ALGEBRA II 1987

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Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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1. The larger root of $4x^2 + x - 3 = 0$ is

   a) -1

   b) -3/4

   c) -1/4

   d) 3/4

   e) 1

2. The curves $x^2 + y^2 = 25$ and $x^2 + y = 19$ intersect in how many points?

   a) 0

   b) 1

   c) 2

   d) 3

   e) 4

3. If $2x + 3y - z = 0$
      $x + 2y + 3z = -3$
      $2x + y - 4z = 1$,   then $x + y - z = ?$

   a) -2

   b) -1

   c) 0

   d) 1

   e) 2

4. A piece of wire 100 inches long is cut into two pieces, and then each piece is bent into the shape of a square. If the sum of the enclosed areas is 397 sq. in., find the length of the shorter piece of wire.

   a) 16

   b) 24

   c) 32

   d) 40

   e) 48

5. The center of the circle $4x^2 + 4y^2 + 8x - 8y + 7 = 0$ is

   a) (1, 1)

   b) (-1, -1)

   c) (1, -1)

   d) (-1, 1)

   e) (4, 8)

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6. A water tank can be emptied by using one pump for 5 hours. A second, smaller pump can empty the tank in 8 hours. If the larger pump is started at 1:00 p.m., at what time should the smaller pump be started so that the tank will be emptied at 5:00 p.m.?

   a) 1:36 p.m.　　　　　　　　d) 4:00 p.m.

   b) 3:30 p.m.　　　　　　　　e) 4:10 p.m.

   c) 3:24 p.m.

7. Let w vary jointly as u and v and inversely as x. What happens to w if u is doubled, v tripled, and x doubled?

   a) **w** is halved　　　　　　d) w is quadrupled

   b) w is doubled　　　　　　　e) w remains the same

   c) w is tripled

8. $\begin{vmatrix} \tan x & 1 \\ \sec^2 x & \tan x \end{vmatrix} =$

   a) $\cos^2 x$　　　　　　　　d) -1

   b) $\csc^2 x$　　　　　　　　e) $\sin^2 x$

   c) 1

9. The solutions of $x^2 - 3x + 1 \geq 29$ are

   a) $-4 \leq x \leq 7$　　　　　d) $x < -4$ or $x > 7$

   b) $x \leq -4$ or $x \geq 7$　　　e) $x < -2$ or $x > 14$

   c) $4 \leq x \leq -7$

10. The graph of the equation $25y^2 - 9x^2 - 100y - 54x + 10 = 0$ is

   a) a line　　　　　　　　　d) an ellipse

   b) a parabola　　　　　　　e) a hyperbola

   c) a circle

11. If $\theta = 60^{\circ}$, then $\sin \theta + \csc \theta =$

a) $3\sqrt{3}$

b) $\dfrac{7\sqrt{3}}{6}$

c) $5/2$

d) $\dfrac{4 + \sqrt{3}}{2}$

e) 1

12. Write this complex number $\dfrac{2 + 3i}{2 - 3i}$ in standard form.

a) $\dfrac{-5 + 12i}{13}$

b) $\dfrac{-5}{13}$

c) $\dfrac{13 + 12i}{13}$

d) $\dfrac{13 + 12i}{-5}$

e) $\dfrac{-5}{13} + 12i$

13. An automobile radiator contains 16 quarts of a 20% solution of antifreeze. How much of the original must be drawn off and replaced by pure antifreeze to make a 25% antifreeze?

a) 0.2 quarts

b) 0.8 quarts

c) 1 quart

d) 8 quarts

e) 1.5 quarts

14. This system of equations $6x - 3y = 9$
     $4x - 2y = -10$ has

a) one solution

b) two solutions

c) no solution

d) an infinite number of solutions

e) three solutions

15. If 4 coins are tossed in the air, what is the probability that at least one will turn up heads?

a) 1/16

b) 1/4

c) 3/4

d) 1/2

e) 15/16

16. If $3^{x + 4} = 2^{1 -3x}$, then x =

    a) $\dfrac{1 - \ln 81}{1 + \ln 8}$

    b) $\dfrac{\ln 2 - \ln 81}{\ln 24}$

    c) $\dfrac{\log_{24} 2 - \log_{24} 81}{2}$

    d) no real solution

    e) $\dfrac{\ln 2 + \ln 81}{\ln 24}$

17. The solutions to the inequality $|2x + 4| > 8$ are

    a) $-6 > x > 2$

    b) $x > 2$ or $x < -6$

    c) $x > 2$

    d) $x < -6$

    e) $x > 6$ or $x < -6$

18. The fourth term in the expansion of $(2x^3 - y^2)^6$ is

    a) $-20x^9 y^6$

    b) $160x^9 y^6$

    c) $-120x^9 y^6$

    d) $-160x^9 y^6$

    e) $-160x^6 y^5$

19. If $\log_4 (x + 1) = 2 + \log_4 (3x - 2)$, x =

    a) $33/47$

    b) $\dfrac{-1 \pm \sqrt{217}}{6}$

    c) $\dfrac{-1 - \sqrt{217}}{6}$

    d) $\dfrac{-1 + \sqrt{217}}{6}$

    e) $9/11$

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20. How many three-digit numbers can be formed from the digits 2, 4, 6, 8, 9 if repetitions are not allowed?

    a) 30   d) 125

    b) 60   e) 160

    c) 120

21. If $f(x) = 2x^2 + 4x - 3$ then $\dfrac{f(x + h) - f(x)}{h}$ is

    a) $4x - 2h - 4$ d) $4x + 2h - 4$

    b) $4x + 2h + 4$ e) $4x - h - 4$

    c) $4x - 2h + 4$

22. The sum of the roots of $x^2 + 9x - 22 = 0$ is

    a) -13 d) 9

    b) 13   e) 0

    c) -9

23. The slope of the straight line parallel to $3x - 2y = 1$ at the point $(1/2, 0)$ is

    a) 2/3   d) 3/2

    b) $-2/3$ e) no slope

    c) $-3/2$

24. Determine k so that $f(x) = x^3 + kx^2 - kx + 10$ is divisible by $x + 3$

    a) 3/10   d) 17/12

    b) 9/16   e) 15/4

    c) 11/8

25. The graphs of $y = \log_2 4x$ and $y = 3^{x/2}$ intersect at the point

    a)   (4, 9)   d)   (2, 3)

    b)   (2, 4)   e)   (4, 4)

    c)   (4, 3)

26. Find all roots of $P(x) = x^3 - 2x^2 + 4x - 8$

    a) $x = 2, 2, 2$ d) $x = -2, \pm 2i$

    b) $x = 2, 2, -2$ e) $x = 2, -2, 2i$

    c) $x = 2, \pm 2i$

27. The graph of $P(x) = x^3 + ax^2 + bx + c$ intersects the x-axis at $x = -2$ and $x = 1$ and the y-axis at $y = 0$. Find $P(12)$.

    a)   1848   d)   1232

    b)   1428   e)   1056

    c)   2346

28. Evaluate. $\begin{vmatrix} 1 & -2 & 3 \\ 3 & -1 & 4 \\ 2 & 1 & 0 \end{vmatrix}$

    a)   3   d)   -35

    b)   -5   e)   5

    c)   -3

29. If five black, three red, two white, and two green discs are to be arranged in a row, find the number of possible color arrangements for the discs.

    a)   233,160   d)   149,700

    b)   151,200   e)   216,280

    c)   166,320

30. Given A(3, -1) and B(-2, 6), the equation of the perpendicular bisector of line segment AB is

a) $7x + 5y - 15 = 0$ d) $7x - 5y + 15 = 0$

b) $5x - 7y - 15 = 0$ e) $5x - 7y + 15 = 0$

c) $7x + 5y + 15 = 0$

31. $\dfrac{6}{3 - \sqrt{3 - x}} =$

a) $\dfrac{3 + \sqrt{3 - x}}{1 + x}$ d) $\dfrac{18 + \sqrt{3 - x}}{1 + x}$

b) $\dfrac{18 + 6\sqrt{3 - x}}{6 + x}$ e) $\dfrac{4\sqrt{3 - x}}{1 + x}$

c) $\dfrac{3 + 6\sqrt{3 - x}}{1 + x}$

32. If $g(x)$ is a real valued function, the domain of $g(x) = \dfrac{\sqrt{4 + x}}{1 - x}$ is

a) $x \neq 1$ d) $x \leq -4$

b) $x > -4$ e) $x \leq 4$ and $x \neq 1$

c) $x \geq -4$ and $x \neq 1$

33. How many imaginary roots does $3x^5 + 4x^3 + 2x - 5 = 0$ have?

a) 0   d) 3

b) 1   e) 4

c) 2

34. If the fourth term of an arithmetic sequence is 5 and the ninth term is 20, the sixth term is

a) 9   d) 15

b) 11   e) 17

c) 13

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35. If $f(x) = x^2 - 6x + 5$, then $f\left[f(2)\right] =$

a)   -3   d)   32

b)   18   e)   -22

c)   14

36. The equation $x^2 + y^2 - 4x + 6y = 3$ describes a circle of radius

a)   1   d)   4

b)   2   e)   5

c)   3

37. The minimum value of $f(x) = -5x^2 - 10x - 4$ is

a)   -1   d)   -2

b)   0   e)   2

c)   1

38. If $\log_a 3 = 0.4771$ and $\log_a 2 = 0.3010$, $\log_a \dfrac{3}{2} =$

a)   0.7781   d) ≈ 1.585

b)   0.1761   e)   1.347

c)   0.1505

39. If $56x^2 + 11x - 15$ is factored as $(ax - b)(cx + d)$ where a, b, c, d are positive integers, then b = ?

a)   3   d)   8

b)   5   e)   11

c)   7

40. If matrix $A = \begin{bmatrix} 2 & -1 & 3 \\ 1 & 4 & -1 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 1 & 3 \\ 2 & -4 \\ -1 & 1 \end{bmatrix}$

then A x B =

a) B x A

b) $\begin{bmatrix} -3 & 10 \\ 13 & -14 \end{bmatrix}$

c) $\begin{bmatrix} 7 \\ -1 \end{bmatrix}$

d) $\begin{bmatrix} -3 & 13 \\ 10 & -14 \end{bmatrix}$

e) does not exist