TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-FIFTH ANNUAL MATHEMATICS CONTEST

2023

Algebra I/Integrated Math I

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Scoring Formula: $4 \times$ (Number Right) $-$ (Number Wrong) $+ 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. For $f(x) = |x - 3|$, what is the value of $f(2) - f(3) - f(4)$?
    a. -3
    b. -2
    c. -1
    d. 0
    e. 1

2. What is the next term of the sequence, 2, 3, 4, 6, 6, 12, 8, 24, 10, ___?
    a. 12
    b. 24
    c. 36
    d. 48
    e. 60

3. What is the value for x that satisfies the equation: $\frac{2}{3}x - 9 = \frac{5}{7}x - 12$?
    a. $x = 3$
    b. $x = 4$
    c. $x = 28$
    d. $x = 63$
    e. $x = 72$

4. What are all solutions to the equation $x^4 - x^2 - 6 = 0$?
    a. $x = -3, -2, 2, 3$
    b. $x = -\sqrt{3}, -\sqrt{2}, \sqrt{2}, \sqrt{3}$
    c. $x = -3, x = 2$
    d. $x = -2, x = 3$
    e. $x=-\sqrt{3}, x = \sqrt{3}$

5. The population of a city in 2000 was 615,901. The population of the city decreases by 2.5% per year. What function models the city's population when x = number of years after 2000 and f(x) = city's population x years after 2000?
    a. $f(x) = 615901(-2.5)^x$
    b. $f(x) = 615901(1.025)^x$
    c. $f(x) = 615901(1.025)^{x-2000}$
    d. $f(x) = 615901(0.975)^x$
    e. $f(x) = 615901(0.975)^{x-2000}$

6. What is the value for x that satisfies the equation: $9^{3x+1} = 3^{4x+15}$?
    a. $x = 14$
    b. $x = 7$
    c. $x = 1.4$
    d. $x = 6.5$
    e. $x = 2$

7. The sum of two integers is 10 and the sum of their squares is 52. What are the integers?
    a. 4 and 6
    b. -6 and -4
    c. 3 and 7
    d. -7 and -3
    e. 5 and 5

8. In the second quarter of a recent game, then Bruins scored 21 points by making only two- and three-point baskets. If they made nine baskets, how many of each type did they score?
    a. 3 two-point baskets, 6 three-point baskets
    b. 4 two-point baskets, 5 three-point baskets
    c. 5 two-point baskets, 4 three-point baskets
    d. 6 two-point baskets, 3 three-point baskets
    e. 7 two-point baskets, 2 three-point baskets

9. My car has an 18-gallon gas tank. For the first quarter of the tank of gas, I drove 160 miles. For the second quarter of the tank, I drove 150 miles. For the third quarter of the tank, I drove 100 miles. Assuming I could end the tank of gas at a gas station, how many miles could I drive and maintain an average fuel economy of 30 miles driven per gallon of gas used?
    a. 135 miles
    b. 230 miles
    c. 130 miles
    d. 140 miles
    e. 240 miles

10. For the data set below, what could the missing value be so that the mean and median are equal?

65, 72, 83, 94, ___

I. 11
II. 46
III. 101
IV. 156

a. II only
b. III only
c. II & III only
d. II, III, & IV only
e. I, II, III, & IV

11. What is an equation of the line passing through the point (3, 5) and through the vertex of the parabola given by $y = (x - 7)^2 + 2$?

a. $y - 5 = \frac{3}{10}(x - 3)$
b. $y - 5 = \frac{10}{3}(x - 3)$
c. $y - 5 = \frac{-3}{4}(x - 3)$
d. $y - 5 = \frac{-4}{3}(x - 3)$
e. $y - 5 = \frac{3}{4}(x - 3)$

12. Which function best models the data in the table below?

x	y
2	2
3	4
4	8
5	16

a. $y = 2x - 2$
b. $y = x^2 - 2x$
c. $y = 2^{x-1}$
d. $y = 2^x$
e. $y = 4x - 4$

13. If a and b are the zeros to the function $f(x) = x^2 - 11x + 30$. What is the value of $a + b - ab$?
    a. 19
    b. -19
    c. 41
    d. -41
    e. 1

14. An Uber ride costs $2.25 per mile, rounded up to the next mile, plus a flat fee of $4.75. Which of these is not a possible fare for this ride?
    a. $7.00
    b. $16.00
    c. $25.00
    d. $33.00
    e. $43.00

15. What is the minimum value of the function $f(x) = 3x^2 - 9x + 8$?
    a. 62
    b. 28.25
    c. 8
    d. 1.5
    e. 1.25

16. What is a recursive definition for the sequence for $\frac{27}{2}, 9, 6, 4, \frac{8}{3}, \ldots$?
    a. $\begin{cases} t_1 = \frac{27}{2} \\ t_n = \frac{2}{3} t_{n-1} \end{cases}$
    b. $\begin{cases} t_1 = \frac{27}{2} \\ t_n = \frac{3}{2} t_{n-1} \end{cases}$
    c. $\begin{cases} t_1 = \frac{27}{2} \\ t_n = t_{n-1} - 2 \end{cases}$
    d. $\begin{cases} t_1 = \frac{27}{2} \\ t_n = t_{n-1} - 3 \end{cases}$
    e. $\begin{cases} t_1 = \frac{27}{2} \\ t_n = \frac{4}{3} t_{n-1} \end{cases}$

17. For $f(x) = 4x + 11$, what is the value of $\frac{f(x)-f(3)}{x-3}$?
    a. 1
    b. 4
    c. 8
    d. 23
    e. cannot be determined

18. What happens to selected measures of central tendency for the data set 3, 5, 8, 12, 17, 23 when 5 is added to all the values?
    a. The mean does not change
    b. The median does not change
    c. Only the mean increases by 5
    d. Both the mean and median increase by 5
    e. The mode increases by 5

19. Which of the following best describes the solutions to $f(x) = 3x^2 - 4x + 7$?
    a. Two imaginary solutions (no real solutions)
    b. One repeated irrational solution
    c. One repeated rational solution
    d. Two rational solutions
    e. Two irrational solutions

20. Which of these is **not** in the range of the function $f(x) = 4x + 2$, if the domain is all natural numbers
    a. 2354
    b. 1878
    c. 932
    d. 1094
    e. 3118

21. What is the equation of the line that is the axis of symmetry for the parabola $f(x) = 6x^2 + 18x + 7$?
    a. $x = 3$
    b. $x = -3$
    c. $x = 1.5$
    d. $x = -1.5$
    e. $y = 7$

22. Which of these coordinates points is equidistant from the coordinate points (3, 7) and (8, 19)?
    a. $(0.5, 12)$
    b. $(2.5, 6)$
    c. $(-6.5,\ 18)$
    d. $(17.5, 17)$
    e. $(8, 7)$

23. What is the product $(x^3 - 7x + 2)(x^2 + 9x - 6)$ when simplified?
    a. $x^4 + 2x^3 - 67x^2 + 60x - 12$
    b. $x^5 + 9x^4 + x^3 + 65x^2 - 24x - 12$
    c. $x^5 - 2x^4 - x^3 - 61x^2 - 24x + 12$
    d. $x^5 + 9x^4 - 13x^3 - 61x^2 + 60x - 12$
    e. $x^6 + 2x^4 - 4x^3 - 63x^2 + 60x - 12$

24. According to a recent Wikipedia search, the population of India is approximately 1.375×10^9. The population of one of its neighboring countries, Bhutan, is approximately 7.77×10^5. Approximately, how many times greater is India's population than Bhutan's?
    a. 1.77×10^3
    b. 1.77×10^4
    c. 5.65×10^4
    d. 5.65×10^3
    e. 6.395×10^4

25. Which of these values is one of the many solutions to $\left|-\frac{1}{2}x - 3\right| > 10$?
    a. 6
    b. -6
    c. -16
    d. -26
    e. -36

26. Which of these numbers does not contain at least one factor in the form of a prime number raised to the fifth power?
    a. 2592
    b. 3888
    c. 38416
    d. 50000
    e. 253125

27. Which of these points is a solution to the system of linear inequalities $\begin{cases} y > -4x + 8 \\ y < 5x - 3 \end{cases}$?
    a. $(5, -12)$
    b. $(2, 0)$
    c. $(0, -3)$
    d. $(3,12)$
    e. $(3,0)$

28. What set of numbers has the largest standard deviation?
    a. First 10 positive multiples of 6
    b. First 10 prime numbers
    c. First 10 positive perfect squares
    d. First 10 positive powers of 2
    e. First 10 terms (starting with n = 1) of the sequence $(-1)^n(9n + 3)$

29. For what integer values of x will $3^x > 100$ but $2^x < 100$?
    a. x = 5 only
    b. x = 6 only
    c. x = 5 or x = 6 only
    d. x = 6 or x = 7 only
    e. x = 5 or x = 6 or x = 7 only

30. What can be said of the axis of symmetry from $f(x) = 3(x - 4)^2 + 5$ compared to $g(x) = -(x + 7)^2 - 6$?
    a. The axis moves 11 spaces left as we look from f to g.
    b. The axis moves 11 spaces right as we look from f to g.
    c. The axis moves 11 spaces up as we look from f to g.
    d. The axis moves 11 spaces down as we look from f to g.
    e. The axis of symmetry does not move as we look from f to g.

31. What are the zero(s) of the function $f(x) = 6|x - 15|$?
    a. 15
    b. -15 *or* 15
    c. 90
    d. -90 *or* 90
    e. 6

32. The exchange rate recently between United States Dollars and Mexican Pesos was 1 dollar = 18.3976 pesos. What is the equivalent amount of dollars for 1000 pesos?
    a. $1839.76
    b. $0.02
    c. $54.35
    d. $18397.60
    e. $5.44

33. A $10,000 investment is placed into an account earning an average annual interest rate of 5.43%. At this rate, about how long will we need for the investment to double in value?
    a. 13 years
    b. 29 years
    c. 95 years
    d. 18 years
    e. 37 years

34. Points A, B, and C are vertices of a right triangle with coordinate points $A = (1, 6)$, $B = (1, 1)$, and $C = (13, 1)$. M is the midpoint of $\overline{AB}$ and N is the midpoint of $\overline{BC}$. What is the length of $\overline{MN}$?
    a. 10.5
    b. 13
    c. 6.5
    d. 8.5
    e. 9.5

35. For a family earning a particular range of money, the income tax the family is responsible for is modeled by $y = .22x - 4251.5$, where x is the amount of money earned and y is the income tax paid. Identify the correct interpretation for the rate of change.
    a. For each additional 22 cents the family earns, the family pays 1 more dollar in taxes
    b. For each additional 1 dollar the family earns, the family pays 22 more cents in taxes
    c. As the family makes 22 fewer cents, the family pays 1 more dollar in taxes
    d. As the family makes 1 less dollar, the family pays 22 more cents in taxes
    e. For each additional 1 dollar the family earns, the pays 4251.5 more dollars in taxes

36. For the graph of the linear equation $y = 2x + 7$, what is the sum of the values of the x-intercept and the y-intercept?
    a. 10.5
    b. 3.5
    c. 5
    d. 9
    e. 14

37. The total surface area of a right cylinder is given by the formula $V = 2\pi rh + 2\pi r^2$. What is the formula solved for r?
    a. $r = \sqrt{\frac{V}{2\pi} - rh}$
    b. $r = \frac{-h}{2} + \sqrt{\frac{\pi h^2 + V}{4\pi}}$
    c. $r = -\frac{h}{2} + \sqrt{\frac{\pi h^2 + 2V}{4\pi}}$
    d. $r = -\frac{h}{2} + \sqrt{\frac{\pi h^2 + V}{2\pi}}$
    e. $r = \frac{2}{h}\sqrt{\frac{\pi h^2 + 2V}{4\pi}}$

38. What is the average rate of change for the function $f(x) = 3^x$ on the interval $[-2,3]$?
    a. $\frac{242}{9}$
    b. 18
    c. $\frac{242}{18}$
    d. $\frac{242}{45}$
    e. $\frac{18}{5}$

39. What is an equation of a line parallel to $7x + 8y = 15$ and contains the point $(-2, -4)$?
    a. $y = \frac{7}{8}x - \frac{21}{4}$
    b. $y = \frac{8}{7}x - \frac{12}{7}$
    c. $y = -\frac{7}{8}x - \frac{23}{4}$
    d. $y = -\frac{8}{7}x - \frac{44}{7}$
    e. $y = \frac{-15}{8}x - \frac{43}{4}$

40. What best describes the intercepts of the graph of the parabola $y = 3x^2 - 7x + 8$?

a. Positive y-intercept, no x-intercepts
b. Positive y-intercept, one positive x-intercept
c. Positive y-intercept, one positive x-intercept, one negative x-intercept
d. Positive y-intercept, two positive x-intercepts
e. Positive y-intercept, two negative x-intercepts