SEVENTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA II TEST

1963

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Scoring Formula: 4R - W.

DIRECTIONS:

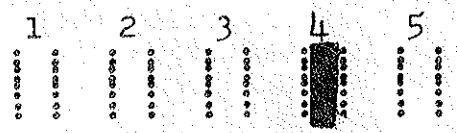
Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) 2/3; (2) 3; (3) 6;

(4) 3/2; (5) none of these.



The correct answer for the sample problem is "3/2", which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If the speed of a railroad train were lessened 4 mph, the train would take 1/2 hour longer to run 180 miles. Which equation satisfies the given conditions?

   (1) $x^2 + 4x - 1440 = 0$ (2) $x^2 - 4x + 1440 = 0$

   (3) $x^2 - 4x - 1440 = 0$ (4) $x^2 + 4x + 1440 = 0$

   (5) None of these

2. When simplified $(y^0)\sqrt{18y^{2n}}\,(3y^n + 1)^{-1}$ reduces to

   (1) $3\sqrt{2}\ y$ (2) 0 (3) $3\sqrt{2}\ y^{-1}$

   (4) y^{-1} (5) none of these

3. If $y = mx + b$, if $y = -1$ when $x = 1$ and if $y = 5$ when $x = 4$, then m and b must have the values

   (1) $m = -2,\ b = 3$ (2) $m = -2,\ b = -3$

   (3) $m = 2,\ b = -3$ (4) $m = 2,\ b = 3$

   (5) None of these

4. Eight students are assigned seats at random in a row. The probability that two particular students A and B, will be seated side by side is

   (1) 1/7 (2) 1/8 (3) 1/56

   (4) 1/4 (5) None of these

5. If P stands for permutations and C for combinations and if $P_4^n = 20(C_2^{n-1})$ then n has the value

   (1) 4 (2) 7 (3) 6

   (4) 5 (5) None of these

6. If one root of $x^2 + px + q = 0$ is 3 times the other root, the relation between p and q is necessarily

   (1) $\frac{p}{q} = \frac{-4}{1}$ (2) $\frac{p}{q} = \frac{-4}{3r}$ (3) $\frac{q^2}{p} = \frac{9}{4}$

   (4) $\frac{p^2}{q} = \frac{16}{3}$ (5) None of these

7. The length of a rectangular field is 3 times the width. The formula for the area A in terms of the perimeter P is

(1) $P = 8\sqrt{\frac{A}{3}}$ (2) $A = \frac{3}{16}P^2$ (3) $A = \frac{3}{64}P^2$

(4) $P = 4\sqrt{\frac{A}{3}}$ (5) None of these

8. When simplified $\dfrac{x - \frac{9}{x}}{\frac{6}{x^2} + \frac{1}{x} - 1}$ reduces to:

(1) $\frac{x(x+3)}{2-x}$ (2) $\frac{-x(x-3)}{2+x}$ (3) $\frac{x(x+3)}{x+2}$

(4) $\frac{-x(x+3)}{x+2}$ (5) None of these

9. The graph of $y = 3x^2 - 2x + k$ cuts the x axis at the point whose abscissa is 3. The value of k is

(1) 21 (2) -21 (3) 33

(4) -33 (5) None of these

10. If each member of the equation $2x - 3 = x - 2$ is multiplied by $x - 4$, what extraneous root is introduced?

(1) -1 (2) 0 (3) 4

(4) -4 (5) None of these

11. Since a circle is a special case of an ellipse it would be reasonable to surmise that the area of an ellipse of major axis a and minor axis b is

(1) πab (2) $\frac{\pi ab}{4}$ (3) $\frac{\pi ab}{2}$

(4) $2\pi ab$ (5) None of these

12. The value of $\log_{32} 56 - \log_{32} 7$ is

(1) 3/5 (2) 8 (3) 1/4

(4) 3/2 (5) None of these

13. If $\log_a b = x$ and $\log_b a = y$, then xy is

(1) ab (2) 0 (3) a + b

(4) 1 (5) None of these

Here 14. The reciprocal of $-3 + 4i$ in the form of $a + bi$ is

(1) $\frac{-3}{25} - \frac{4i}{25}$ (2) $\frac{1}{-3 + 4i}$ (3) $3 - 4i$

(4) $\frac{3}{25} + \frac{4i}{25}$ (5) None of these

15. The point A $(12 + 5i)$ lies on a circle whose center is the origin O. If the radius OA is rotated counterclockwise through an angle of $270°$, what complex number will express the outer extremity of the radius?

(1) $12 - 5i$ (2) $5 + 5i$ (3) $5 - 12i$

(4) $-5 + 12i$ (5) None of these

16. If you know that 1 is a root of $x^3 + ax + 6 = 0$, then the quadratic equation which the other two roots must satisfy is

(1) $x^2 + x - 6 = 0$ (2) $x^2 - ax + 6 = 0$ (3) $x^2 - x - 6 = 0$

(4) $x^2 + x + 6 = 0$ (5) None of these

17. If $3x^4 + kx^3 + x^2 - 16x + 4$ is divided by $x - 2$, for what value of k will the remainder be 8?

(1) $\frac{-16}{5}$ (2) 0 (3) 3

(4) -2 (5) None of these

18. If $\frac{10}{2 + i} = x + 3i + yi$, x and y being real, the value of x and y is

(1) $x = 4$, $y = -5$ (2) $x = 4$, $y = \frac{-16}{5}$

(3) $x = 0$, $y = 5$ (4) $x = \frac{16}{5}$, $y = -4$

(5) None of these

19. The set of all solutions in the form (x, y) of

$$\begin{vmatrix} 0 & x & y \\ x & 0 & y \\ x & y & 0 \end{vmatrix} = 0 \quad \text{is}$$

(1) $\{(0,0)\}$ (2) $\{(0,1), (1,0), (0, 0)\}$

(3) $\{(x, -x)\}$ (4) $\{(0, y), (x, 0)\}$

(5) None of these

20. The set of all x that satisfies $|2x - 3| > 5$ is

(1) $\{x \mid x > 4\}$ (2) $\{x \mid -1 < x < 4\}$ (3) $\{x \mid x < -1 \text{ or } x > 4\}$

(4) $\{x \mid x < -4 \text{ or } x > 1\}$ (5) None of these

21. The set of all x that satisfies $x + 5 < 2x$ is

(1) $\{x \mid x > 5\}$ (2) $\{x \mid x < -5/3\}$ (3) $\{x \mid x < 5\}$

(4) $\{x \mid -5/3 < x < 5\}$ (5) None of these

22. The set of all x that satisfies $\frac{x - 1}{x - 3} > \frac{x + 3}{x + 1}$ is

(1) $\{x \mid -1 < x < 3\}$ (2) $\{x \mid x < -3 \text{ or } x > 1\}$

(3) $\{x \mid x < -1 \text{ or } x > 3\}$ (4) $\{x \mid -3 < x < 1\}$

(5) None of these

23. The middle term of $\left(\frac{2}{3}\sqrt{x^{-1}} - 3\sqrt{x}\right)^6$

(1) $160x$ (2) 240 (3) $-240x^{-1}$

(4) -160 (5) None of these

24. If the square root of x varies directly as y and inversely as the square of z and if $x = 16$ when $y = 24$ and $z = 2$, then when $x = 9$ and $y = 2$, z equals

(1) $\frac{2}{3}$ or $\frac{-2}{3}$ (2) $\frac{2}{3}$ (3) 1 or -1

(4) 1 (5) None of these

25. Given $V = 1/6\ abc$, $a = b = c$ and $a^2 = 2e^2$ then V in terms of e is

(1) $V = \frac{\sqrt{2}\, e^3}{3}$ (2) $V = \frac{e^3}{3}$ (3) $V = \frac{\sqrt{2}\, e^4}{6}$

(4) $V = \frac{\sqrt{-4}\, e^4}{6}$ (5) None of these

26. $(x - x^2)^3 + (x^2 - 1)^3 + (1 - x)^3$ may be factored as follows

(1) $3x(1 - x)^3 (x + 1)$ (2) $3x(x - 1)^3 (x + 1)$

(3) $3x(x - 1)^3 (x - 1)$ (4) $3x(x + 1)^3 (x - 1)$

(5) None of these

27. A set A is said to be mapped into a set B if each member of A is paired with one and only one member of B. If X is the set $\{0, 1, 2\}$ and Y is the set $\{0, 1\}$, then X may be mapped into Y in how many distinct mappings?

(1) 1 (2) 6 (3) 2

(4) 8 (5) None of these

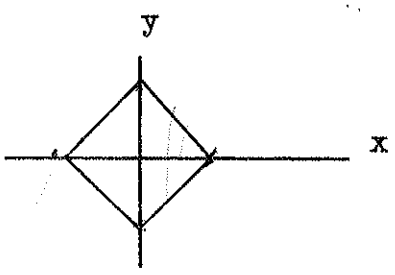
28. The graph of $3x^2 - 5xy - 2y^2 = 0$ is

(1) an ellipse (2) a parabola (3) hyperbola

(4) 2 straight lines (5) None of these

29. The graph of $|x| + |y| = 1$ looks like

(1)



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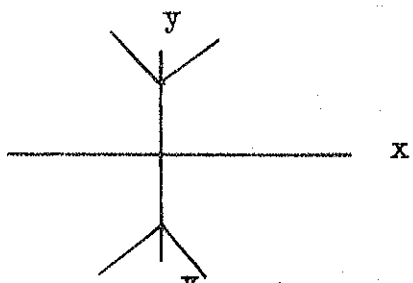

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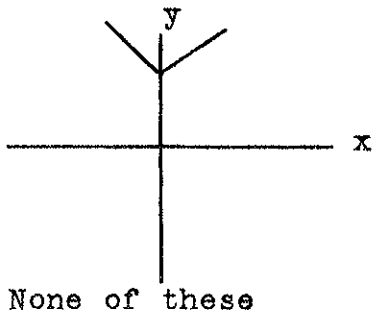

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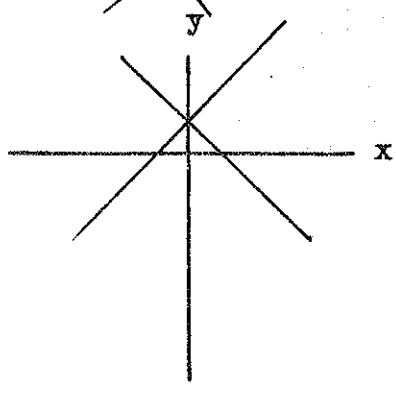

(5) None of these

30. A quadratic expression $ax^2 + bx + c$ has the value 3 when $x = 0$, and the value 1 when x is either $(2 + \sqrt{2})$ or $(2 - \sqrt{2})$. The expression may be written as

(1) $x^2 - 4x + 3$ (2) $4x^2 - x + 3$ (3) $x^2 - 4x + 3 = 0$

(4) $4x^2 - x + 3 = 0$ (5) None of these

31. The function $f(x) = ax^2 + bx + c \geq 0$ for all real x, cuts the x axis at

(1) at least one point (2) no point

(3) at most one point (4) exactly one point

(5) None of these

32. There are 15 points in a plane and no three of these points lie in the same straight line. The number of triangles which can be formed by joining the points is

(1) 455 (2) 45 (3) 500

(4) 276 (5) None of these

33. Find the coefficient of a^4b^3c in the expansion of $(a + b + 2c)^8$

(1) 280 (2) 560 (3) 1120

(4) 140 (5) None of these

34. If eight coins are tossed simultaneously, what is the chance that at least one will turn head up?

(1) $\frac{1}{256}$ (2) $\frac{7}{8}$ (3) $\frac{63}{64}$

(4) $\frac{511}{512}$ (5) None of these

35. Two gamblers A and B are playing a game of chance and each player has staked 32 dollars. They are playing for 3 points, but when A has gained 2 points and B one point, they decide to stop playing. In what ratio should they divide the 64 dollars?

(1) 3: 1 (2) 2 : 1 (3) 1 : 1

(4) 4 : 1 (5) None of these

36. Let f be a function whose domain is the set of all real numbers. Which of the following statements is always true?

(1) $f(x - y) = f(x) - f(y)$ (2) $f(\sqrt{x}) = \sqrt{f(x)}$

(3) $f(|x|) = |f(x)|$ (4) $f(x) \cdot f(y) = f(xy)$

(5) None of these

37. The sum of the roots of $x^2 + bx + c = 0$ is 4 and their product is 1. The correct expression is

(1) $x^2 + 4x + 1 = 0$ (2) $x^2 - 4x + 1 = 0$

(3) $x^2 - 5x - 1 = 0$ (4) $x^2 - 4x - 1 = 0$

(5) None of these

38. The operations of $\cup$ and $\cap$ in the algebra of sets may be considered somewhat analagous to the operations of $+$ and x in the Algebra of real numbers. Which of the following true properties in the algebra of sets is false for the corresponding property in the algebra of real numbers.

(1) $A \cap B = B \cap A$ (2) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(3) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(4) $A \cup (B \cup C) = (A \cup B) \cup C$

(5) None of these

39. If $5x - 3$, $x + 2$ and $3x - 11$ form an arithmetic progression, then the sum of the first six terms is

(1) -87 (2) -33 (3) 177

(4) 54 (5) None of these

40. The geometric mean between $5\sqrt{2} + 1$ and $5\sqrt{2} - 1$ is

(1) 1 (2) $5\sqrt{2}$ (3) 5

(4) 7 (5) None of these