ALGEBRA II TEST, 1981   Edited by: Richard Harville
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Scoring Formula: 4R - W + 40   Billy F. Bryant

This test was prepared from a list of Algebra II questions submitted by Volunteer State Community College.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

1. If $-a < 0 < -b$, which of the following is true?

   (a) $a < b < 0$

   (b) $a < 0 < b$

   (c) $b < a < 0$

   (d) $b < 0 < a$

   (e) $0 < b < a$

2. If the functions $y = |x + 1|$ and $y = -|x| + 2$ are graphed relative to the same coordinate system, the number of points in their intersection is

   (a) 1

   (b) 2

   (c) 3

   (d) an infinite number

   (e) They do not intersect.

3. A pair of balanced dice is rolled. If the sum of the upward faces is six, the payoff is $72.00. What is the most one should pay to play the game?

   (a) $5.00

   (b) $10.00

   (c) $12.00

   (d) $15.00

   (e) $72.00

4. If $x^4 + x^2 + 1$ is factored into two second degree factors with integral coefficients, the difference of the factors could be

   (a) 0

   (b) $2x$

   (c) $2x^2 + 2$

   (d) 2

   (e) $x^4 + x^2 + 1$ cannot be factored.

5. The sum of all real values of x for which $3^{2x} - 4(3^{x+1}) = -27$ is

    (a) -3

    (b) 0

    (c) 3

    (d) 9

    (e) Impossible to find over the real numbers for the given equation.

6. Evaluate $\sqrt[4]{3^{11} + 3^{11} + 3^{11}}$. The result, when reduced and simplified, is

    (a) 3^3

    (b) 9

    (c) 3

    (d) $3\sqrt[4]{3^2}$

    (e) $\sqrt[4]{3^{33}}$

7. If $211_x = 821_{10}$, the subscripts referring to the number base, then x is a multiple of

    (a) 3

    (b) 5

    (c) 7

    (d) 11

    (e) 13

8. A store owner found that if she charges x dollars each for a certain toy, she can sell 300 - 100x of them. The toys cost her $2 each. What should she charge for each toy to maximize her profit?

    (a) $5.00

    (b) $2.50

    (c) $2.00

    (d) $4.25

    (e) Cannot be determined from this information.

9. If x is a real number and the value of $\sqrt{\dfrac{x+1}{1-x}}$ is to be real, then x must be such that

    (a) $x \geq -1$

    (b) $x < 1$

    (c) $-1 \leq x < 1$

    (d) $x \leq -1$ or $x > 1$

    (e) x is any number other than 1

10. The coefficient of the term of the expansion of $(b - b^{-1})^{12}$ containing b^{-6} (<u>after</u> <u>simplification</u>) is

    (a) -1320

    (b) -220

    (c) 220

    (d) -66

    (e) 66

11. The sum of all real values of x for which $3^{x^2 - 4x - 5} = 1$ is

    (a) -4

    (b) 0

    (c) 3

    (d) 4

    (e) 6

12. Which of the following equations has roots which are twice the roots of $x^4 - \dfrac{1}{2}x^3 + \dfrac{3}{4}x^2 - 5x + 2 = 0$?

    (a) $2x^4 - x^3 + \dfrac{3}{2}x^2 - 10x + 4 = 0$

    (b) $x^4 - \dfrac{1}{2}x^3 + \dfrac{3}{4}x^2 - 5x + 4 = 0$

    (c) $x^4 - \dfrac{1}{2}x^3 + \dfrac{3}{4}x^2 - 5x + 32 = 0$

    (d) $x^4 - x^3 + 3x^2 - 40x + 32 = 0$

    (e) $4x^4 - 2x^3 + 3x^2 - 20x + 8 = 0$

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13. What is the probability of throwing two balanced dice such that the absolute value of the difference in face value between the two dice is three, and the product of the numbers thrown is equal to twice the sum of the two numbers?

    (a) 1/18

    (b) 1/36

    (c) 1/6

    (d) 1/9

    (e) none of the above

14. The statement $2x^2 + 5 > 7x$ is equivalent to the statement

    (a) $-\frac{5}{2} < x < 1$

    (b) $1 < x < \frac{5}{2}$

    (c) $x < -\frac{5}{2}$ or $x > 1$

    (d) $x < 1$ or $x > \frac{5}{2}$

    (e) All values of x are irrational

15. If $\sqrt{-1} = i$, then $8 + 3i + 6i^2 + 7i^3 + 2i^4$ is equal to

    (a) 26

    (b) -4i

    (c) -3 + 3i

    (d) 4 - 4i

    (e) 4 - 10i

16. The circle whose equation is $4x^2 + 4y^2 - 4x + 9y + 1 = 0$ has a radius of

    (a) 9/4

    (b) 9/16

    (c) 81/16

    (d) 6

    (e) 9/2

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17. If $x = \sqrt{yz}$, where $x > 0$, $y > 0$, and $z > 0$, then log y is equal to

    (a). $2(\log x - \log z)$

    (b) $\dfrac{\log x^2}{\log z}$

    (c) $\dfrac{x^2}{z}$

    (d) $2x - z$

    (e) $\log x^2 - \log z$

18. If $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$, then $f(\frac{1}{x})$ is equal to

    (a) $f(x)$

    (b) $-f(x)$

    (c) $\dfrac{1}{f(x)}$

    (d) $\dfrac{1}{-f(x)}$

    (e) $\dfrac{x - 1}{x + 1}$

19. If $\log_8 m + \log_8 \frac{1}{6} = \frac{2}{3}$, then m is equal to

    (a) 1/2

    (b) 2/3

    (c) 23/6

    (d) 4

    (e) 24

20. The hands of a clock indicate that it is between 4 and 5 o'clock. After the elapse of slightly less than one hour, the position of the hands will be interchanged. What time is it now?

    (a) 4:26 5/13

    (b) 4:22 34/124

    (c) 4:25 5/13

    (d) 4:26 122/143

    (e) cannot be determined

21. The sum of the rational roots of $x^3 - 21x + 20 = 0$ is equal to

    (a) 0

    (b) 1

    (c) 10

    (d) -20

    (e) 21

22. If the repeating decimal $.0003131\overline{31}$ is expressed as a common fraction in lowest terms, the sum of the numerator and denominator is

    (a) 9931

    (b) 9942

    (c) 9991

    (d) 9999

    (e) 99031

23. If $\dfrac{5}{(x - 2)(x - 5)} = \dfrac{G}{x - 2} + \dfrac{H}{x - 5}$ is an identity, then the product of G and H is

    (a) -36/9

    (b) -25/9

    (c) -9/25

    (d) -9/36

    (e) 1

24. If $\left|x^2 - 9\right| < k$ whenever $\left|x - 3\right| < \dfrac{1}{10}$, then the smallest value which can be used for k is

    (a) 0.61

    (b) 0.63

    (c) 0.65

    (d) 0.7

    (e) 0.75

33. It is known that the maximum load, L, of a beam supported at both ends varies jointly as breadth, b, and the square of its depth, d, and inversely as the length between supports, 1. A beam 15 ft. long, 2 inches wide, and 6 inches deep when supported at both ends can carry a maximum load of 900 lbs. The maximum load for a beam of similar material 12 ft. long, 3 inches wide and 4 inches deep is

   (a) 1080 lbs.

   (b) 480 lbs.

   (c) 720 lbs.

   (d) 1225 lbs.

   (e) 750 lbs.

34. If $p(x) = (x + 1)(x + k)$ and $p(3) = 4$, then k is equal to

   (a) 1

   (b) -1

   (c) 4

   (d) -2

   (e) none of the above

35. If r and s are the roots of the equation $ax^2 + bx + c = 0$, the value of $\dfrac{1}{r^2} + \dfrac{1}{s^2}$ is equal to

   (a) $b^2 - 4ac$

   (b) $\dfrac{b^2 - 4ac}{2a}$

   (c) $\dfrac{b^2 - 4ac}{c^2}$

   (d) $\dfrac{b^2 - 2ac}{c^2}$

   (e) none of the above

36. The expression $(x^4 - 16)$ when factored into prime factors is

   (a) $(x^2 - 4)^2$

   (b) $(x^2 + 4)^2$

   (c) $(2x - 1)^4$

   (d) $(x^2 - 4)(x + 1)(x - 1)$

   (e) none of the above

37. Assume that a thin band is placed around the earth's equator so that it fits tightly all around, and if the band's circumference is increased it will stand out from the earth equally all around. If the earth is a sphere of circumference 25,000 miles and the band's circumference is increased by 10 feet, which is the best approximation of the distance the band will stand out from the earth?

    (a) 1.6 feet

    (b) .0001 feet

    (c) 10.5 feet

    (d) 1000 feet

    (e) 5000 feet

38. If $\frac{7}{x} - \frac{5}{x} = \frac{1}{3}$, then x is equal to

    (a) 1/6

    (b) 2/3

    (c) 6

    (d) 2

    (e) 3

39. The complete solution set for the inequality $|3x + 9| - |x - 2| > 7$ is

    (a) $-3 < x < 3$

    (b) $x < -3$ or $x > 2$

    (c) $x < -9$ or $x > 2$

    (d) $x < -9$ or $x > 0$

    (e) $x < -9$ or $0 < x < 2$

40. If $(a + \frac{1}{a})^2 = 3$, then $a^3 + \frac{1}{a^3}$ is equal to

    (a) $\frac{10\sqrt{3}}{3}$

    (b) $3\sqrt{3}$

    (c) 0

    (d) $7\sqrt{7}$

    (e) $6\sqrt{3}$

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