ELEVENTH ANNUAL MATHEMATICS CONTEST

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ALGEBRA II TEST

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Scoring Formula: 4R - W.

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) $\frac{2}{3}$ (2) 3 (3) 6

(4) $\frac{3}{2}$ (5) none of these.

1. 

The correct answer for the sample problem is $\frac{3}{2}$, which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. A simpler form of $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{3}}$ is:

   (1) $\frac{\sqrt{1}+\sqrt{3}}{\sqrt{1}-\sqrt{3}}$

   (2) $4+\sqrt{15}$

   (3) $\frac{\sqrt{8}}{\sqrt{2}}$

   (4) ∞

   (5) 2.

2. The factors of $5a^2-6ac-15ab+18bc$ are:

   (1) $(a-3b)(5a+6c)$

   (2) $(5a-3b)(a-2c)$

   (3) $(a+3b)(5a-6c)$

   (4) $(a-3b)(5a-6c)$

   (5) none of these.

3. When $\left[\left(\frac{9}{4}\right)^{-\frac{1}{2}}+\left(\frac{2}{3}\right)^{-2}+\frac{1}{3}\left(\frac{x^0}{2}\right)^2\right]^{-\frac{3}{2}}$ is simplified, its value is:

   (1) $\frac{1}{3\sqrt{3}}$

   (2) $3\sqrt{3}$

   (3) 9

   (4) $\frac{1}{9}$

   (5) $\frac{3}{\sqrt{3}}$.

4. If $a+3i=2+3bi$, where i is the imaginary unit, then (a,b) =

   (1) (2,3)

   (2) (2,1)

   (3) (b,a)

   (4) (2,-3)

   (5) (2,2).

5. If the points A(-10, -2) and B(-2, 10) are given, then the coordinates of the midpoint of the line segment AB are:

(1) (-4, -6) (2) (-5,6) (3) (5, -6) (4) (-6, 4) (5) (-4, 6).

6. One factor of $16x^4 + 7x^2y^2 + y4$ is:

(1) $4x^2 + y^2$ (2) $4x^2 + xy - y^2$ (3) $4x^2 - xy - y^2$ (4) $2x + y$ (5) $4x^2 - xy + y^2$.

7. $\sqrt{(x^2 - x^{-2})^2 + 4}$ simplifies to:

(1) $x^2 + x^{-2}$

(2) $x^2 - x^{-2} + 2$

(3) $\frac{1}{x}\sqrt{(x^4 - 1)^2 + 4}$

(4) $\frac{x^2 - 1}{x}$

(5) none of these.

8. If $x = \frac{1}{y}$, $y = 1 - \frac{1}{r}$, and $r = 1 + \frac{1}{s - 1}$, then x equals:

(1) $\frac{r - 1}{r}$

(2) $\frac{s}{s - 1}$

(3) $s - \frac{1}{s}$

(4) $\frac{y}{s - r}$

(5) s.

9. If the order of the digits of a two-digit base ten numeral is reversed, the resulting number is 4 less than twice the original number. If the sum of the digits is 13, the number is:

(1) 49 (2) 76 (3) 85 (4) 67 (5) 58.

10. Given $f(x) = x^2 - 1$ and $g(x) = \frac{x - 1}{x + 1}$. The numerical value of $f\left[\frac{1}{g(3)}\right]$ is:

(1) 3 (2) -2 (3) -3 (4) 8 (5) $\frac{-3}{4}$.

11. If $(x^2 - x)^2 - 6(x^2 - x) = 0$, x equals:

(1) 3, -2 (2) -3, 2 (3) 0,1, -2,3 (4) 0,1,2, -3 (5) none of these.

12. Only one of the following sets is closed with respect to addition, subtraction, multiplication, and division (except by 0). Which one is it?

(1) the set of natural numbers

(2) the set of integers

(3) the set of irrational numbers

(4) the set of real numbers

(5) the set of whole numbers.

13. A solution (a,b,c) of the system $\begin{cases} -a + b + 4c = 0 \\ a + 3b + 8c = 0 \\ a + 2b + 5c = 0 \end{cases}$ is:

(1) (1,-3,1) (2) (-1,-2,1) (3) (2,-2,1) (4) (1,1,0) (5) none of these.

14. The roots of the equation $x + \sqrt{x} = 0$ are:

(1) -1,1 (2) 1,0 (3) 0 (4) 1,1 (5) -2,2.

15. The sum of the infinite progression; 2, 0.2, 0.02, 0.002, ... is:

(1) 2 (2) $\frac{21}{10}$ (3) $\frac{20}{9}$ (4) 3 (5) 2.2.

16. The curves represented by $x^2 + y^2 = 25$ and $x^2 - 3y = 21$ have:

(1) no points in common

(2) four points in common

(3) three points in common

(4) two points in common

(5) none of these.

17. One train traveling 5 miles an hour faster than another train requires 24 minutes less time to travel 180 miles. The rates of the trains are:

(1) 4 mph and 9 mph

(2) 40 mph and 45 mph

(3) -45 mph and -50 mph

(4) 45 mph and 50 mph

(5) -4 mph and -9 mph.

18. The graph of $x^2 + 9y^2 - 6xy - 9 = 0$ is a:

(1) circle (2) ellipse (3) parabola (4) hyperbola (5) pair of parallel lines.

19. The graphs of the functions $y = 2^x$ and $y = \log_2 x$

(1) intersect when x is infinitely large

(2) do not intersect

(3) intersect in a single point

(4) intersect in exactly two points

(5) intersect in an infinite number of points.

20. If $\log_2 x^3 = 2 \log_2 x - 1$, the value of x is?

(1) $\frac{2}{3}$ (2) 1 (3) -4 (4) $\frac{1}{4}$ (5) $\frac{1}{2}$.

21. If a, b, c, and d are selected from the set of natural numbers and $\frac{a}{b} < \frac{c}{d}$, then which of the following statements is not true?

(1) $\frac{a+b}{b} < \frac{c+d}{d}$

(2) $\frac{a}{b} < \frac{a+c}{b+d}$

(3) $\frac{a+c}{b+d} < \frac{c}{d}$

(4) $a + b(d) \geq b(c + d)$

(5) $\frac{c-d}{d} > \frac{a-b}{b}$.

22. The coordinates of the vertex of the parabola $y = x^2 + 6x$ are:

(1) (6,0) (2) (-3, -9) (3) (3,9) (4) (0,0) (5) (1,7).

23. If the lines $3x + by = 6$ and $ax + 2y = 8$ are parallel, then the product ab has the value of:

(1) -6 (2) -1 (3) 6 (4) 1 (5) $\frac{3}{2}$.

24. If $(x - 2)^2 < 4$, then the solution set is:

(1) $0 < x < 2$

(2) $x > 2$

(3) $2 < x$

(4) $x - 2 < 2$

(5) $0 < x < 4$.

25. The solution set of $|2x + 3| \geq 5$ is:

(1) $-4 < x < 1$

(2) $-4 \leq x \leq 1$

(3) $x \geq 1$ or $x \leq -4$

(4) $x \geq 1$ and $x \leq -4$

(5) $x > 1$ and $x < -4$.

26. The square root of 0.0000016 is:

(1) 4×10^{4} (2) 0.4×10^{-4} (3) 4×10^{-4} (4) $\sqrt{1.6 \times 10^{-3}}$ (5) $\sqrt{1.6 \times 10^{3}}$.

27. The 4th term of an arithmetic progression is 7 and the 9th term is 22. The 15th term is:

(1) 24 (2) 40 (3) 32 (4) 30 (5) 14.

28. If the greatest common divisor of a and b is c then the least common multiple of a and b is:

(1) ab (2) $\frac{c}{a}$ (3) $\frac{ab}{c}$ (4) $\frac{bc}{a}$ (5) ac.

29. Which one of the following statements concerning inequalities is not always true?

(1) If $a > b$, then $an > bn$.

(2) If $a > b$, then $a - n > b - n$.

(3) If $a > b$, then $a + n > b + n$.

(4) If $a > b$, then $\frac{a}{n} > \frac{b}{n}$, if $n > 0$.

(5) If $a \leq b$, then $\frac{a}{n} \geq \frac{b}{n}$, if $n < 0$.

30. If only one of the following is not a property of an equivalence relation, which is it?

(1) symmetric

(2) transitive

(3) infinitive

(4) reflexive

(5) none of these is a property of an equivalence relation.

31. The set of real numbers for which $|3x - 1| < 5$ is:

(1) $x > \frac{-4}{3}$

(2) $x < 2$

(3) $-2 < x < \frac{4}{3}$

(4) $\frac{-4}{3} < x < 2$

(5) none of these.

32. If $f(x) = ax^2 + bx + c \geq 0$ for all real x then the graph of the function intersects the x-axis in:

(1) no point

(2) two distinct points

(3) at most one point

(4) exactly one point

(5) at least one point.

33. If twelve distinct points in the plane are located so that no three are collinear, how many different quadrilaterals (not necessarily convex) may be formed using any four of them as vertices?

(1) 12 (2) 144 (3) 1320 (4) 64 (5) 495.

34. The middle term of $\left[\frac{2}{3\sqrt{x}} - 3\sqrt{x}\right]^6$ is:

(1) $160x$ (2) -160 (3) $-20x$ (4) 20 (5) 120.

35. If a cubic equation, with real coefficients, has a root of $3 - i$ and if the product of the roots is -5, then the real root is:

(1) $3 - i$ (2) 5 (3) $\frac{1}{2}$ (4) -5 (5) $\frac{-1}{2}$.

36. A box contains m red balls and n white balls. One ball is drawn at random, its color is noted and then it is placed back in the box. Then a second ball is drawn at random. What is the probability that the two balls are of different colors?

(1) $\frac{m + n}{m^2 + n^2}$

(2) $\frac{2mn}{(m + n)^2}$

(3) $\frac{2mn}{m^2 + n^2}$

(4) $\frac{mn}{(m + n)^2}$

(5) $\frac{2mn}{m + n}$.

37. Given $Ex^2 + Fx + D = 0$ and $F = ED$. The roots are real if:

(1) F is positive

(2) F is imaginary

(3) $F > 2$

(4) $F \geq 4$

(5) $E = E = C$

38. The solution of the system, $2x^2 - 3y^2 = 5$ and $3x^2 - 2y^2 = 30$ are the coordinates of the:

(1) two points of intersection of a line and a circle

(2) four points of intersection of an ellipse and a circle

(3) four points of intersection of two ellipses

(4) four points of intersection of two hyperbolas

(5) four points of intersection of a parabola and an ellipse.

39. One root of the equation $x^3 - 6x^2 + 9x - 2 = 0$ is.

(1) 3

(2) -2

(3) $-2 - \sqrt{3}$

(4) $1 + \sqrt{2}$

(5) $2 + \sqrt{3}$.

40. The surface area of a sphere varies as the square of the radius, and volume of a sphere varies as the cube of its radius. If the surface area of the sphere is trebled, then the volume is multiplied by:

(1) $\sqrt{3}$

(2) 3

(3) $3\sqrt{3}$

(4) 9

(5) none of these.