ALGEBRA II TEST   EDITED BY:

1972   Hugh L. McHenry
                                         Department of Mathematics
Scoring Formula: 4R-W   Memphis State University

   This test was prepared from a list of Algebra II questions

submitted by eleven colleges and universities across Tennessee.

DIRECTIONS:

   Do not open this booklet until you are told to do so.

   This is a test of your competence in high school algebra. For
each problem there are listed 5 possible answers; one and only
is correct. You are to work each problem, determine the correct
answer, and indicate your choice by making a heavy black mark in the
correct place on the separate answer sheet provided. You must use
a pencil with soft lead (No. 2 lead or softer). A sample problem
follows:

1. If $2x = 3$, then x equals   1   2   3   4   5

    (1). 2/3   (2). 3   (3). 6
                                       1. □   □   □   ■   □
    (4). 3/2   (5). none of these

   The correct answer for the sample problem is 3/2. which is answer
(4); so you would answer this problem by making a heavy black mark
under space 4 as indicated above.

   If you should change your mind about an answer, be sure to erase
completely. Avoid wild guessing, as wrong answers count against you.
Do not mark more than one answer for any problem. Make no stray
marks of any kind on your answer sheet.

   When told to do so, open your test booklet to page 2 and begin.
When you have finished one page, go on to the next. The working
time for the entire test is 80 minutes.

1. Find the value of the following determinant: $\begin{vmatrix} -2 & 0 & 6 \\ 5 & 3 & -4 \\ -1 & 0 & 7 \end{vmatrix}$

    (1). -24

    (2). 14

    (3). 0

    (4). -60

    (5). -8

2. $.72\overline{72}$ is the same as

    (1). 720/999

    (2). 7272/10,000

    (3). 24/33

    (4). 72/100

    (5). none of these

3. When $x^{13} + 1$ is divided by $x - 1$, the remainder is:

    (1). 1

    (2). -1

    (3). 0

    (4). 2

    (5). none of the above

4. The prime factored form of $m^3n + m^3 + m^2n + m^2$ is:

    (1). $m^2(mn + m + n + 1)$

    (2). $(m^2 + mn)(m + m^2n)$

    (3). $(m^3 + m^2)(n + 1)$

    (4). $m^2(m + 1)(n + 1)$

    (5). none of these

5. Which one of these relations is a function if x is in the domain?

    (1). $y = x^2 + 1$

    (2). $y^2 = x - 1$

    (3). $y^2 = 25 - x^2$

    (4). $x = 3$

    (5). $x = |y|$

6. The equation, $2x^2 - 3x = 7$, has:

    (1). two real roots that are equal

    (2). two imaginary roots

    (3). two integral roots

    (4). two irrational roots

    (5). no solution

7. The maximum safe load w of a horizontal beam supported at both
    ends varies directly as the breadth x and the square of the
    depth y and inversely as the length z between the supports.
    If a beam 10 ft. long, 2 in. wide, and 4 in. deep can support
    a load of 800 pounds, what is the maximum safe load of a beam
    of the same material 15 ft. long, 3 in. wide, and 6 in. deep?

    (1). 2550 lbs.

    (2). 3600 lbs.

    (3). 2500 lbs.

    (4). 1800 lbs.

    (5). none of these

8. If 6 identical coins are tossed together on a table, in how
    many different ways can heads and tails show up?

    (1). 720

    (2). 20

    (3). 64

    (4). 36

    (5). 120

9. Which of the following statements is true with regard to the solution of: $|x|^2 + |x| - 6 = 0$

   (1). The sum of the roots is 0.

   (2). The product of the roots is -6.

   (3). The sum of the roots is -1.

   (4). There is only one root.

   (5). None of the above

10. If m men can do a job in d days, then m + r men can do the job in:

   (1). d + r days

   (2). d - r days

   (3). md/(m + r) days

   (4). d/(m + r) days

   (5). none of the above

11. The graph of the equation 2x - y = 6 is perpendicular to the graph of which of these equations?

   (1). 2x + y = 6

   (2). 2y + x = 4

   (3). -2x + y = -6

   (4). 4x - 2y = 12

   (5). 2y - x = 8

12. The solution of the equation $2x/(2x + 3) = (4x^2 + 1)/(4x^2 + 4x - 3)$ is:

   (1). -1/2

   (2). 1/2

   (3). 3/5

   (4). 4/3

   (5). none of the above

13. The solution set for $\sqrt{x + 5} + \sqrt{x} = 1$ is:

   (1). $\{4\}$

   (2). ϕ

   (3). $\{-4\}$

   (4). $\{4i\}$

   (5).   none of these

14. $\text{Log}_5 \sqrt[3]{25}$ is equal to:

   (1). $5/3$

   (2). $2/3$

   (3). $3/2$

   (4). $3/5$

   (5).   none of these

15. Let f be a function whose domain is the set of all real numbers. Which of the following statements is always true?

   (1). $f(x - y) = f(x) - f(y)$

   (2). $f(\sqrt{x}) = \sqrt{f(x)}$

   (3). $f(|x|) = |f(x)|$

   (4). $f(xy) = f(x)\, f(y)$

   (5).   none of these

16. Let S be the sum of the first nine terms of the sequence:

$x + a,\ x^2 + 2a,\ x^3 + 3a,\ \ldots,\ x^k + ka,\ \ldots.$ Then S equals:

   (1). $(50a + x + x^2)/(x + 1)$

   (2). $50a - \left[(x + x^{10})/(x - 1)\right]$

   (3). $\left[(x^9 - 1)/(x + 1)\right] + 45a$

   (4). $\left[(x^{10} - x)/(x - 1)\right] + 45a$

   (5). $\left[(x^{11} - x)/(x - 1)\right] + 45a$

17. Define an operation * for positive real numbers as
    $a*b = ab/(a + b)$; then $4 * (4*4)$ equals:

    (1). 3/4

    (2). 1

    (3). 4/3

    (4). 2

    (5). 16/3

18. Find all values of k such that $kx^2 - kx + 2 = 0$ has real roots.

    (1). $\{k \mid k \geq 8\}$

    (2). $\{k \mid k \leq 0\}$

    (3). $\{k \mid k \geq 8\} \cap \{k \mid k \leq 0\}$

    (4). $\{k \mid 0 \leq k \leq 8\}$

    (5). $\{k \mid k \leq 0\} \cup \{k \mid k \geq 8\}$

19. The intersection of the solutions sets to $x^2 - 3x + 2 < 0$

    and $x^2 - 3x/2 < 0$ is the set:

    (1). ϕ

    (2). $\{x \mid 0 < x < 3/2\}$

    (3). $\{x \mid 1 < x < 3/2\}$

    (4). $\{x \mid x > 0\}$

    (5). none of these

20. Given that $(x + y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$, what is the value of $\sum_{k=0}^{n} \binom{n}{k}$?

    (1). n^n

    (2). n^k

    (3). 2^n

    (4). nxy

    (5). none of these

21. Solve the following system of equations for x: $1/x + 1/y - 1/z = 6$

$$3/x - 2/y + 1/z = -1$$

$$2/x - 1/y + 2/z = -1$$

   (1). $x = 1/2$

   (2). $x = 1/3$

   (3). $x = 2$

   (4). $x = 1$

   (5). $x = -3$

22. The coefficient of $x^8 y^2$ in the expansion of $(x - 2y)^{10}$ is:

   (1). 45

   (2). -45

   (3). -180

   (4). 90

   (5). 180

23. If $4^x - 4^{x-1} = 24$, then $(2x)^x$ equals:

   (1). $5/2$

   (2). $\sqrt{5}$

   (3). $25\sqrt{5}$

   (4). $3\sqrt{3}$

   (5). $3/2$

24. If x is positive and $\log x \geq \log 2 + (1/2)\log x$, then:

   (1). x can be any real number

   (2). the maximum value of x is 1

   (3). the minimum value of x is 1

   (4). the maximum value of x is 4

   (5). the minimum value of x is 4

25. What is the sum of the digits in the solution to the
     equation $8^{1/6} + x^{1/3} = 7/(3 - \sqrt{2})$?

    (1). 3

    (2). 8

    (3). 9

    (4). 13

    (5). none of these

26. In an experiment, a group of trees is set out in an ell with
     5 trees in each arm plus the common corner tree (for a total
     of 11). The corner tree is diseased. If each of the remaining
     trees has the same chance of becoming infected, what is the
     probability that the next two trees affected will be adjacent
     to each other and the corner tree or on either side of the
     corner tree?

    (1). 1/3

    (2). 3/10

    (3). 3/11

    (4). 1/15

    (5). 1/30

27. Find the sum of the odd integers from 1 to 3,001 inclusive.

    (1). 2,253,001

    (2). 1,362,703

    (3).   543,727

    (4).   252,523

    (5).   50,451

28. A ball is dropped from a height of 18 ft. On the first bounce
     it rises 12 ft. and on each bounce rises 2/3 of the height of
     the preceding bounce. How far will it travel in coming to rest?

    (1). 60 ft.

    (2). 45 ft.

    (3). 90 ft.

    (4). 70 ft.

    (5). none of these

29. If A = {(x,y) | x,y ∈ real numbers and x - y > 0} and

B = {(x,y) | x,y ∈ real numbers and $x^2 + y^2 < \sqrt{2}$} then

the graph of A ∩ B is all points inside:

(1). a circle

(2). a semi-circle

(3). a triangle

(4). a square

(5). a quadrant of a circle

30. If $\tan\left(\dfrac{x}{y}\right) = \sqrt{\dfrac{1 - \cos x}{1 + \cos x}}$, then $\tan\left(\dfrac{x}{y}\right)$ is also equal to:

(1). (1/2) tan x

(2). (sin x) / (1 + cos x)

(3). (cos x) / (1 + sin x)

(4). sec x

(5). none of the above

31. The fraction $\dfrac{a^2 + b^2 - c^2 + 2ab}{a^2 + c^2 - b^2 + 2ac}$ is (with suitable restrictions

on the value of a, b, and c):

(1). reducible to -1

(2). reducible to a polynomial of three terms

(3). irreducible

(4). reducible to $\dfrac{a - b + c}{a + b - c}$

(5). reducible to $\dfrac{a + b - c}{a - b + c}$

32. If the foci and the vertices of the hyperbola $x^2 - y^2 = 16$ become respectively the vertices and foci of an ellipse, the equation of the resulting ellipse is

(1). $x^2 + y^2 = 16$

(2). $\dfrac{x^2}{16} + \dfrac{y^2}{32} = 1$

(3). $\dfrac{x^2}{32} + \dfrac{y^2}{16} = 1$

(4). $\dfrac{x^2}{4\sqrt{2}} + \dfrac{y^2}{4} = 1$

(5). none of these

33. In how many points will the graph for $y = \dfrac{x^2 - 4}{x - 2}$ intersect the graph for $y = 2x$?

(1). one

(2). two

(3). none

(4). several

(5). cannot be determined

34. A milk truck left a town for a city 20 miles away at 8 a.m. A mail truck left the city for the town at 8:15 a.m. The two trucks met at 8:30 a.m., and they met again on their return trip at 9:40 a.m. If each used 30 minutes to make deliveries before starting its return trip, find the average speed of each.

(1). milk truck = 20   mail truck = 40

(2). milk truck = 30   mail truck = 30

(3). milk truck = 40   mail truck = 20

(4). milk truck = 15   mail truck = 50

(5). milk truck = 50   mail truck = 30

35. There are four roads between cities A and B, and three roads between B and C. In how many ways may a person travel from A to B to C and return without passing over any road twice?

    (1). 36

    (2). 12

    (3). 72

    (4). 144

    (5). 14

36. If $3^{2x} - (2) 3^x + 1 = 0$, then $\log_e x$ is:

    (1). 1/2

    (2). -1

    (3). 0

    (4). 1

    (5). none of these.

37. The graph of the equation $4x^2 - 9y^2 - 8x + 36y - 68 = 0$ is:

    (1). a circle with center at (1, 2).

    (2). a hyperbola with center at (1, 2).

    (3). an ellipse with center at (2, 2).

    (4). a paraobla with center at (1, 1).

    (5). none of these.

38. If $4^{\log_9 x} = 1/2$, then $(x) (12)^x$ is equal to:

    (1). $\sqrt{3}$

    (2). $(-\sqrt{3})/12$

    (3). $(\sqrt[4]{12})/4$

    (4). $(\sqrt[3]{12})/3$

    (5). none of these

39. Consider the following system of equations:

$$x^2 - 2xy = 0$$

$$2x^2 - xy + 2y^2 = 18.$$

Which one of the following is not obtained by computing a + b where (a, b) is a solution to the system?

(1). 3

(2). -3

(3). 3/2

(4). -9/2

(5). 9/2

40. $\sin^{-1}x + \cos^{-1}x$ equals:

(1). π

(2). 2π

(3). 0

(4). $\pi/2$

(5). none of these.