

FOURTEENTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHER'S ASSOCIATION

ALGEBRA II TEST

1970

Scoring Formula: 4R-W

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals
- (1) $\frac{2}{3}$ (2) 3 (3) 6
- (4) $\frac{3}{2}$ (5) none of these



The correct answer for the sample problem is $\frac{3}{2}$, which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 30 minutes.

1. Which of the following is not a rational number?

- (1) $\sqrt{4}$ (2) $6/8$ (3) 1.414 (4) $\sqrt[3]{5}$ (5) $0.333 \dots$

2. Which of the following is a factor of $x^3 + 16$?

- (1) $x^2 + 4$ (2) $x + 4$ (3) $x + 2$ (4) $x + 8$ (5) none of these

3. Factor completely: $x^4 - 81$

- (1) $(x^2 + 9)(x - 3)(x + 3)$
(2) $(x - 3)(x^3 + 27)$
(3) $(x + 3)(x^3 - 27)$
(4) $(x^2 - 9)(x^2 + 9)$
(5) None of these

4. If $f(x) = 3x - 1$ and the range of f is the set $\{0, 2, 5\}$, then f expressed as a set of ordered pairs is

- (1) $\{(1/3, 0), (1, 2), (2, 5)\}$
(2) $\{(-1/3, 0), (2, 1), (2, 5)\}$
(3) $\{(0, 1/3), (2, 1), (5, 2)\}$
(4) $\{(3, 0), (1, 2), (2, 5)\}$
(5) Not enough information given

5. Which of the following best characterizes the relation $S = \{(x, y) \mid y = 2x + 3\}$?

- (1) S is a function
(2) S is not a function
(3) S is a constant function
(4) S is a linear function
(5) None of the above

6. How many gallons of 90% alcohol solution must be mixed with an 80% alcohol solution to obtain 1000 gallons of 88% alcohol?

- (1) 200 (2) 500 (3) 800 (4) 1200 (5) None of these

7. The twelfth term of the geometric Sequence $9, 3\sqrt{3}, 3, \dots$, is

(1) $9\sqrt{3}$

(2) $\frac{\sqrt{3}}{81}$

(3) $\frac{\sqrt{3}}{243}$

(4) $\frac{1}{\sqrt{3}}$

(5) none of the above.

8. The expression $\frac{\sqrt{3}-\sqrt{6}}{\sqrt{6}+\sqrt{3}}$ when simplified is

(1) $\frac{-\sqrt{3}}{3\sqrt{3}}$

(2) $-1/3$

(3) $\frac{-\sqrt{3}}{\sqrt{9}}$

(4) $2\sqrt{2}-3$

(5) $3-2\sqrt{2}$

9. Express 0.0002597 in Scientific notation rounded off to three significant figures.

(1) 2.6×10^{-4}

(2) 2.60×10^{-3}

(3) 2.6×10^{-3}

(4) 2.60×10^{-4}

(5) None of these.

10. If $\log_{10}x = .4871 + 3$ and $\log_{10} 3.07 = .4871$, then x equals

(1) 6.07 (2) 9.21 (3) 3070 (4) 0.00307 (5) none of the above.

11. $\log_{10} [\log_5 (\log_2 32)]$ equals

(1) 5 (2) 1 (3) 0 (4) 10 (5) 2

12. Divide $\frac{4-3i}{2+5i}$, write the answer in a + bi form

(1) $\frac{7}{29} + \frac{26i}{29}$ (2) $-7 - 26i$ (3) $\frac{-7}{29} - \frac{26i}{29}$ (4) $\frac{7}{29} - \frac{26i}{29}$

(5) none of these

13. Find a value of k so that the roots of $2x^2 + kx + 18 = 0$ are positive and equal.

- (1) 10 (2) 12 (3) 8 (4) 16 (5) none of these.

14. When one finds the solution set of the system $\begin{cases} x + y + z = 1 \\ 2x + 3y - 4z = 2 \end{cases}$ one finds the

- (1) solution set is empty.
(2) solution set is finite.
(3) solution set is infinite.
(4) solution set consists only of $(1, 0, 0)$.
(5) none of the above.

15. Solve the following system $\begin{cases} 12x^2 + 48y^2 = 192 \\ x - 2y = 4 \end{cases}$

- (1) $\{(0, 4), (-2, 0)\}$
(2) $\{(4, 0), (0, -2)\}$
(3) $\{(4, -2), (0, 0)\}$
(4) \emptyset
(5) none of these.

16. Solve the inequality $x^2 - 2x - 3 < 0$.

- (1) $-1 < x < 3$
(2) $-3 < x < 1$
(3) $0 < x < 3$
(4) $-1 < x < 0$
(5) none of these.

17. Solve for x when $|x - 4| < 10$

- (1) $0 < x < 14$ (2) $-6 < x < 14$ (3) $-6 < x < 0$ (4) $6 < x < 14$
(5) none of these.

18. T varies directly as the square of x and inversely as p . A set of corresponding values is $T = 10$, $x = 6$, and $p = 20$. An appropriate formula for T in terms of x and p is:

(1) $T = \frac{9x^2}{50p}$

(2) $T = \frac{50x}{9p^2}$

(3) $T = \frac{9x}{50p^2}$

(4) $T = \frac{50x^2}{9p}$

(5) None of these

19. If f is a real valued function with an inverse f^{-1} , the graph of f is symmetric to the graph of f^{-1}

(1) With respect to the line $y = x$.

(2) With respect to the line $y = -x$

(3) With respect to the x -axis

(4) With respect to the y -axis

(5) None of the above

20. The shaded portion of the coordinate plane to the right is the graph of

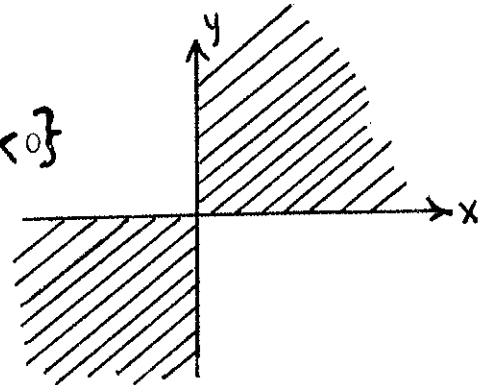
(1) $\{(x, y) \mid xy < 0\}$

(2) $\{(x, y) \mid \frac{x}{y} < 0\}$

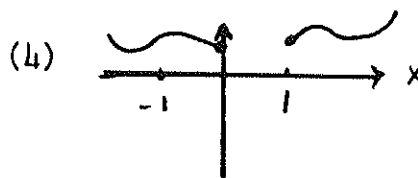
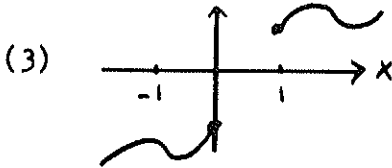
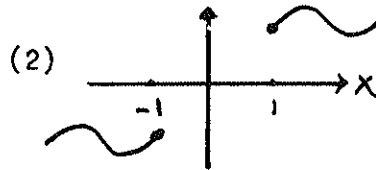
(3) $\{(x, y) \mid x > 0 \text{ and } y > 0\} \cup \{(x, y) \mid x < 0 \text{ and } y < 0\}$

(4) $\{(x, y) \mid |x| \cdot |y| > 0\}$

(5) None of the above



21. Let f be a real valued function with domain D and the properties if $x \in D$ then $-x \in D$ and $f(-x) = -f(x)$. Which of the following could be the graph of f ?



(5) all of the above.

22. Let $A = \{(x, y) \mid |x \cdot y| > 0\}$. If $P = (a, b)$ is an element of A , then

(1) $a = 0$

(2) $b = 0$

(3) P is the origin

(4) $a = 0$ or $b = 0$

(5) $a \neq 0$ and $b \neq 0$

23. For what value of x does $25^x = 1/5$?

(1) 2 (2) -2 (3) 1 (4) 1/2 (5) -1/2

24. If the zeros of a polynomial function f are -3, 0, 1, then

(1) $f(x) = x^2 + 2x - 3$

(2) $f(x) = x^3 - 2x^2 - 3x - 3$

(3) $f(x) = x^3 + 2x^2 - 3x$

(4) $f(x) = x^3 - 2x^2 + 3x$

(5) $f(x) = (x + 1)(x - 3)x$

25. If P is the polynomial $x^2 + x + 1$, then the factors of P are:

(1) $(x + 1)(x + 1)$

(2) $(x + 1)(x - 1)$

(3) $(x - 1)(x - 1)$

(4) P is irreducible with respect to integers.

(5) none of the above.

26. Which of the following is $(x + y)^3$ not equal to:

(1) $x^3 + y^3$

(2) $x^3 + 3x^2y + 3xy^2 + y^3$

(3) $(x + y)^2(x + y)$

(4) $(x + y)(x^2 + 2xy + y^2)$

(5) $(x + y)(x + y)(x + y)$

27. Let $f(x) = x$ and $g(x) = x^2 + 2$. Then $\frac{g(f(x))}{f(x)}$ is

(1) g

(2) x

(3) $\frac{x^2 + 2}{x}$

(4) $x + 2$

(5) $g(x)$

28. In the Binomial Expansion of $(1 - a)^5$ the third term is

(1) $\frac{5!}{2!3!} a^2$

(2) $\frac{-5!}{2!3!} a^3$

(3) $\frac{5!}{2!3!} a^3$

(4) $\frac{-5!}{2!3!} a^2$

(5) none of the above.

29. The prime factorization of $12x^3 - 2x^2 - 4x$ is

- (1) $2x(2x + 1)(3x + 2)$
- (2) $(6x^2 + 4x)(2x + 1)$
- (3) $x(4x + 2)(3x + 2)$
- (4) $(6x + 2)(2x^2 + 2)$
- (5) None of the above

30. $3\sqrt{8} \div 3\sqrt{2}$ equals

- (1) $3\sqrt{4}$
- (2) 3^2
- (3) $3\sqrt{2}$
- (4) $1\sqrt{4}$
- (5) $1\sqrt{6}$

31. Expressed as a single radical with least possible index, $\sqrt[3]{3}\sqrt{2}$ equals

- (1) $\sqrt[6]{6}$
- (2) $\sqrt[3]{108}$
- (3) $\sqrt{6}$
- (4) $\sqrt[4]{64}$
- (5) $\sqrt[6]{72}$

32. Which term in the arithmetic progression 4, 1, . . . is -77?

- (1) 28th
- (2) 16th
- (3) 24th
- (4) 12th
- (5) can not be determined from given information

33. Let $R = \{(1, 2), (2, 1), (3, 3)\}$. Then

- (1) R is an equivalence relation on $X = \{1, 2, 3\}$
- (2) R^{-1} is a function
- (3) R is not a function
- (4) R^{-1} is an equivalence relation on $X = \{1, 2, 3\}$
- (5) $R \neq R^{-1}$

34. The solution set of $x = \sqrt{\frac{3-5x}{2}}$ is
(1) \emptyset (2) $\{-3, 1/2\}$ (3) $\{-3\}$ (4) $\{1/2\}$ (5) $\{3, 1/2\}$
35. Which of the following is the complete real solution of the inequality $(x + 2)^2 (3x - 4) < 0$?
(1) $\{x | x < -2\} \cup \{x | x > -2\} \cup \{x | x \leq 4/3\}$
(2) $\{x | (x + 2)^2 < 0\} \cup \{x | x > 4/3\}$
(3) $\{x | x < 4/3\} \cap \{x | (x + 2)^2 > 0\}$
(4) $\{x | x > -2\} \cup \{x | x \leq 4/3\}$
(5) none of the above.
36. If $S = \{n \in \text{integers} \mid 2 + 4 + \dots + 2n = n(n + 1) + 2\}$, then
(1) $1 \in S$
(2) If $k \in S$, then $k + 1 \in S$.
(3) S is the set of Natural numbers.
(4) All of the above.
(5) None of the above.
37. Solve the following system:
$$\begin{cases} 4x + 2y - 3z = 8 \\ x - 3y - 5z = 0 \\ 3x - y + z = -8 \end{cases}$$

(1) $(-1, 3, 2)$
(2) $(1, 3, -2)$
(3) $(-1, -3, -2)$
(4) $(-1, 3, -2)$
(5) None of these.
38. How many ways can 3 letters be selected from the word WISDOM if repetition of letters is not allowed?
(1) 120 (2) 240 (3) 216 (4) 60 (5) None of these

39. One bag contains 4 red and 6 black balls. Another contains 3 red and 2 black balls. One ball is to be drawn from each bag. What is the probability that both balls are red?

- (1) 1 (2) $6/25$ (3) $7/15$ (4) $13/25$ (5) None of these.

40. A boat, which goes 12 miles per hour in still water, travels on a river where the current is 3 miles per hour. What is the average velocity for a round trip, 60 miles downstream and return, in miles per hour?

(1) $12 \frac{3}{4}$

(2) 12

(3) $11 \frac{1}{4}$

(4) Insufficient data to solve this problem.

(5) None of these.

