TWENTY-EIGHTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II 1984

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Scoring Formula: 4R - W + 40

This test was prepared from a list of Algebra II questions submitted by the Tri-Cities State Technical Institute.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. If $\begin{aligned} 4x + 5y &= 3 \\ 7x + 9y &= 5 \end{aligned}$ then x =

a) -2
b) -1
c) 0
d) 1
e) 2

2. The equation of the line through the points (1,1) and (10,13) is of the form ax + by = 1. What is a?

a) 1
b) 2
c) 3
d) 4
e) 5

3. Let the point A be (1,1), B be (4,1) and C be (3,11). What is the area of triangle ABC?

a) 15
b) 22
c) 30
d) 33
e) 44

4. Let the point A be (1,1), B be (4,1) and C be (5,11). Which angle of triangle ABC is largest?

a) A
b) B
c) C
d) A and B are equal.
e) Equilateral triangle

5. The numbers 34 and 45 are written in base 8. Their product, written in base 8, is

a) 1036
b) 1066
c) 1530
d) 2014
e) 2772

6. The larger root of $x^2 + x - 72 = 0$ is

   a) 6
   b) 8
   c) 9
   d) 12
   e) 24

7. A circle passses through the points (4,2), (-2,2) and (4,-6). What is the center of the circle?

   a) (1,2)
   b) (1,-2)
   c) (-1,2)
   d) (-1,-2)
   e) (1,1)

8. What is the radius of the circle of problem 7?

   a) 4
   b) $\sqrt{20}$
   c) 5
   d) 16
   e) 25

9. The inequality $|x + 1| > \sqrt{x^2 + 1}$ is valid when

   a) $x > 0$
   b) $x < 0$
   c) $x < -1$
   d) $-1 < x < 1$
   e) Never valid

10. If
$$\begin{aligned} 2x + y + z &= 3 \\ 2x - 2y + 3z &= -5 \\ 4x + 5y + z &= 13 \end{aligned}$$
then $x + y + z =$

    a) -2
    b) -1
    c) 0
    d) 1
    e) 2

11. If $8x^2 + 14x - 15$ is factored $(ax + b)(cx - d)$ where a, b, c and d are positive integers, then a =

    a) 1
    b) 2
    c) 3
    d) 4
    e) 8

12. If $x^4 - 9x^2 + 12x - 4$ is factored $(x^2 + ax - b)(x^2 - cx + d)$ where a, b, c and d are positive integers, then a =

a) 1
b) 2
c) 3
d) 4
e) 5

13. A man drives the first 3 miles of a 4 mile course at 40 mph. At what rate must he drive the last mile in order to average 50 mph for the entire course?

a) 60 mph
b) 80 mph
c) 100 mph
d) 160 mph
e) 200 mph

14. The curves $y = 4x^2$ and $y = x^2 + 4$ intersect in how many points?

a) 0
b) 1
c) 2
d) 3
e) 4

15. Mary's father, John, will be 5 times Mary's age in 4 more years. At present, the difference in their ages is 32. Mary's age is

a) 2
b) 3
c) 4
d) 5
e) 6

16. Let $a_0 = 1$ and $a_{i+1} = 2a_i + 1$ for $i \geq 0$. What is a_{100}?

a) $2^{100} - 1$
b) $2^{100} + 1$
c) $2^{101} - 1$
d) $2^{101} + 1$
e) $2^{102} - 1$

17. The equation $x^3 + 3x^2 + 3x + 2 = 0$ has how many real roots.

a) 0
b) 1
c) 2
d) 3
e) 4

18. The equation $\sqrt{x + 5} = x - 1$ has how many real roots?

a) 0
b) 1
c) 2
d) 3
e) 4

19. In the state of confusion, license plates have exactly 3 digits, chosen from the set {0,1,2,3,4}. No digit can appear more than once. How many different license plates are possible?

a) 10
b) 60
c) 120
d) 125
e) 243

20. Refer to problem 19. In a neighboring state the license plates also have 3 digits, but each digit can appear at most two times. How many plates are possible now?

a) 10
b) 60
c) 120
d) 125
e) 243

21. What is the remainder when $x^3 + 2x + 1$ is divided by $x^2 + x + 1$?

a) $2x + 2$
b) $-2x - 2$
c) $2x$
d) 2
e) -2

22. What is the remainder when $x^9 - x^8 + x^7 - x^6 + x^5 - x^4 + x^3 - x^2 + x - 1$ is divided by $x - 1$?

a) -10
b) -1
c) 0
d) 1
e) 10

23. Let $p(x)$ be a polynomial such that $p(1) = 2$ and $p(-1) = 0$. What is the remainder when $p(x)$ is divided by $x^2 - 1$?

a) $x + 1$
b) $x - 1$
c) $x + 2$
d) x
e) 0

24. Let $f(x) = 1 + 2x$. What is $f(f(x))$?

a) $3 + 4x$
b) $4 + 3x$
c) $2 + 4x + 4x^2$
d) $1 + 2x$
e) $1 + 4x + 4x^2$

25. If $y = \frac{1 - x}{3 + x^2}$ where x varies over all real numbers, what is the set of possible values of y?

a) $1/6 \leq y \leq 1/2$
b) $-1/6 \leq y \leq 1/2$
c) $-1/2 \leq y \leq 1/6$
d) $-6 \leq y \leq 2$
e) All real numbers

26. What is the difference of the roots of $x^2 - 6x - 27$?

a) 0
b) 3
c) 6
d) 9
e) 12

27. If $\log_{10}(x) = a$, what is $\log_x(100)$?

a) 2a
b) a/2
c) 2/a
d) a^2
e) $1 + a$

28. Given that $\log(2) = .301$ and $\log(3) = .477$, what is $\log(9)$?

a) .176
b) .564
c) .778
d) .800
e) .954

29. Given that $\log_{10}(2) = .301$ and $\log_{10}(3) = .477$, what is $10^{.176}$?

a) .667
b) 1
c) 1.333
d) 1.5
e) 6

30. What is the coefficient of x^7y^3 in $(2x^2 + 3xy)^5$?

a) 10
b) 18
c) 108
d) 540
e) 1080

31. Evaluate $\begin{vmatrix} 1 & 4 \\ 2 & 7 \end{vmatrix}$.

a) -13
b) -1
c) 1
d) 13
e) 15

32. Evaluate $\begin{vmatrix} 1 & 0 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 2 & 0 & 0 & 7 \\ 0 & 1 & 0 & 0 \end{vmatrix}$

a) -13
b) -1
c) 1
d) 13
e) 15

33. The greatest common divisor of the numbers 2540 and 2667 is

a) 1
b) 3
c) 7
d) 113
e) 127

34. Three men and three women are at a party. Two persons are selected at random to receive a prize. What is the probability that two women are selected?

a) 1/6
b) 1/5
c) 1/4
d) 1/3
e) 1/2

35. What is a good approximation to $\sqrt{2121}$?

a) 44
b) 45
c) 46
d) 47
e) 48

36. Three men and three women are seated at random at a circular table. What is the probability that the men and women are seated alternately?

a) 1/10
b) 1/6
c) 1/5
d) 1/3
e) 1/2

37. The greatest common divisor of the polynomials $x^3 + 6x^2 + 12x + 7$ and $x^3 + 8x^2 + 12x + 5$ is

a) 1
b) x
c) x + 1
d) x - 1
e) $x^2 + 1$

38. The operation $a \odot b$ is defined by $a \odot b = (a^2 + b^2)/(a + b)$. The operation is

a) Associative and commutative
b) Associative but not commutative
c) Commutative but not associative
d) Neither commutative nor associative

39. For what positive values of a and b is

$$\left(a^2 + b^2\right)/(a + b) \geq 2\left(a^{-1} + b^{-1}\right)^{-1}.$$

a) All positive a,b

b) When $0 < a \leq 1$, $0 < b \leq 1$

c) When $a^2 + b^2 \leq 1$

d) When $a^2 + b^2 \geq 1$

e) Only when $a = b$

40. For what value of k does the system below have a solution?

$$2x + 3y = 1$$
$$3x + 5y = 1$$
$$x + y = k$$

a) -2

b) -1

c) 0

e) 1

f) 2