TWELFTH ANNUAL MATHEMATICS CONTEST

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ALGEBRA II TEST

1968
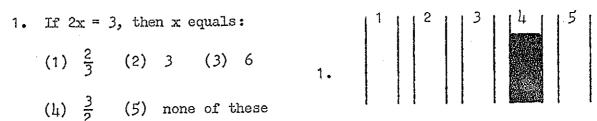
Scoring Formula: 4R - W.

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

   (1) $\frac{2}{3}$ (2) 3   (3) 6

   (4) $\frac{3}{2}$ (5) none of these

1.

The correct answer for the sample problem is $\frac{3}{2}$, which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. When $(0.000016)^{\frac{1}{2}}$ is divided by 400,000, the value is

    (1) 10^{-3} (2) 10^2 (3) .4   (4) 4   (5) 10^{-8}.

2. $x^6 - 64$ is factored completely into real factors as:

    (1) $(x^3 + 8)(x^3 - 8)$

    (2) $(x^3 + 8)(x - 2)(x^2 + 2x + 4)$

    (3) $(x + 2)(x - 2)(x^2 + 2x + 4)(x^2 - 2x + 4)$

    (4) $(x^3 - 8)(x + 2)(x^2 - 2x + 4)$

    (5) none of these.

3. The complete factorization of $a^2 - 2ab + b^2 - x^2 - 2xy - y^2$ is

    (1) $(a - b)^2 (x + y)^2$

    (2) $(a + b - x + y)(a + b - x - y)$

    (3) $(a - b + x + y)(a - b - x - y)$

    (4) $(a - b - x + y)(a + b - x + y)$

    (5) none of these.

4. Determine the value of $(16)^{-\frac{3}{4}} + 5x^0 - (-8)^{\frac{5}{3}}$, where $x \neq 0$.

    (1) $\frac{-287}{8}$ (2) $\frac{297}{8}$ (3) $\frac{224}{8}$ (4) $\frac{-256}{8}$ (5) $\frac{-257}{8}$.

5. Another form of $\left(\dfrac{3x^{-1} - y^{-2}}{x^{-2} + 2y^{-1}}\right)^{-2}$ is:

    (1) $\dfrac{y^2(y + 2x^2)^2}{x^2(3y^2 - x)^2}$

    (2) $\dfrac{3x^2 - y^4}{x^4 + 2y^2}$

    (3) $\dfrac{3x^{-3} - y^{-4}}{x^{-4} + 2y^{-3}}$

    (4) $\dfrac{3x - y^0}{x^0 + 2y}$

    (5) none of these.

6. The numerator of a fraction is 5 less than the denominator. If the numerator is increased by 1 and the denominator by 6, the value of the fraction is unchanged. The fraction is:

(1) $\frac{1}{7}$ (2) $\frac{3}{8}$ (3) $\frac{7}{12}$ (4) $\frac{8}{13}$ (5) $\frac{1}{6}$.

7. When simplified $\dfrac{\dfrac{r^2}{r-t} - 4t}{\dfrac{r-2t}{r^2-t^2}} =$

(1) $(r + 2t)^2 (r + t)$

(2) $(r - 2t) (r + t)$

(3) $\dfrac{(r-2t)^3}{(r+t)}$

(4) $(r + t)$

(5) $(r - t) (r + 2t)$.

8. The quantity H varies jointly as m and t and inversely as d. This statement expressed as an equation would be:

(1) $H = \dfrac{k(m + t)}{d}$

(2) $H = \dfrac{kmt}{d}$

(3) $H = mtd$

(4) $H = \dfrac{kd}{mt}$

(5) none of these.

9. The equation $\sqrt{x - 2} = x - 4$ has

(1) two roots.

(2) one root.

(3) no roots.

(4) three roots.

(5) none of these.

21. The intercepts of the graph of $4x^2 + 9y^2 = 36$ are:

    (1) $(3,2)$, $(-3, -2)$.

    (2) $(0, 2)$, $(0, -2)$, $(0, 3)$, $(0, -3)$.

    (3) $(2, -2)$, $(3, -3)$.

    (4) $(0, 2)$, $(0, -2)$, $(3, 0)$, $(-3, 0)$.

    (5) $(3, -2)$, $(-3, 2)$.

22. If $f(x) = (1 - x)(1 + x)x$ for any real number x, then

    (1) the domain of x is all positive real numbers.

    (2) the range is all positive real numbers.

    (3) the range is all real numbers.

    (4) the range is all negative numbers.

    (5) the range is all real numbers except, 1, 0, -1.

23. If $f : x \longrightarrow \dfrac{x + 1}{x + 2}$, with the necessary restrictions on the domain and range, then:

    (1) $f^{-1} : x \longrightarrow \dfrac{(2x - 1)}{(-x + 1)}$

    (2) $f^{-1} : x \longrightarrow \dfrac{1}{x}$

    (3) $f^{-1} : x \longrightarrow \dfrac{(x + 2)}{(x + 1)}$

    (4) $f^{-1} : x \longrightarrow \dfrac{(x - 1)}{(x - 2)}$

    (5) none of these.

24. The value of k in the equation $x^2 - 4kx + 3k^2 = 0$, such that the sum of the roots is two-thirds of the product of the roots is:

    (1) 6   (2) 2   (3) 6 or -2   (4) -2   (5) 2 or 0.

25. Which of the following relations is a function?

(1) $\{(x,y) \mid x = y^2\}$

(2) $\{(x,y) \mid x^2 \leq y\}$

(3) $\{(x,y) \mid x^2 = y\}$

(4) $\{(x,y) \mid x = 5\}$

(5) $\{(x,y) \mid y > x\}$.

26. If $f(x) = a^x$, where a and x are real numbers, then $f(x + 2) - f(x)$ equals

(1) a^2 (2) $2a$ (3) $2f(x)$ (4) $(a^2 - 1)f(x)$ (5) none of these.

27. The solution of the equation $\log_3 x^2 + \log_3 (x/3) = 0$ is

(1) $\frac{1}{3}$ (2) $\log_3 (1/3)$ (3) 3 (4) 9 (5) $\sqrt[3]{3}$.

28. How many cubic centimeters of a 40% solution of sulfuric acid must be added to 100cc of a 20% solution to obtain a 30% solution of the acid?

(1) 10   (2) 50   (3) 40   (4) 150   (5) 100.

29. A, B and C working together can do a piece of work in one day, A and C together in 1½ days, and B and C together in 2 days. How many days would each of them require to do the work alone?

(1) A in 3 days, B in 2 days, C in 6 days.

(2) A in 3 days, B in 6 days, C in 2 days.

(3) A in 2 days, B in 3 days, C in 6 days.

(4) A in 6 days, B in 3 days, C in 2 days.

(5) None of these.

30. The graph of $x^2 - y^2 = 7$ is

(1) a circle.

(2) an ellipse.

(3) a line.

(4) a parabola.

(5) a hyperbola.

40. Let S be the set of positive integers and have defined on it the usual operations of addition and multiplication. Which of the following pairs of properties hold true on S?

    (1)   Existence of additive identity element and associativity for multiplication.

    (2)   Associativity for multiplication and addition.

    (3)   Existence of multiplicative identity element and multiplicative inverses.

    (4)   Distributive law and existence of additive identity element.

    (5)   Commutativity for subtraction and division.