TWENTY-SEVENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II TEST

Scoring Formula: 4R - W + 40

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This test was prepared from a list of Algebra II questions submitted by Volunteer State Community College.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. If $\sqrt{3x + 10} - \sqrt{2x + 5} = 1$, then the sum of the roots is

   a) -4.
   b) -16.
   c) 16.
   d) 8.
   e) 0.

2. $(i + 1)(i - 1)$ is equal to

   a) 2i.
   b) 0.
   c) -2.
   d) -1.
   d) i.

3. The exposure necessary to obtain a good photographic negative varies directly as the square of the f number of the camera lens. If an exposure of $\frac{1}{50}$ sec produces a good negative at f 16, what exposure time would be necessary at a lens setting of f 8?

   a) $\frac{2}{25}$ sec
   b) $\frac{1}{200}$ sec
   c) approximately 328 sec
   d) 5.12
   e) $\frac{1}{30}$ sec

4. Connie and Donna made a 1210-mile trip in 23 hours. Connie drove at 55 mph, while Donna drove at 50 mph. How much time did each of them drive?

   a) Connie 14 hours and Donna 9 hours
   b) Connie 12 hours and Donna 11 hours
   c) Connie 13 hours and Donna 10 hours
   d) Connie 12.5 hours and Donna 10.5 hours
   e) Each drove 11.5 hours.

5. The sum of the roots for the system $\begin{cases} 4x + 5y - 6z = 31 \\ y - 2z = 7 \\ 5y + z = 2 \end{cases}$ is

   a) $-\frac{2}{11}$.

   b) -2.

   c) $-\frac{5}{2}$.

   d) 0.

   e) not unique.

6. If a column of men 3 miles long is marching at 5 mph, how long will it take a courier on a horse traveling at 25 mph to deliver a message from the end of the column to the front and then return?

   a) 10 minutes

   b) 12 minutes

   c) 15 minutes

   d) 20 minutes

   e) 30 minutes

7. The solution of the inequality $|3x - 15| \geq 15$ is

   a) $x \leq -10$ or $x \geq 0$.

   b) $x \geq 10$ or $x \leq 0$.

   c) $-10 \leq x \leq 0$.

   c) $0 \leq x \leq 10$.

   d) $0 \leq x \leq 10$.

   e) all real numbers.

8. How much pure alcohol must be added to 2 liters of a 30% alcohol solution to strengthen it to a 50% solution?

   a) $\frac{5}{4}$ liters

   b) $\frac{4}{5}$ liters

   c) 10 liters

   d) $\frac{14}{5}$ liters

   e) $\frac{4}{995}$ liters

9. How many distinguishable permutations can be made from the letters in the word MATHEMATICS?

a) $11!$

b) $\frac{11!}{2!\ 2!\ 2!}$

c) $\frac{2!\ 3!}{11!}$

d) $\frac{3!\ 3!}{11!}$

e) $\frac{2!\ 2!\ 2!}{11!}$

10. The complete solution set of $\frac{5}{x-3} > \frac{4}{x-2}$ is given by

a) $\{x|\ 2 < x < 3\}$.

b) $\{x|\ x > -2\}$.

c) $\{x|\ -2 < x < 0 \text{ or } x > 3\}$.

d) $\{x|\ -2 < x < 2 \text{ or } x > 3\}$.

e) $\{x|\ x < -2 \text{ or } 2 < x < 3\}$.

11. In simplified radical form, $x^{3/4}(x^{1/3} - x^{-2/3})$ is equal to

a) $\sqrt[4]{x} - \sqrt{x}$.

b) $\sqrt[12]{x^5}$.

c) $(x - 1)\sqrt[12]{x}$.

d) $x(\sqrt[12]{x} - \sqrt[12]{x^5})$.

e) $\sqrt[7]{x^4} - \sqrt[7]{x^5}$.

12. The sum of the values of x and y for which

$3x - 5 + (4x + 3)i = y - 4 + (3x + y)i$ is

a) 7.

b) -9.

c) 15.

d) -17.

e) 0.

13. The simplified form for $\dfrac{x - \dfrac{1}{1 + \dfrac{1}{x}}}{x + \dfrac{1}{x - \dfrac{1}{x}}}$ is

a) -1.

b) $\dfrac{x^3 - x - 1}{x^3 - x + 1}$.

c) $\dfrac{x - 1}{x + 1}$.

d) $\dfrac{x - 1}{x}$.

e) $\dfrac{-x^2 + x + 1}{2x}$.

14. Find the fourth term of the expression $(a - x)^7$.

a) $70a^2x^3$

b) $-7a^3x^4$

c) $-35a^4x^3$

d) $14a^4x^3$

e) $-35a^3x^4$

15. The sum of the first six terms of the following progression:

2, -6, 18, ..., is

a) -486.

b) -364.

c) 365.

d) 364.

e) 486.

16. If $f(x) = \frac{2 + x}{2 - x}$, then $\frac{f(q) - f(-q)}{1 + f(q)f(-q)}$ is equal to

a) $\frac{8q}{4 - q^2}$.

b) $\frac{-8q}{(q - 2)^2}$.

c) $\frac{4q}{(2 + q)^2}$

d) $\frac{8q}{2 - q}$.

e) $\frac{4q}{4 - q^2}$.

17. Find the equation of a circle tangent to the x-axis at P(5,0) with the center of the circle on the line $y = 2x + 3$.

a) $(x - 1)^2 + (y - 5)^2 = 25$.

b) $(x - 5)^2 + (y - 13)^2 = 169$.

c) $(x - 13)^2 + (y - 5)^2 = 169$.

d) $(x - 1)^2 + (y - 5)^2 = 1$.

e) $(x - 5)^2 + (y - 13)^2 = 25$.

18. If $f(x) = x^3 - 2x^2 + 5x + 30$, then $\frac{f(x)}{x - 2 + 3i}$ will have a remainder of

a) $46 + 48i$.

b) $48 + 12i$.

c) $48 - 12i$.

d) 4.

e) $4i$.

19. If $\log_{10}2 = 0.301$ and $\log_{10}3 = 0.477$, then $\log_{10}5$ is equal to

a) 0.778.

b) 0.699.

c) 0.586.

d) 0.725.

e) none of the above

20. If $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & -1 \\ 2 & 3 & 0 \end{bmatrix}$, then its inverse matrix, A^{-1}, is equal to

a) $\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$.

b) $\begin{bmatrix} -2 & -2 & 1 \\ -4 & -5 & 2 \\ 3 & 3 & -1 \end{bmatrix}$.

c) $\begin{bmatrix} -4 & -2 & 3 \\ -5 & -2 & 3 \\ 2 & 1 & -1 \end{bmatrix}$.

d) $\begin{bmatrix} -2 & 3 & -4 \\ -2 & 3 & -5 \\ 1 & -1 & 2 \end{bmatrix}$.

e) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$.

21. Four cards are randomly drawn from an ordinary deck of cards. The probability that each will be of a different suit is

a) $\left(\frac{1}{4}\right)^4$.

b) $\left(\frac{1}{4}\right)^4 (4!)$.

c) $\left(\frac{13}{52}\right)\left(\frac{12}{51}\right)\left(\frac{11}{50}\right)\left(\frac{10}{49}\right)(4!)$.

d) $\left(\frac{13}{52}\right)\left(\frac{13}{51}\right)\left(\frac{13}{50}\right)\left(\frac{13}{49}\right)(4!)$.

e) $\left(\frac{13}{52}\right)\left(\frac{12}{52}\right)\left(\frac{11}{52}\right)\left(\frac{10}{52}\right)(4!)$.

22. Two dice are rolled. The probability that at least one of them shows an even number is

a) $\frac{1}{2}$.

b) $\frac{3}{4}$.

c) $\frac{1}{4}$.

d) $\frac{5}{6}$.

e) $\frac{2}{3}$.

23. The solution for the system $\begin{cases} \log_3(y^2 - x^2) = 1 \\ \log_4 xy = \frac{1}{2} \end{cases}$ is

a) $(\log_2 1, \log_4 16), (\log_2 \frac{1}{2}, \log_3 \frac{1}{9})$.

b) $(\log_4 \frac{1}{16}, \log_3 1), (\log_2 2, \log_3 \frac{1}{3})$.

c) $(\log_2 \frac{1}{2}, \log_5 \frac{1}{25}), (\log_3 3, \log_3 9)$.

d) $(\log_3 9, \log_4 \frac{1}{4}), (\log_4 4, \log_3 \frac{1}{9})$.

e) $(\log_3 3, \log_4 \frac{1}{16})$, $(\log_3 \frac{1}{3}, \log_4 \frac{1}{16})$.

24. The set of all points equidistant from the point (-2,5) and the line y = -3 is

a) (-2,1).

b) $(x - 2)^2 + (y - 5)^2 = 9$.

c) $(x - 2)^2 + (y - 1)^2 = 9$.

d) $(x + 2)^2 = 16(y - 1)$.

e) $(x + 2)^2 = 32(y - 1)$.

25. If $x^4 + 3x^2 + 4$ is factored over the integers, the difference of the factors could be

a) 0.

b) 2.

c) 3.

d) 2x.

e) Cannot be factored over the integers.

26. A statistical study has shown that if a store charges x dollars each for toy trucks, 300 - 100x will be sold. If the trucks cost the store \$2.00 each, determine what the store should charge to maximize profit.

a) \$1.50

b) \$2.50

c) \$3.00

d) \$3.50

e) \$4.00

27. The sum of the roots of the equation $3^{2x} - 3^{x+1} + 2 = 0$ is equal to

a) $\log_3 6$.

b) 2.

c) 0.

d) 3.

e) $\log_3 2$.

28. Three couples go together to a concert and buy tickets for six adjacent seats. How many ways can they seat themselves if no one cares who sits next to whom as long as couples stay together?

a) 48

b) 36

c) 72

d) 720

e) 12

29. The domain of the function $f(x) = \frac{\sqrt{x - 3}}{x^2 - 3x - 4}$ is equal to

a) $\{x \mid x \neq 3\}$.

b) $\{x \mid x \neq 4, x \neq -1\}$.

c) $\{x \mid x \geq 3\}$.

d) $\{x \mid -1 < x < 4\}$.

e) $\{x \mid x \geq 3, x \neq 4\}$.

30. The least value of $2x^2 - 8x + 3$ is equal to

a) -2.

b) 2.

c) -5.

d) 5.

e) 3.

31. The graph of $x^2 - 4y^2 = 0$ is

a) one straight line.

b) one point.

c) two straight lines.

d) an ellipse.

e) a hyperbola.

32. If $\log_6 3 = 0.6131$, find x if $\log_6 x = -1.3869$.

a) $\frac{1}{6}$

b) 6

c) 36

d) 12

e) $\frac{1}{12}$

33. If $\log_{10} 2 = 0.3010$, then $\log_5 2$ is equal to

a) 0.1505.

b) 0.6020.

c) 0.4306.

d) 0.3010.

e) 2.3010.

34. A straight line is tangent to the circle $4x^2 + 4y^2 - 16x + 40y + 16 = 0$ at the point $(0, -5+\sqrt{21})$. The slope of this line is

a) $\frac{2\sqrt{21}}{21}$.

b) $\frac{-10 - \sqrt{21}}{2}$.

c) $\frac{-\sqrt{21}}{2}$.

d) $\frac{2}{10 + \sqrt{21}}$.

e) $\frac{-2}{\sqrt{21}}$.

35. If a straight line passes through the points P(a,b) and Q(c,d), then which of the following statements is not true?

a) $(b-d)x - (a - c)y + ad - bc = 0$.

b) Slope $m = -\frac{d - b}{a - c}$.

c) The x-intercept is $-\frac{bc - ad}{d - b}$.

d) $\frac{(b - d)x}{bc - ad} + \frac{(c - a)y}{bc - ad} = 1$.

e) The distance PQ is $\sqrt{a^2 - 2ac + c^2 - b^2 + 2bd - d^2}$.

36. If $x^3 - 3x^2 - 3x + 1 = 0$, then the sum of the roots is

a) 0.

b) $4 + 2\sqrt{3}$.

c) 5.

d) 3.

e) imaginary.

37. When simplified, $\sqrt{7 + 4\sqrt{3}}$ is equal to

a) $3 + \sqrt{3}$.

b) $-2 + \sqrt{3}$.

c) $3 - \sqrt{3}$.

d) $2 - \sqrt{3}$.

e) none of the above.

38. If $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots}}}}$, then

a) $x = 1$.

b) $0 < x < 1$.

c) $1 < x < 2$.

d) x is infinite.

e) $x > 2$ but finite.

39. Ozone and Cosma live at opposite ends of the same lane. Ozone wanted to deliver a box of candy at Cosma's house, and Cosma wanted to leave a tennis racket at Ozone's house. They started at the same moment, each walking at a constant speed. They met the first time 300 meters from Ozone's house. On their return trip, they met 400 meters from Cosma's house. How long is the lane? (Assume that neither loitered at the other house nor when they met.)

a) 450 meters

b) 500 meters

c) 550 meters

d) 600 meters

e) 800 meters

40. A system of two second-degree equations $\begin{cases} x^2 - y^2 = 0, \\ (x - a)^2 + y^2 = 1 \end{cases}$ has, in general, four solutions. For what values of a is the number of solutions of this system decreased to two?

a) 0

b) ± 1

c) ± 2

d) $\pm \sqrt{2}$

e) $\pm \sqrt{3}$