EIGHTH ANNUAL MATHEMATICS CONTEST

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ALGEBRA II TEST

1964

Scoring Formula: 4R - W.

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

   (1) 2/3; (2) 3; (3) 6;

   (4) 3/2; (5) none of these.

The correct answer for the sample problem is "3/2", which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Evaluate: $(9^{1/2} + 4^{1/2})^2$

(1) 5; (2) 13; (3) 25; (4) 14/3; (5) 20.

2. The difference between $(\sqrt{125} + \sqrt{24})$ and $(\sqrt{96} + \sqrt{20})$ may be written:

(1) $3\sqrt{5} - 2\sqrt{6}$ (2) $3\sqrt{5} - 6\sqrt{6}$ (3) $7\sqrt{5} + 2\sqrt{6}$

(4) $\sqrt{149} - \sqrt{116}$ (5) $\sqrt{33}$

3. Which of the following is largest?

(1) $\frac{3}{2^{4/3}}$ (2) $2^{2/3}$ (3) $\frac{2}{3^{1/3}}$

(4) $\frac{4}{2^{5/3}}$ (5) $(1/2)^{3/2}$

4. If $N = \frac{u^{1/3}v^{2/3}}{w^{1/2}}$ and u = 8, v = 27, and w = 36, then N equals

(1) $\sqrt{6}$ (2) 9 (3) $3\sqrt{6}$ (4) 3 (5) 1

5. Which of the following satisfy $\sqrt{2x - 5} + (2x + 5)^{1/2} = \sqrt{4x + 20}$

(1) $\sqrt{5}$ (2) $\pm\frac{5\sqrt{5}}{2}$ (3) $\frac{5\sqrt{5}}{2}$ (4) $-\frac{5\sqrt{5}}{2}$ (5) $5\sqrt{5}$

6. The quantity $1 + y$ can be divided without a remainder by

(1) $1 + y^{1/3}$ (2) $1 + \sqrt[3]{y} + \sqrt[3]{y^2}$ (3) $y^{1/3}$

(4) $1 - y^{1/3}$ (5) $1 + \sqrt[3]{y} - \sqrt[3]{y^2}$

7. The sum of the prime factors of $2x^3 - x^2 - 3x$ is

(1) $4x - 4$ (2) $4x - 2$ (3) $x(2x - 3)(x + 1)$

(4) $2x^2 - 2x + 1$ (5) None of these

8. An expression that is equivalent to $\frac{r^{-1} + s^{-1}}{r^{-3} + s^{-3}}$ is

(1) $-\frac{1}{r^{-2} + s^{-2}}$ (2) $\frac{(s^3 - r^3)(s - r)}{r^4 s^4}$ (3) $-(x^{-4} - s^{-4})$

(4) $\frac{r^2 s^2}{s^2 + sr + r^2}$ (5) $\frac{r^2 s^2}{s^2 - sr + r^2}$

9. An expression equivalent to $\frac{2\sqrt[3]{2}}{5\sqrt{3}}$ is

(1) $\frac{2\sqrt[3]{6}}{45}$ (2) $\frac{4}{5}$ (3) $\frac{6\sqrt[3]{2}}{15}$

(4) $\frac{2\sqrt[3]{6}}{15}$ (5) $\frac{2\sqrt[3]{18}}{15}$

10. If $2^{\frac{n}{2}+1} = 32$, then n equals

(1) 5 (2) 6 (3) 7 (4) 8 (5) None of these

11. Log $(b\sqrt[3]{a})$ equals

(1) $\frac{3}{2}\log(a + b)$ (2) $\frac{1}{2}\log a + 3\log b$ (3) $\frac{3}{2}\log(ab)$

(4) $3\log(\sqrt{a} \cdot b)$ (5) None of these

12. $8a^3 - 27b^3$ may be expressed as

(1) $(2a + 3b)(4a^2 - 6ab + 9b^2)$ (2) $(2a + 3b)(4a^2 - 9b^2)$

(3) $(2a - 3b)(4a^2 + 6ab + 9b^2)$ (4) $(4a^2 + 9b^2)(2a - 3b)$

(5) $(2a - 3b)(4a^2 + 12ab + 9b^2)$

13. The solution set for $2x + 10 = x^2$ is

(1) $\{-1 + \sqrt{11}, -1 - \sqrt{11}\}$ (2) $\{-2 + 2\sqrt{44}, -2 - 2\sqrt{44}\}$

(3) $\{2 + 2\sqrt{11}, 2 - 2\sqrt{11}\}$ (4) $\{1 + \sqrt{11}, 1 - \sqrt{11}\}$

(5) $\{1 + 2\sqrt{11}, 1 - 2\sqrt{11}\}$

14. Simplify: $\left(\frac{64\ r^{-6}s^3t^6}{343\ a^3b^{-9}c^0}\right)^{2/3}$

(1) $\frac{16\ b^6s^2t^4}{49\ a^2r^4}$ (2) $\frac{16\ r^{-4}s^2t^2}{49\ a^2b^{-6}}$ (3) $\frac{16\ b^2s^2t^4}{49\ a^2r^4}$

(4) $\frac{4\ r^6s^2t^4}{7\ a^2b^4}$ (5) $\frac{4\ b^6s^2t^4}{7\ a^2r^4}$

15. Find the coefficient of x^3 in the expansion of $(x - 1/3)^8$.

(1) $\frac{243}{56}$ (2) $\frac{56}{243}$ (3) $-\frac{14}{243}$ (4) $-\frac{56}{243}$

(5) None of these

16. The middle term of the expansion of $\left(\frac{a}{2} + \frac{2}{a}\right)^6$ is

(1) 1 (2) 0 (3) $\frac{2}{a}$ (4) $\frac{a}{2}$ (5) 20

17. If $x^5 - 2x^4 - x^2 + 6$ is divided by $x - 2$ the remainder is

(1) 16 (2) -2 (3) 4 (4) 2 (5) None of these

18. The factored form of $a^3 - 9a^2b + 27ab^2 - 27b^3$ is

(1) $(a + 3b)(a^2 + 6ab + 9b^2)$ (2) $(a - 3b)(a^2 - 3ab + 9b^2)$

(3) $(a + 3b)^3$ (4) $(a - 3b)^3$ (5) $a^3 - (3b)^3$

19. For the roots of the equation $9x^2 + 4x + k = 0$ to be equal, k must be

(1) 0 (2) 1 (3) 1/4 (4) - 3/4 (5) 4/9

20. If the domain of the function $f(x) = 2x^2 + x + 1/8$ is the set of all real numbers, then the range of the function is the set of all

(1) positive real numbers (2) negative real numbers

(3) non-negative real numbers (4) real numbers

(5) non-positive real numbers

21. If m and n are positive integers which of the following is always a positive integer?

(1) $m \cdot n$ (2) m/n (3) $m - n$ (4) $n - m$

(5) all of the above

22. If one solution of a quadratic equation is $x = 2 - i$, then the equation is

(1) $x^2 - 2ix - 7 = 0$ (2) $x^2 - 4x + 5 = 0$ (3) $x^2 + 4x + 7 = 0$

(4) $x^2 - 2x - 1 = 0$ (5) $x^2 - 4x + 3 = 0$

23. Which equation represents a real ellipse?

(1) $3x^2 - 4 = 2y^2$ (2) $3y^2 = 7 - 2x^2$ (3) $(x-3)(x+3y) = 9$

(4) $x^2 + 3x = y - 4$ (5) $x^2 + 2y^2 + 3 = 0$

24. The sum of the roots of the quadratic equation $9x = 4 - 3x^2$ is

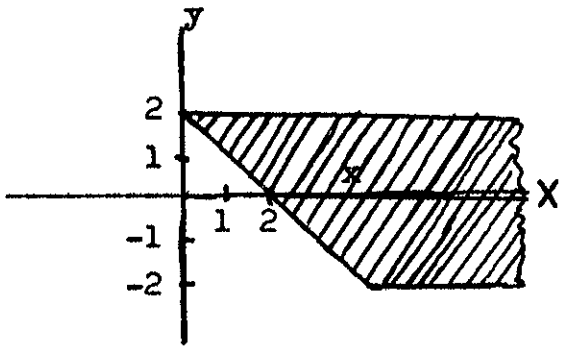
(1) -3 (2) 3 (3) 9 (4) -9 (5) $-4/3$

25. Abe noted that during the first half of a trip that he had averaged 20 miles per hour. What average rate must he make during the second half of the trip if he is to average 30 miles per hour for the entire trip?

(1) 25 (2) 40 (3) 50 (4) 60 (5) none of these

26. The graph below represents

(1) $\{(x,y) \mid x + y \geq 2 \text{ and } |y| \leq 2\}$
(2) $\{(x,y) \mid x + y \geq 2 \text{ or } |y| \leq 2\}$
(3) $\{(x,y) \mid x + y \leq 2 \text{ and } |y| \geq 2\}$
(4) $\{(x,y) \mid x + y \leq 2 \text{ or } |y| \geq 2\}$
(5) none of the above



27. The solutions set of the system of equations $x^2 - 2y^2 = 1$; $x^2 + 4y^2 = 25$ is

(1) $\{(1,0)(3,-2)\}$ (2) $\{(2,5/2)(3,-5/2)\}$ (3) $\{(0,1)(0,0)\}$

(4) $\{(3,4)(-3,-4)\}$ (5) none of these

28. The quadratic equation $4x^2 - 3x + 1 = 0$ has roots that are

(1) conjugate imaginary (2) equal and rational

(3) unequal and rational (4) equal and irrational

(5) unequal and irrational

29. $\log_4 8$ equals

(1) 2 (2) 3/2 (3) 32 (4) 1/2 (5) none of these

30. The set of all x that satisfies $3x + 4 < 5x - 8$ is

(1) $\{x \mid x < 6\}$ (2) $\{x \mid x > 6\}$ (3) $\{x \mid x < 12\}$

(4) $\{x \mid -6 < x < 6\}$ (5) none of these

31. If the sum of two numbers is 10 and their product is 20, the sum of their reciprocals is

(1) 1/10 (2) 1/2 (3) 2 (4) 10 (5) none of these

32. The diagonal of square I is $x + y$. The perimeter of square II with <u>twice</u> the area of I is

(1) $(x + y)^2$ (2) $\sqrt{2}\,(x + y)^2$ (3) $2\,(x + y)$

(4) $\sqrt{8}\,(x + y)$ (5) $4\,(x + y)$

33. John can run around a circular track in 40 seconds. Bill, running in the opposite direction, meets John every 15 seconds. What is Bill's time to run around the track, expressed in seconds?

(1) 12 1/2 (2) 24 (3) 25 (4) 27 1/2 (5) 55

34. If the set $S = \{1, 2, 3\}$ and an operation $*$ is defined by the table

$*$	1	2	3
1	1	2	3
2	2	0	2
3	3	3	1

then a basic property under $*$ which holds is

(1) existence of an identity element (2) commutativity

(3) closure (4) associativity (5) none of these

35. If $A = \{1, 3, 4, 5, 7, 8\}$ $B = \{2, 3, 4, 5, 6, 9\}$ $C = \{1, 3, 5, 9, 10\}$ then $A \cup (B \cap C)$ is

(1) $\{1, 3, 4, 5, 9\}$ (2) $\{3, 5\}$ (3) $\{1, 3, 4, 5, 7, 8, 9\}$

(4) $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ (5) $\{1, 3, 4, 5\}$

36. The 50th term of the arithmetic progression $\frac{1}{2} + \frac{3}{2} + \frac{5}{2} + \ldots$

(1) 51 (2) 50 1/2 (3) 50 (4) 49 1/2 (5) 49

37. The first term of a geometric progression having a sum of 23 1/4, a common ratio of 2, and having 5 terms is

(1) 1 (2) 3/4 (3) 1/2 (4) 1/4 (5) none of these

38. Two swimmers, at opposite ends of a 90 foot pool, start to swim the length of the pool, one at the rate of 3 feet per second, the other at 2 feet per second. They swim back and forth for 12 minutes. Allowing no loss of time at the turns, find the number of times they pass each other.

(1) 24 (2) 21 (3) 20 (4) 16 (5) 18

39. Five times the amount of Al's money added to the amount of Bill's money is more than \$51.00. Three times Al's money minus Bill's money is \$21.00. If A represents Al's money and B represents Bill's money in dollars, then

(1) $A > 9$, $B > 6$ (2) $A > 9$, $B < 6$ (3) $A > 9$, $B = 6$

(4) $A > 9$, but we can put no bounds on B (5) $2A = 3B$

40. If $(x + 1)^2$ is greater than $(5x - 1)$ and less than $(7x - 3)$, find the smallest possible integral value of x.

(1) 1 (2) 2 (3) 3 (4) 4 (5) none of these