NINETEENTH ANNUAL MATHEMATICS CONTEST
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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II TEST

1975

Scoring Formula: 4R - W

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This test was prepared from a list of Algebra II questions submitted by East Tennessee State University.

DIRECTIONS:

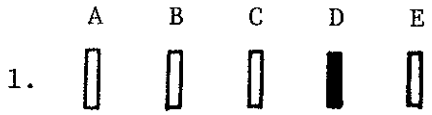
Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

(a). 2/3. (b). 3. (c). 6.

(d). 3/2. (e). none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. $\frac{2}{1 + \sqrt{2}}$ is equal to

   (a) 1

   (b) $-2(1 + \sqrt{2})$

   (c) $\sqrt{2}$

   (d) $\frac{2}{3}$

   (e) $2(\sqrt{2} - 1)$

2. Factor completely, in the Real Number system, $(x^4 - 16)$

   (a) $(x^2 + 4)\ (x + 2)\ (x - 2)$

   (b) $(x + 2)\ (x^3 - 8)$

   (c) $(x - 2)\ (x^3 + 8)$

   (d) $(x^2 + 4)\ (x^2 - 4)$

   (e) none of these

3. If $\frac{x + y + z}{7} = \frac{2x}{3} = \frac{y}{2}$ then $\frac{x + y + z}{z}$ is equal to

   (a) 0

   (b) 1

   (c) 2

   (d) 4

   (e) -4

4. $\sqrt{-2} + \sqrt{-8}$ is equal to

   (a) $2\sqrt{2}i$

   (b) $\sqrt{10}i$

   (c) $2\sqrt{2}$

   (d) $3i$

   (e) $3\sqrt{2}i$

5. Solve the inequality $\frac{1}{x} + 3 > \frac{2}{x}$

(a) $x > \frac{1}{3}$ or $x < \frac{1}{3}$

(b) $x > 0$

(c) $x > \frac{1}{3}$ or $x < 0$

(d) x is any real number

(e) $x < 3$

6. Solve the equation $(3 - 2i)x + 4 = 0$

(a) $x = \frac{-12 - 8i}{5}$

(b) $x = -\frac{9}{13}$

(c) $x = \frac{-3 - 2i}{9}$

(d) $x = \frac{-12 - 8i}{13}$

(e) $x = -12 + 8i$

7. $\dfrac{\sqrt{a^2 + x^2} - \dfrac{x^2 - a^2}{\sqrt{a^2 + x^2}}}{a^2 + x^2}$ is equal to

(a) 0

(b) $\dfrac{2a^2}{a^2 + x^2}$

(c) $\dfrac{2x^2}{(a^2 + x^2)^{3/2}}$

(d) $\dfrac{2a^2}{(a^2 + x^2)^{3/2}}$

(e) $\dfrac{2x^2}{a^2 + x^2}$

8. The value of $\left(\frac{1}{16}\right)a^0 + \left(\frac{1}{16a}\right)^0 - 64^{-1/2} - (-32)^{-4/5}$, $a \neq 0$, is

(a) $\frac{7}{8}$

(b) $1\frac{13}{16}$

(c) $1\frac{3}{16}$

(d) 1

(e) $\frac{1}{16}$

9. When simplified and expressed with negative exponents, the expression $(x^{-1}y^{-1})(x + y)^{-1}(x^{-1} + y^{-1})$ is equal to

(a) $x^{-3}y^{-1} + 2x^{-2}y^{-2} + x^{-1}y^{-3}$

(b) $x^{-3}y^{-1} + 2^{-1}x^{-2}y^{-2} + x^{-1}y^{-3}$

(c) $x^{-2}y^{-2}$

(d) $\frac{1}{x^{-2}y^{-2}}$

(e) none of these

10. The expression $\sqrt{\frac{x^3 + 3x^2 + 3x + 1}{x^3 - x^2 - x + 1}}$ is equal to

(a) $\sqrt{2x(x + 1)}$

(b) $\frac{x + 1}{x - 1}$

(c) $\left|\frac{x + 1}{x - 1}\right|$

(d) $\left(\sqrt{\frac{x + 1}{x - 1}}\right)^3$

(e) none of these

11. $\dfrac{7 - \dfrac{6}{x + 1}}{x - 7 - \dfrac{x^2 - 4}{x + \dfrac{2}{x + 3}}}$ is equal to

(a) -1

(b) -2

(c) 2

(d) $7x + 1$

(e) $\dfrac{1}{x + 1}$

12. The solution set of $\dfrac{x^2}{x - 2} - \dfrac{x - 1}{2} + \dfrac{2 - 3x}{x - 2} + 1 = 0$ is

(a) $\{-1\}$

(b) $\{2\}$

(c) $\{2, -1\}$

(d) $\emptyset$, the empty set

(e) none of these

13. The solution set of $2x^3 - 3x^2 - x + 1 = 0$ is

(a) $\{\frac{1}{2}, -\frac{1}{2}, 1\}$

(b) $\{-\frac{1}{2}, \frac{1 + \sqrt{7}}{2}, \frac{1 - \sqrt{7}}{2}\}$

(c) $\{-\frac{1}{2}, \frac{1 + \sqrt{6}}{2}, \frac{1 - \sqrt{6}}{2}\}$

(d) $\{1, -1, 2\}$

(e) $\{\frac{1}{2}, \frac{1 + \sqrt{5}}{2}, \frac{1 - \sqrt{5}}{2}\}$

14. It is given that the discriminant is zero in the equation $ax^2 - 2x\sqrt{2} + c = 0$, with a and c real constants. The roots are necessarily:

(a) equal and real

(b) equal and irrational

(c) equal and imaginary

(d) equal and integral

(e) none of these

15. The graph of $y = x^2 + 6$ and the graph of $y = x + 6$ meet in two points. The distance between these two points is:

(a) 1

(b) less than 1

(c) 2

(d) more than 2

(e) $\sqrt{2}$

16. The graph of $x^2 - 4y^2 = 0$ is

(a) a hyperbola intersecting only the x-axis

(b) a hyperbola intersecting only the y-axis

(c) a hyperbola intersecting neither axis

(d) a pair of straight lines

(e) none of these

17. Find an equation of the circle which has center $(-2, 3)$ and which passes through $(4, 5)$.

(a) $(x + 2)^2 + (y - 3)^2 = 36$

(b) $(x - 2)^2 + (y + 3)^2 = 40$

(c) $x^2 + y^2 = 40$

(d) $x^2 + y^2 + 4x - 6y - 27 = 0$

(e) none of these

18. The electrical resistance of a wire varies directly as its length and inversely as the square of its diameter. If a wire 100 feet long of diameter 0.01 inches has a resistance of 25 ohms, find the resistance in the wire made of the same material which has a diameter of 0.015 inches and is 50 feet long.

(a) $8\frac{1}{3}$ ohms

(b) $5\frac{5}{9}$ ohms

(c) $\frac{250\sqrt{0.015}}{3}$ ohms

(d) 15 ohms

(e) none of these

19. The distance a ball rolls down an inclined plane varies as the square of the time. If a ball rolls 6 feet in the first two seconds, how far will it roll in 5 seconds?

(a) 10 feet

(b) 15 feet

(c) 37.5 feet

(d) 75 feet

(e) 150 feet

20. If $\log_{10} x - 5 \log_{10} 3 = -2$ then x is equal to

(a) 1.25

(b) 0.81

(c) 2.43

(d) 0.8

(e) either 0.8 or 1.25

21. If $\log_4 x = 2.5$, the value of x is

(a) 16

(b) 32

(c) $16\sqrt{2}$

(d) 0.5

(e) none of these

22. Given $a_0 = 1$, $a_1 = 3$ and the general relation $a_n^2 - a_{n-1}a_{n+1} = (-1)^n$ for $n = 1, 2, 3, \ldots$. Then a_3 is equal to

(a) $\frac{13}{27}$

(b) 33

(c) 21

(d) 10

(e) -17

23. The 12th term of the geometric progression $9, 3\sqrt{3}, 3, \ldots$ is

(a) $9\sqrt{3}$

(b) $\frac{\sqrt{3}}{81}$

(c) $\frac{\sqrt{3}}{243}$

(d) $\frac{1}{\sqrt{3}}$

(e) none of these

24. Solve the inequality $|2x - 5| \geq 10$

(a) $x = \frac{5}{2}$ or $x = \frac{15}{2}$

(b) $-\frac{5}{2} \leq x \leq -\frac{15}{2}$

(c) $x \geq \frac{15}{2}$ or $x \leq -\frac{5}{2}$

(d) $x = -\frac{5}{2}$ or $x = \frac{15}{2}$

(e) $-\frac{15}{2} \leq x \leq \frac{5}{2}$

25. Solve $\frac{3}{2(x-1)} + (x + 1) \leq \frac{1}{2}$

(a) $x < 1$

(b) $x > 1$

(c) $x \leq 1$

(d) $x \geq 1$

(e) none of these

26. If r_1 and r_2 are two roots of $x^2 - 4x - 2 = 0$, then $r_1^2 + r_2^2$ is equal to

(a) 0

(b) 20

(c) 16

(d) 12

(e) none of these

27. If $3(x - 1) = 2(y + 1)$, the minimum value of the expression $5x^2 + 4y^2$ is

(a) $\left(\frac{15}{14}\right)^2$

(b) $\frac{15}{14}$

(c) $\frac{125}{14}$

(d) 25

(e) 0

28. The sum of the first n terms of the series,

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \ldots ,$$ is

(a) $\dfrac{1}{\sqrt{n}+\sqrt{n+1}}$

(b) $\dfrac{2}{\sqrt{n}}$

(c) $\dfrac{n}{\sqrt{n}+\sqrt{n+1}}$

(d) $\sqrt{n+1} - 1$

(e) none of these

29. The number of distinct permutations of all the letters of the word EQUIVALENCE is

(a) $\dfrac{11!}{8!}$

(b) $11!$

(c) $9!$

(d) $10!$

(e) $\dfrac{11!}{3!}$

30. Consider the 12 regions in the following fixed figure formed by the concentric circles and three diameters. In how many ways can these 12 regions be colored with 11 different colors if the following two conditions must be satisfied:

(1) One of the colors must be used twice and each of the others must be used exactly once.

(2) Two adjacent regions whose boundaries have more than one common point cannot have the same color.

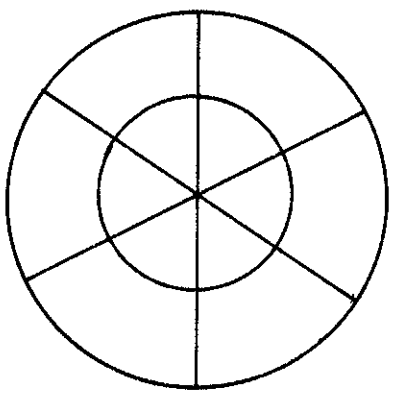
(a) $8 \times 11!$

(b) $11!$

(c) $\dfrac{11!}{2}$

(d) $12!$

(e) 132



31. If five cards are drawn at random from a normal deck of 52 cards, what is the probability that they consist of two aces and three kings?

(a) 1

(b) 0

(c) $\dfrac{\binom{5}{2}\binom{5}{3}}{\binom{52}{5}}$

(d) $\dfrac{\binom{4}{2}\binom{4}{3}}{\binom{52}{5}}$

(e) none of these

32. If $\log_6 (x + 5) = \frac{1}{2}$, then x is equal to

(a) $5 - \frac{1}{2}$

(b) $\frac{1}{2} - 5$

(c) $\sqrt{6} - 5$

(d) $5 - \sqrt{6}$

(e) none of these

33. The inverse $f^{-1}(x)$ of a function $f(x) = -2x + 1$ is

(a) $2x - 1$

(b) $-\frac{1}{2}x + \frac{1}{2}$

(c) $-\frac{1}{2}x + 1$

(d) $-2x + 1$

(e) $-x + \frac{1}{2}$

34. The center of the circle described by $x^2 + y^2 - 4x + 6y + 9 = 0$ is

(a) (3, 2)

(b) (-2, -3)

(c) (4, 6)

(d) (2, 3)

(e) (2, -3)

35. If $a_1b_2 = a_2b_1 \neq 0$, then the graphs of $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are

(a) parallel to the x-axis

(b) parallel to the y-axis

(c) perpendicular

(d) parallel or identical

(e) meeting at a point

36. Determine the solution set of the system:

$$\begin{cases} x - y + 3z = 3 \\ x - 2y - 3z = 0 \\ 2x - 3y + 6z = 5 \end{cases}$$

(a) $x = 0,\ y = 1,\ z = \frac{4}{3}$

(b) $x = 0,\ y = -1,\ z = \frac{2}{3}$

(c) $x = 2,\ y = 1,\ z = \frac{2}{3}$

(d) $x = 3,\ y = 1,\ z = \frac{1}{3}$

(e) $x = 1,\ y = -3,\ z = -\frac{1}{3}$

37. Determine the number of integers between 10 and 500 satisfying two conditions: (1) The remainder when divided by 6 is 1. (2) The remainder when divided by 8 is 1.

(a) 81

(b) 70

(c) 61

(d) 490

(e) 20

38. Determine the value or values for x so that $x - 4$, $x - 1$ and $2x - 2$, in that given order, form a geometric progression.

(a) $x = 4$ or $x = 1$

(b) $x = 7$

(c) $x = -7$

(d) $x = -4$ or $x = -1$

(e) $x = 7$ or $x = 1$

39. Solve the equation $5^{2x} = 5^x + 6$. The value of x is

(a) $\log_5 6$

(b) $\log_5 3$

(c) $\log_5 2$

(d) 0

(e) none of these

40. One day Charlie Brown was piloting his motor boat upstream (against the current). He had gone one mile from the dock when he passed a bottle floating downstream. He continued to go upstream for 5 minutes until he became curious about the contents of the bottle. He quickly turned around and headed downstream (at the same throttle) and overtook the bottle exactly opposite the dock from which he had started his trip. How fast was the current?

(a) 30 mph

(b) 12 mph

(c) $\sqrt{10}$ mph

(d) 6 mph

(e) none of the above