\ V\ (2 \pm 3, 3)$

33. How many committees consisting of 4 men and 2 women can be chosen from 10 men and 7 women?

A) 4410 B) $\frac{10!}{6!} \cdot \frac{7!}{5!}$ C) 2205 D) $(10!) \cdot (4!)$

E) 560

34. Simplify: $\dfrac{\dfrac{x+2}{x-2} + \dfrac{x-2}{x+2}}{\dfrac{x+2}{x-2} - \dfrac{x-2}{x+2}}$

A) 1 B) $\dfrac{2x^2+1}{x}$ C) $\dfrac{x^2+4}{4x}$ D) $\dfrac{2x^2+8}{8}$

E) $2x^2+1$

35. Five years ago, a woman was 4 times as old as her son. Six years from now, she will be 9 years more than twice her son's age. Find the woman's age and her son's age now.

A) Woman is 25 and son is 5
B) woman is 40 and son is 10
C) Woman is 36 and son is 9
D) Woman is 45 and son is 15
E) Woman is 20 and son is 5

36. A dealer mixed some coffee worth 80 cents a pound with coffee worth 95 cents a pound to make a mixture to be sold for 85 cents a pound. If the number of pounds of 80 cent coffee was 10 more than the number of pounds of the 95 cent coffee, how many pounds of each kind did he use?

A) **10** pounds of **80** cent and **20** pounds of **95** cent
B) **15** pounds of **80** cent and **25** pounds of **95** cent
C) **10** pounds of **95** cent and **20** pounds of **80** cent
D) **25** pounds of **80** cent and **15** pounds of **95** cent
E) **30** pounds of **80** cent and **20** pounds of **95** cent

37. If tan A = -1 and sin A is negative, then the measure of A is:

A) $\dfrac{\pi}{4}$ B) $\dfrac{3\pi}{4}$ C) $\dfrac{5\pi}{4}$ D) $\dfrac{9\pi}{4}$ E) $\dfrac{-\pi}{4}$

38. The graph of the equation $4x^2 - 3y^2 + 8x + 12y = 32$ is:

A) a line B) a parabola C) a hyperbola D) a circle
E) an ellipse

39. The graphs of the equations $x^2 + y^2 = 4$ and $x^2 + 4y^2 = 16$ intersect in:

A) No points B) exactly one point C) exactly two points
D) exactly three points E) exactly four points

40. A ball is dropped from a height of 72 inches. The ball rebounds 1/2 of the distance it falls on each bounce. How far has the ball travelled vertically when it ceases bouncing?

A) 144 inches B) 288 inches C) 216 inches
D) 72 inches E) 96 inches