THIRTY-FOURTH ANNUAL MATHEMATICS CONTEST
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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II   1990

Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the <u>best</u> answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

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1. A is inversely proportional to D and directly proportional to K. If A = 8 when D = 2 and K = 2, find A when D = 3 and K = 6.

   a. 24   b. 16/3   c. 2   d. 4   e. 16

2. Simplify the expression below to an equivalent one without negative exponents:

$$\frac{x^{-3}y^{-5}z^2}{x^{-5}y^2z^{-3}}$$

   a. $x^2/(y^3z)$

   b. x^2z^5/y^7

   c. $y^3/(x^2z)$

   d. $z/(x^8y^3)$

   e. $1/(x^{-8}y^3z)$

3. The sum of two numbers is 6 and their product is -24. Find a quadratic polynomial with the two numbers as roots.

   a. $6x^2 - 24$

   b. $x^2 - 24x + 6$

   c. $x^2 - 6x - 24$

   d. $24x^2 + 6x + 1$

   e. $(x - 6)(x + 24)$

4. Determine the nature of the solutions of the equation without solving: $7y^2 + 2 = 2\sqrt{14}y$.

   a. Two unequal rational solutions

   b. Two unequal real solutions

   c. One real solution

   d. Two conjugate imaginary solutions

   e. No solutions

5. Which pair of equations represents two perpendicular lines?

   a. $y = -x + 2$; $y + x = 5$

   b. $x + 3y = 3$; $x - 3y = 1$

   c. $2x + y = 6$; $4x + 2y = 5$

   d. $x + 2y = 1$; $2x + y = 3$

   e. $3y = x + 9$; $y = -3x + 1$

6. $\dfrac{2x^2 + x - 1}{2x^2 + 3x - 2} \div \dfrac{x^2 - 2x + 1}{x^2 + x - 2} = $?

 a. $(x + 1)^2/(x + 2)^2$

 b. $(x + 1)/(x - 1)$

 c. $(x + 2)^2/(x + 1)^2$

 d. $(x - 1)/(x + 1)$

 e. $(x + 1)^2$

7. How many ways can four people be arranged around a card table if the arrangements $\begin{matrix} N \\ W\;E, \\ S \end{matrix}$ $\begin{matrix} W \\ S\;N, \\ E \end{matrix}$ $\begin{matrix} S \\ E\;W, \\ N \end{matrix}$ and $\begin{matrix} E \\ N\;S \\ W \end{matrix}$ are all considered to be the same?

 a. 8   b. 24   c. 4   d. 6   e. 1

8. Evaluate $81^{3/4}$.

 a. $\sqrt[3]{81^4}$ b. 108   c. 60.75   d. 27   e. $1/81^{4/3}$

9. Which polynomial has a factor of $(x - 1)$?

 a. $x^5 + x^4 + x^3 + x^2 + x + 1$

 b. $3x^{50} - 2x^{25} + i^{100}$

 c. $x^4 - x^3 + 2x^2 - 2x - 4$

 d. $x^3 - 11x + 20$

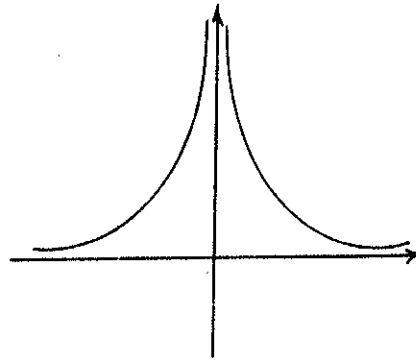 e. $3x^{50} - 2x^{25} + i^{150}$

10. Simplify: $-\sqrt{-121}$
 a. $-11i$ b. $11i$ c. 11 d. -11 e. $i\sqrt{121}$

11. What is the solution to the system: $y = (7/2)x + 6$
 $4(x + 3y) = 2x + 4y$?

 a. $(0,0)$ c. $(-57/30, 399/60)$ e. $(13/10, 211/20)$

 b. $(2,-3)$ d. $(-8/5, 2/5)$

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12. What equation best depicts the following graph:

    a. $f(x) = 2^{-x}$

    b. $f(x) = 2^{x/2}$

    c. $f(x) = 2^{1/x^2}$

    d. $f(x) = (2/3)^{-x}$

    e. $f(x) = 1/2^x$

13. Simplify $(3 + i)^3$.

    a. 18

    b. $18 + 26i$

    c. $11 + 7i$

    d. $27 - i$

    e. $27 + 28i + 9i^2$

14. Solve the inequality: $5 - 2|1.5x + 3| \geq 1$.

    a. $2/3 \leq x \leq 10/3$

    b. $x \leq -2/3$

    c. $x \geq -10/3$

    d. $x \leq -10/3$ or $-2/3 \leq x$

    e. $-10/3 \leq x \leq -2/3$

15. If $\sqrt{z} + 2\sqrt{2z + 1} - \sqrt{3z + 4} = 0$, what is the solution set for z?

    a. $\{2\sqrt{7}/7\}$ b. $\{4\}$ c. $\{0\}$ d. $\{4, 0\}$ e. $\emptyset$

16. Given $3x^5 + 5x^3 + x - 2 = 0$, which possibility for solutions is correct?

    a. 3 positive roots and 2 negative roots

    b. 1 positive and 2 pairs of conjugate imaginary roots

    c. 1 positive, 2 negative, and 1 pair of conjugate imaginary roots

    d. 2 positive, 2 negative, and 1 imaginary root

    e. no positive roots

17. Find the solution set for $4x^{-4} + 9 = 13x^{-2}$.

    a. $\{\pm i, \pm 2/3\}$

    b. $\{\pm 1, \pm 9/4\}$

    c. $\{\pm\sqrt{2}, \pm 2\sqrt{2/3}\}$

    d. $\{\pm 1/2, \pm 9/8\}$

    e. $\{\pm 1, \pm 2/3\}$

Algebra II

18. Which answer below represents the solution to the system?

$$3x + y - z = 11$$
$$x + 3y - z = 13$$
$$x + y - 3z = 11$$

a. $$\dfrac{\begin{vmatrix} 11 & 1 & -1 \\ 13 & 3 & -1 \\ 11 & 1 & -3 \end{vmatrix}}{-20} = x, \quad \dfrac{\begin{vmatrix} 3 & 11 & -1 \\ 1 & 13 & -1 \\ 1 & 11 & -3 \end{vmatrix}}{-20} = y, \quad \dfrac{\begin{vmatrix} 3 & 1 & 11 \\ 1 & 3 & 13 \\ 1 & 1 & 11 \end{vmatrix}}{-20} = z$$

b. $$\begin{vmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ 1 & 1 & 3 \end{vmatrix} = x, \quad \begin{vmatrix} 1 & -1 & 3 \\ 3 & -1 & 1 \\ 1 & 3 & 1 \end{vmatrix} = y, \quad \begin{vmatrix} -1 & 3 & 1 \\ -1 & 1 & 3 \\ -3 & 1 & 1 \end{vmatrix} = z$$

c. $$\begin{vmatrix} 11 & 1 & -1 \\ 13 & 3 & -1 \\ 11 & 1 & 3 \end{vmatrix} = x, \quad \begin{vmatrix} 3 & 11 & -1 \\ 1 & 13 & -1 \\ 1 & 11 & 3 \end{vmatrix} = y, \quad \begin{vmatrix} 3 & 1 & 11 \\ 1 & 3 & 13 \\ 1 & 1 & 11 \end{vmatrix} = z$$

d. $$\begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & -3 \end{vmatrix} = x, \quad \begin{vmatrix} 1 & 1 & 3 \\ 3 & 1 & 1 \\ -1 & -3 & -1 \end{vmatrix} = y, \quad \begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 3 \\ -3 & 1 & -1 \end{vmatrix} = z$$

e. $$\dfrac{\begin{vmatrix} 3 & 1 & 1 \\ 1 & 3 & 1 \\ -1 & -1 & -3 \end{vmatrix}}{-20} = x, \quad \dfrac{\begin{vmatrix} 1 & 1 & 3 \\ 3 & 1 & 1 \\ -1 & -3 & 1 \end{vmatrix}}{-20} = y, \quad \dfrac{\begin{vmatrix} 1 & 3 & 1 \\ 1 & 1 & 3 \\ 3 & 1 & -1 \end{vmatrix}}{-20} = z$$

19. Find x when $\dfrac{2x + 1}{x + 2} - \dfrac{x + 1}{x + 3} = \dfrac{-3}{x^2 + 5x + 6}$

a. -3 b. 2 c. -2 d. -5 e. $\emptyset$

20. Suppose a pair of dice are thrown together two times in succession. What is the probability of at least one of the dice showing a 3-spot on the first throw and the sum of both dice being 7 on the second throw?

a. $17/36$ b. $8/36$ c. $12/36^2$ d. $66/36^2$ e. $42/36^2$

21. Find A and B if $3x^2 - 5x - 6 = 3(x + A)^2 + B$.

a. $-5/6 = A, -97/12 = B$ d. $-5/3 = A, 97/12 = B$

b. $5/6 = A, -97/36 = B$ e. $-5/3 = A, 79/9 = B$

c. $5/3 = A, 241/36 = B$

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22. What is the graph of $y = \dfrac{2(x^2 - 9)}{4 - x^2}$?

a.

d.

b.

e.

c.
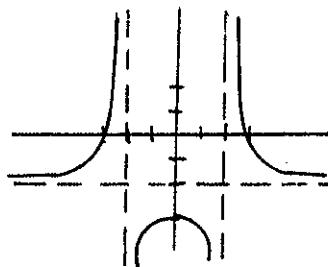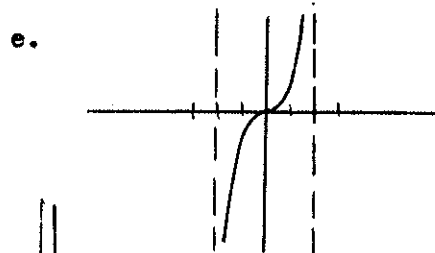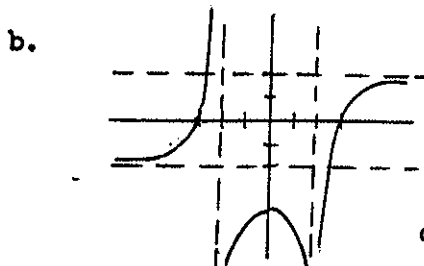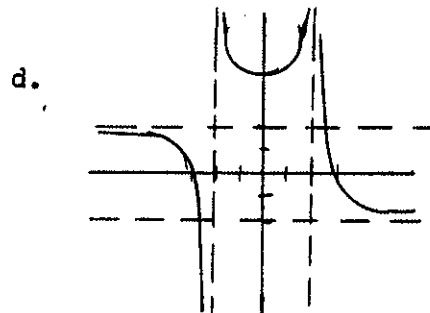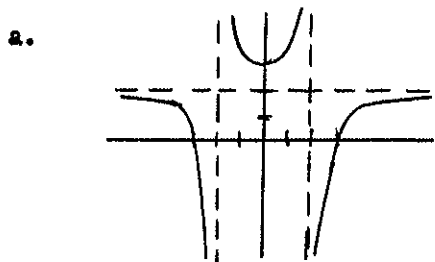
23. A child needs special instruction if he makes above 130 or below 70 on the Wechsler IQ test. What percentage of children <u>do not</u> require special instruction?

0.13%   2.14% 13.59% 34.13% 34.13% 13.59% 2.14%   0.13%

50   70   85   100   115   130   145
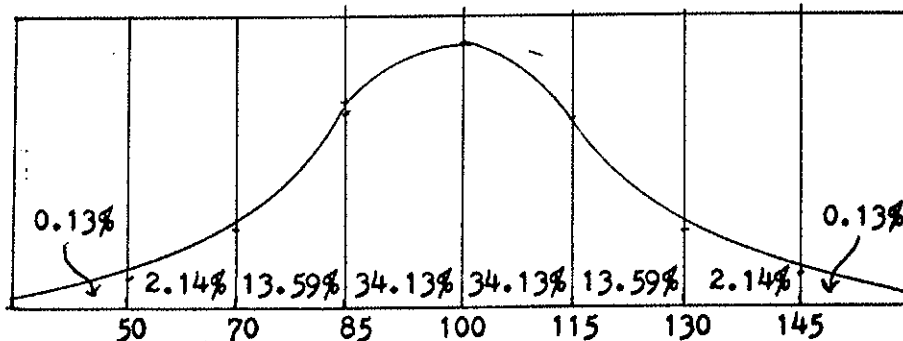
a. 34.13%   b. 95.46%   c. 4.54%   d. 2.27%   e. 47.72%

24. Simplify: $\dfrac{3 - 7i}{4 + 5i}$

a. 47/9 + 13/40i   c. −58/23 + 58/43i   e. −23/58 + 43/58i

b. 23/9 + 43/40i   d. −23/41 − 43/41i

25. What is the coefficient on the $x^4 y^5$ term in the expansion of $(2x - y)^9$?

a. −16   b. 126   c. −126(16)   d. −126   e. 16

26. Some roots of a polynomial equation are 2, −1, and $2 - 3i\sqrt{5}$. All of the coefficients are integers. Another root must be

    a. −2

    b. $2 - 3i\sqrt{5}$

    c. $3\sqrt{5} + 2i$

    d. 1

    e. $2 + 3i\sqrt{5}$

27. The inequality $4x^2 - 9y^2 > 36$ goes with which graph below?

    a.

    b.

    c.

    d.

    e.

28. Which of the following systems is both dependent and consistent (that is, has an infinite number of solutions)?

    a. $x - y = 21$
       $3y - 3x = 7$

    b. $x + 3y = 10$
       $x + 9y = 22$

    c. $2x - y = 1$
       $3x + y = 11$

    d. $2x - 4y = 17$
       $-6x + 12y = -51$

    e. $4x - y = 10$
       $2x + 3y = -2$

29. A class of 12 consists of 7 guys and 5 gals. Three students are to be chosen as class officers. If the selection of any one student is just as likely as the selection of any other student, what is the probability that at least 2 of the officers will be guys?

    a. 28/55   b. 5/9   c. 38/65   d. 12/27   e. 7/11

30. Which relation below is not a function?

    a. $y = 3 - x$ c. $y^2 = x - 2$ e. $y^3 = x$

    b. $y = \sqrt{2x} + 3$ d. $y = |x - 2|$

31. A certain fifth degree polynomial has leading term $2x^5$ and constant term 3. Which of the numbers given below could never be a root of the polynomial?

    a. ± 1 b. ± 3 c. $\pm 1/2$ d. $\pm 2/3$ e. $\pm 3/2$

32. $(5 + 4i)(3 + 7i) = ?$

    a. $13 - 47i$ c. $8 + 11i$ e. $8 - 11i$

    b. $-13 + 47i$ d. $-8 - 11i$

33. If $f(x) = 5x - 3$ and $g(x) = 2x + 7$, then $f \circ g(x) = ?$

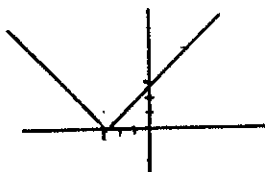   a. $10x + 32$ c. $10x + 1$ e. $3x - 11$
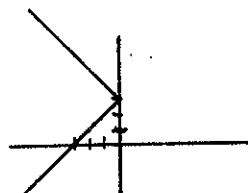
    b. $2(5x - 3) + 7$ d. $7x + 4$
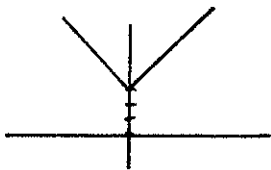
34. Which is the graph of $y = |3 - x|$?

    a.

    d.

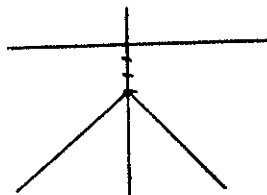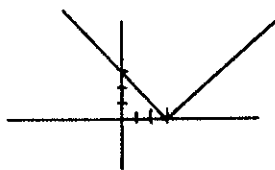
    b.

    e.

    c.

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35. Suppose the function $f(x) = 2x^3 + 4x^2 - 3x + 5$ is divided by $x + 2$. Then the remainder is

    a. 31   b. -11   c. $f(2)$ d. -31   e. $f(-2)$

36. Simplify: $\dfrac{15x^{1/3}y^3z^{1/2}}{3x^{1/2}y^2z^{3/2}} \cdot (4xy^{5/2}z^4)$

    a. $5x^5y^7z^3$ c. $20x^{1/3}yz^3$ e. $20x^{5/6}y^{7/2}z^3$
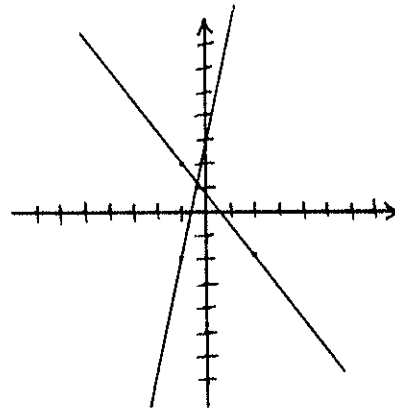
    b. $20x^2y^7z^{1/3}$ d. $12x^{1/3}y^{1/2}z^3$

37. Which parabola below has its vertex at $(3,4)$?

    a. $y = 3(x - 5)^2 - 4$ d. $y = 5(x - 4)^2 + 3$

    b. $y = 5(x - 3)^2 + 4$ e. $y = 4(x - 5)^2 - 3$

    c. $y = 4(x - 3)^2 - 5$

38. Which method shown below would not lead to the correct solution to the system:
    $$y = 5x + 3$$
    $$4x + 3y = 2 \quad ?$$

    a. $4x + 3(5x + 3) = 2$ e.

    b. $\begin{array}{r} 4x + 3y = 2 \\ + (15x - 3y = -9) \end{array}$

    c. $\begin{array}{r} y = 5x + 3 \\ + (3y = -5x + 2) \end{array}$

    d. $4(y - 3)/5 + 3y = 2$

39. In how many ways can a banquet committee comprised of 20 students choose a subcommittee of 4 students to be in charge of decorations?

    a. 4!   b. 20!   c. 20!/4!   d. 20!/16!   e. 20!/(4!16!)

40. Solve the inequality $x^2 - 3x - 7 \le x - 2$.

    a. $-5 \le x \le 1$ c. $x \le -1$ or $5 \le x$ e. $x \ge -1$

    b. $x \le 5$ d. $-1 \le x \le 5$