FORTY-NINTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra I 2005

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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ALGEBRA I 2005

1. Simplify: $10+8\div(-2)-4\bullet3$.

   (a) -21 (b) -39 (c) -6 (d) 6 (e) -9

2. Evaluate: $\left(5^2-3^2\right)^{\frac{1}{2}}$.

   (a) $\frac{1}{2}$ (b) 2 (c) 8 (d) $\frac{1}{4}$ (e) 4

3. The value of -3^{-2} is:

   (a) 6 (b) $-\frac{3}{2}$ (c) $\frac{3}{2}$ (d) $-\frac{1}{9}$ (e) $\frac{1}{9}$

4. Evaluate: $(8-4)\cdot(12-3)\div(2+1\cdot2)$

   (a) 14 (b) 6 (c) 12 (d) 9 (e) 20

5. Which of the following is the only true statement?

   (a) Any positive number is rational.
   (b) Some positive integers are irrational.
   (c) All whole numbers are positive integers.
   (d) Some rational numbers are positive integers.
   (e) An irrational number is not a real number.

6. Simplify: $2(a-4)-3(-2-a)$

   (a) $-a-2$ (b) $5a-2$ (c) $5a-14$ (d) $-a+2$ (e) $a+2$

7. Evaluate the expression $a(a+b)(b-d)^2$ for $a=1$, $b=-2$, and $d=1$.

(a) 9 (b) -9 (c) 1 (d) -1 (e) 27

8. Simplify $\frac{mn-mp}{-m}$.

(a) $n-mp$ (b) $mn+p$ (c) $-mmn+mmp$

(d) $p-n$ (e) $\frac{m^2n-m^2p}{m^2}$

9. Solve this equation for x: $\frac{1}{2}x-3=\frac{1}{4}x-9$.

(a) 6 (b) -6 (c) -24 (d) 36 (e) -48

10. Simplify: $\frac{ab^2-ba^2}{a^2b-b^2a}$

(a) 0 (b) 1 (c) -1 (d) ab (e) -ab

11. Which of the following is a true statement?

(a) $\frac{3}{4}\geq\frac{2}{5}$ and $\frac{-3}{4}\leq\frac{-2}{5}$

(b) $\frac{3}{4}\leq\frac{2}{5}$ and $\frac{-3}{4}\geq\frac{-2}{5}$

(c) $\frac{3}{4}\geq\frac{2}{5}$ and $\frac{-3}{4}\geq\frac{-2}{5}$

(d) $\frac{3}{4}\leq\frac{2}{5}$ and $\frac{-3}{4}\leq\frac{-2}{5}$

(e) $\frac{-3}{4}\geq\frac{-2}{5}$

12. What is the selling price for a car if the dealer's cost is \$7,000 and the percent markup is 8%.

(a) \$15,000 (b) \$7.056 (c) \$6,440

(d) \$7,800 (e) \$7,560

13. Write an equation for the following: Seven more than the cube of x is equal to five less than the square of the sum of x and y.

(a) $x^3+7=x^2+y^2-5$

(b) $x^3+7=5-(x+y)^2$

(c) $x^3+7=(x+y)^2-5$

(d) $(x+7)^3=(x+y)^2-5$

(e) $x^3+7=x^2+y-5$

14. An equivalent expression for $9a^4b^2-12a^3b^3+4a^2b^4$ is:

(a) $ab(3a-2b)^2$

(b) $a^2b^2(3a-2b)(3a+2b)$

(c) $ab(3a-2b)(3a+2b)$

(d) $a^2b^2(3a-2b)^2$

(e) $-a^2b^2(3a+2b)^2$

15. Two cars leave a gas station at the same time, both traveling due west on the same road. One car travels at 50 miles per hour and the other at 70 miles per hour. After how many hours will the cars be 12 miles apart?

(a) $\frac{1}{10}$ (b) $\frac{3}{5}$ (c) $1\frac{2}{3}$ (d) 10 (e) 1

16. Evaluate: $|3-4|-|5-9|$.

(a) 3 (b) -5 (c) 5 (d) -3 (e) -15

17. Simplify the following: $\frac{x+1}{x-3}-\frac{x-3}{x+1}$

(a) $\frac{4}{x^2-2x-3}$ (b) $\frac{-2}{x^2-2x-3}$ (c) 0

(d) $\frac{8}{x-3}$ (e) $\frac{8x-8}{x^2-2x-3}$

18. Determine the next term of the sequence, -3, $-\frac{7}{3}$, $-\frac{5}{3}$, -1, ...

(a) 3 (b) $-\frac{1}{3}$ (c) $\frac{1}{3}$ (d) $-\frac{4}{3}$ (e) 0

19. Which of the following pay-rates would be the best?

(a) \$5 for 2 hours and 30 minutes of work

(b) \$7 for 3 hours and 15 minutes of work

(c) \$10 for 4 hours and 45 minutes of work

(d) The above three pay-rates are the same.

(e) The best pay-rate cannot be determined from the information given.

20. Two less than six times the sum of a number and its square is equal to the opposite of the sum of the number and four. Which of the following is one of the possible values of the number?

(a) $\frac{1}{2}$ (b) $\frac{-1}{3}$ (c) -1 (d) $\frac{-1}{2}$ (e) $\frac{2}{3}$

21. Which of the following represents the relationship of the numbers 0.7777 and $\frac{7}{9}$?

(a) $0.7777 > \frac{7}{9}$ (b) $0.7777 < \frac{7}{9}$ (c) $0.7777 = \frac{7}{9}$

(d) $0.7777 > \frac{7}{9}$ or $0.7777 = \frac{7}{9}$ (e) $0.7777 < \frac{7}{9}$ and $0.7777 = \frac{7}{9}$

22. Simplify: $\frac{6x^2+7x-3}{12x^2+11x-5}$

(a) $\frac{x+7}{2x+11}$ (b) $\frac{2x+3}{4x+5}$ (c) $\frac{2x-3}{4x+5}$ (d) $\frac{2x-3}{4x-5}$ (e) $\frac{7x-3}{22x-10}$

23. A bookstore has 122 books for sale. There are 50 more paperbacks than twice the number of hardbacks. How many hardback books are for sale in the bookstore?

(a) 24 (b) 86 (c) 36 (d) 11 (e) 72

24. If $r+t=4$ and $m+s=7$, determine the value of $\frac{mr+rs+mt+st}{2}$.

(a) 11 (b) 3 (c) 56 (d) 28 (e) 14

25. Five less than three times a number is at most twice the number. What set of numbers satisfies these conditions?

(a) $\{x|x \le 5\}$ (b) $\{x|x \ge 5\}$ (c) $\{x|x > 5\}$

(d) $\{x|x < 5\}$ (e) $\{x|-5 \le x \le 5\}$

26. Evaluate the expression: $-|-5t-16s|$ given that $s=3$ and $t=-2$.

(a) 38 (b) 58 (c) -58 (d) -38 (e) -17

27. Determine the solution set for this equation: $\frac{1}{x-1}=\frac{1}{1-x}$

(a) $x=1$ (b) $x=-1$ (c) all real numbers

(d) all real numbers except -1 (e) empty set

28. Solve this inequality: $\frac{-2x}{6}\geq\frac{-1}{6}$

(a) $x\geq\frac{-1}{2}$ (b) $x\leq 2$ (c) $x\geq\frac{1}{2}$ (d) $x\leq\frac{1}{2}$ (e) $x\leq\frac{-1}{2}$

29. Solve this absolute value inequality: $|3-5x|\leq 2$

(a) $x\geq\frac{1}{5}$ (b) $x\leq 1$ (c) $x\geq\frac{1}{5}$ and $x\leq 1$

(d) $x\leq\frac{1}{5}$ and $x\geq 1$ (e) $x\geq\frac{1}{5}$ or $x\leq 1$

30. Determine the solution set of the equation, $|7-4x|=-15$.

(a) $\left\{2,\frac{-11}{2}\right\}$ (b) $\left\{-2,\frac{11}{2}\right\}$ (c) $\{2\}$ (d) $\left\{\frac{11}{2}\right\}$ (e) $\{\ \}$

31. Which of the following numbers is not a solution of the absolute value inequality, $|3x+7|\geq 5$.

(a) 0 (b) 7 (c) -4 (d) -2 (e) -10

32. If y varies inversely with the square of x and if $y = 0.4$ when $x = 4$, determine the equation that correctly depicts this relation.

(a) $y = \frac{32}{5x^2}$ (b) $y = \frac{x^2}{40}$ (c) $y = \frac{\sqrt{x}}{5}$ (d) $y = \frac{4}{5x^2}$ (e) $y = \frac{4}{5x\sqrt{x}}$

33. For which of the following relations is y not a function of x?

(a) $x^2 + y^2 = 4$

(b) $\{(3,1),(2,3),(4,6),(-1,7)\}$

(c) $y = x^2 + 4$

(d) $xy = 4$

(e) $2x + 3y = 7$

34. Determine the slope and y-intercept of the line, $3x - 4y - 8 = 0$.

(a) $\frac{3}{4}$; $(0,2)$ (b) $\frac{3}{4}$; $(0,-2)$ (c) $\frac{-3}{4}$; $(0,2)$

(d) $\frac{-3}{4}$; $(0,-2)$ (e) $\frac{4}{3}$; $(0,-2)$

35. Determine the domain of the function, $f(x) = \frac{\sqrt[4]{2x+1}}{\sqrt[3]{2x-2}}$.

(a) $(-0.5,+\infty)$ (b) $[-0.5,+\infty)$ (c) $(-0.5,+\infty)\cup(4,+\infty)$

(d) $[-0.5,4)\cup(4,+\infty)$ (e) $(4,+\infty)$

36. Determine the sum and the product of the roots of the equation, $2x^2 - 3x = 2$.

(a) $\frac{3}{2}$; -1 (b) $\frac{3}{2}$; 1 (c) -1; 2 (d) $-\frac{3}{2}$; 1 (e) $-\frac{3}{2}$; -1

37. The numerator of a fraction is 7 less than the denominator. If 4 is subtracted from the numerator and 1 is added to the denominator, the resulting fraction is equal to 1/3. Find the fraction.

(a) $\frac{5}{11}$ (b) $\frac{4}{5}$ (c) $\frac{10}{17}$ (d) $\frac{5}{4}$ (e) $\frac{11}{21}$

38. Solve this system of equations: $\begin{cases} 2x - y + 3 = 0 \\ x - y + 5 = 0 \end{cases}$

(a) (-2, -7) (b) (2, 7) (c) (-2, 7) (d) (2, -7) (e) $\left(\frac{8}{3}, \frac{23}{3}\right)$

39. Which of the following is a polynomial equation with roots -3, 2, and 5.

(a) $x^3 - 10x^2 + 31x - 30 = 0$

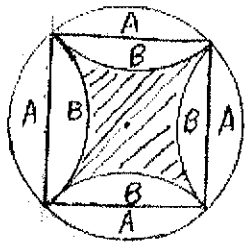
(b) $x^3 - 19x^2 - 30 = 0$

(c) $x^3 - 4x^2 - 11x + 30 = 0$

(d) $x^3 + 4x^2 - 11x - 30 = 0$

(e) $x^3 + 6x^2 - x - 30 = 0$

40. A square is inscribed in a circle as shown in the figure. The length of a side of the square is s and the radius of the circle is r. Sections A and B have equal areas. Determine the area of the shaded region.



(a) $\pi r^2 - \frac{s^2}{8}$ (b) $\pi r^2 - \frac{s^2}{4}$ (c) $s^2 - \frac{\pi r^2}{4}$

(d) $2s^2 - \pi r^2$ (e) $s^2 - 2\pi r^2$