THIRTY-FIFTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 1991

Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

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ALGEBRA II

1. If $x<-2$ then $|-x-2|$ is:

   a) $x+2$ b) $x-2$ c) $-x+2$ d) $-x-2$ e) 0

2. If $f(x) = x^2 + x$ and $g(x) = x-1$ then $g(f(3))$ is:

   a) 6 b) 11 c) 12 d) 24 e) 14

3. When simplified $\log_b 8 \div \log_b \frac{1}{8}$ becomes:

   a) 1 b) 0 c) $\log_b 64$ d) -1 e) $\log_b \frac{63}{8}$

4. If $\tan \theta = 1$ and $\sec \theta = 2$, then $\sin^2 \theta - \cos^2 \theta =$

   a) 1 b) 1/2 c) 0 d) $2/\sqrt{2}$ e) $2\sqrt{2}$

5. $\frac{a^2}{r+t} = m$, and if $m \neq o$ then $r =$

   a) $\frac{m}{a^2} - t$ b) $\frac{a^2-mt}{m}$ c) $\frac{a^2-t}{m}$ d) $\frac{a^2+mt}{m}$ e) a^2m-t

6. If $x^2+y^2=25$ and $x^2-y^2=7$ then x^4-y^4 is:

   a) 7 b) 14 c) 49 d) 175 e) 337

7. Which of the following numbers is a root of $(x^2-2x)^2 + 2(x^2-2x)=3$:

   a) $1+2i$ b) 2 c) $1+\sqrt{2}$ d) $1-2i$ e) -2

8. One of the values of k for which $k\begin{vmatrix} 2 & -2 & 0 \\ k & 1 & -2 \\ 0 & -3 & k \end{vmatrix} = -8$ is:

a) 2 b) 1 c) $\frac{1}{2}$ d) 0 e) -2

9. $\frac{4i}{2-3/i}$ in the form a+bi is:

a) $\frac{-12+8i}{13}$ b) $\frac{12i-8}{13}$ c) $\frac{12+8i}{13}$ d) $\frac{12-8i}{13}$ e) $\frac{-12-8i}{13}$

10. The solution set written as intervals that satisfies both $|x-3| \leq 6$ and $|x+2| < 5$ is:

a) (-7,-3) b) (-3,9] c) (3,9] d) (-7,9] e) [-3,3)

11. Find the fourth term of the recursively defined sequence with $a_1 = 7$ and $a_{k+1} = 3k + a_k$, for any positive integer k.

a) 37 b) 16 c) 51 d) 25 e) 57

12. In an arithmetic sequence, if the eighth term is 10 and the fifteenth term is 23, find the third term.

a) -3 b) $\frac{13}{7}$ c) $\frac{5}{7}$ d) $\frac{75}{14}$ e) $\frac{-34}{7}$

13. If $f(x) = 3x + 5$, then the inverse, $f^{-1}(x) =$

a) $-3x - 5$ b) $\frac{x - 5}{3}$ c) $\frac{1}{3x + 5}$ d) $\frac{x}{3} + \frac{1}{5}$ e) $\frac{x}{3} - \frac{1}{5}$

14. The minimum value of x^2-3x-4 is:

a) -4 b) $-1\frac{3}{4}$ c) $-6\frac{1}{4}$ d) $\frac{3}{2}$ e) -13

15. Solve the inequality and give the solution in interval notation:

$$\frac{5}{2x+1} \leq \frac{3}{x-2}$$

a) $(-\infty, -13] \cup \left(-\frac{1}{2}, 2\right)$ b) $[-13, -\frac{1}{2}] \cup [2, \infty)$

c) $(-\infty, -13] \cup \left[-\frac{1}{2}, 2\right]$ d) $[-13, -\frac{1}{2}) \cup (2, \infty)$

e) $(-\infty, -13) \cup \left[-\frac{1}{2}, 2\right]$

16. If $9^{x+2} = 240+9^x$, then $x =$

a) 0.1 b) 0.2 c) 0.3 d) 0.4 e) 0.5

17. If $x+y=1$, then the largest value of xy is:

a) 1 b) 0.5 c) $\sqrt{2} - 1$ d) 0.25 e) 0

18. $\dfrac{a^{-1}b^{-1}}{a^{-3}-b^{-3}} =$

a) $\dfrac{a^2b^2}{b^2-a^2}$ b) $\dfrac{a^2b^2}{b^3-a^3}$ c) $\dfrac{ab}{b^3-a^3}$

d) $\dfrac{a^3-b^3}{ab}$ e) $\dfrac{a^3-b^3}{a-b}$

19. The sum of two numbers is 10; their product is 20. The sum of their reciprocals is:

a) $\frac{1}{10}$ b) $\frac{1}{2}$ c) 1 d) 2 e) 4

20. If $xy = b$, and $\frac{1}{x^2} + \frac{1}{y^2} = a$, then $(x + y)^2 =$

a) $(a+2b)^2$ b) a^2+b^2 c) $b(ab+2)$ d) $ab(b+2)$ e) $\frac{1}{a} + 2b$

21. The arithmetic mean of the first n positive integers is:

a) $\frac{n}{2}$ b) $\frac{n^2}{2}$ c) n d) $\frac{n-1}{2}$ e) $\frac{n+1}{2}$

22. The number 2.52525... can be written as a fraction. When this fraction is reduced to lowest terms, the sum of the numerator and denominator is:

a) 7 b) 29 c) 141 d) 257 e) 349

23. The coefficient of c^2 in the expansion of $(\sqrt{c} + \sqrt{d})^8$ is

a) 28 b) 56 c) 64 d) 70 e) 256

24. A circle has center C(-3,1) and passes through the point P(7,5). The radius of the circle is:

a) 10 b) 14 c) $\sqrt{58}$ d) $2\sqrt{26}$ e) $2\sqrt{29}$

25. The line ℓ contains the point $\left(-\frac{1}{2}, -\frac{2}{3}\right)$ and is parallel to the line with equation $-2x + \frac{y}{5} = 1$. The y- intercept of ℓ is:

a) $\frac{13}{15}$ b) $\frac{13}{3}$ c) 5 d) $\frac{14}{13}$ e) $-\frac{17}{3}$

26. The number of real solutions of the equation $\sqrt{3x-2} - \sqrt{2x-3} = 1$ is:

a) 0 b) 1 c) 2 d) 3 e) 4

27. The solution set for the equation $2\log_5(x+3) - \log_5(x+1) = 2\log_5 3$ is:

a) empty b) {0,3} c) {1,3} d) {0} e) {3}

28. If a, b, and c are numbers for which $\frac{x^2+10x-36}{x(x-3)^2} = \frac{a}{x} + \frac{b}{x-3} + \frac{c}{(x-3)^2}$ then $a + b + c =$

a) 2 b) 3 c) 10 d) -36 e) 8

29. Which of the following cannot be a rational root of the equation $2x^4 - 16x^3 + 36x^2 - 8x + 17 = 0$?

a) 17 b) $\frac{17}{2}$ c) 2 d) $-\frac{17}{2}$ e) 1

30. The graphs of the equations $x^2 + y^2 = 25$ and $x^2 = 25$ intersect in:

a) no points b) exactly one point c) exactly two points

d) exactly three points e) exactly four points

31. The letters of the acronym TMTA can be arranged in how many distinct ways?

a) 4^4 b) 4! c) $\frac{4!}{2!2!}$ d) $\frac{4!}{2!}$ e) 2!2!

32. The slope of the perpendicular bisector of the line segment joining the points (3,2) and (6,-2) is:

a) $-\frac{4}{3}$ b) $\frac{3}{4}$ c) $-\frac{3}{4}$ d) $\frac{4}{3}$ e) $\frac{9}{2}$

33. If x-3 is a factor of $x^2 + px + 12$, what is the value of p?

a) -7 b) -1 c) 1 d) 7 e) 9

34. If $x^4 - y^4 = 45$ and $x^2 - y^2 = 5$, then $x^2 + y^2 =$

a) 15 b) 9 c) 10 d) 40 e) 50

35. Assuming that girl-boy births are equiprobable, find the probability that a family with 9 children has at least one girl.

a) $\frac{511}{512}$ b) $\frac{1}{2}$ c) $\frac{255}{256}$ d) $\frac{127}{128}$ e) $\frac{1}{512}$

36. An airplane, flying with a tail wind, travels 1200 miles in 5 hours; the return trip, against the wind, takes 6 hours. Find the cruising speed of the plane and the speed of the wind (assume that both are constant).

a) 220 mph, 20 mph b) 220 mph, 40 mph c) 230 mph, 20 mph

d) 230 mph, 40 mph e) 240 mph, 20 mph

37. Find a polynomial with real coefficients of degree 4 that has zeros 2, -2, and $-2 \pm 3i$.

a) $x^4 - 4x^3 + 17x^2 - 16x + 52$ b) $x^4 + 4x^3 + 9x^2 - 16x - 52$

c) $x^4 - 4x^3 + 9x^2 + 16x - 52$ d) $x^4 + 4x^3 + 17x^2 + 16x + 52$

e) $x^4 - 4x^3 - 17x^2 - 16x - 52$

38. Find the domain of $f(x) = \frac{\sqrt{4x-1}}{x^2 - 10x + 24}$. Write your answer in interval notation.

a) $\left[\frac{1}{4}, 4\right) \cup (4, 6) \cup (6, \infty)$ b) $\left[\frac{1}{4}, \infty\right)$

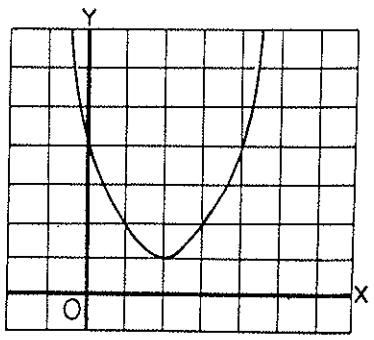
c) $(-\infty, 4) \cup (4, 6) \cup (6, \infty)$ d) $(-\infty, \frac{1}{4}] \cup (4, 6) \cup (6, \infty)$

e) $(-\infty, \frac{-1}{4}] \cup [6, \infty)$

39. The sum of the coordinates of the midpoint of the line segment joining the points P(1,5) and $Q\left(\frac{4}{5}, \frac{-8}{3}\right)$ is:

a) $\frac{-28}{15}$ b) $\frac{4}{15}$ c) $\frac{29}{15}$ d) $\frac{31}{15}$ e) $\frac{61}{30}$

40.



If the figure above shows the graph of the equation $y = ax^2 + bx + c$, then which of the following can be concluded about the roots of this equation for $y = 0$?

I. They are positive.
II. They are equal.
III. They are real.
IV. They are imaginary.

a) III only b) IV only c) I and III only d) II and IV only

e) I, II, and III