FORTY-SEVENTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra I   2003

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the <u>best</u> answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

1.   Which of the following pairs of lines are perpendicular?

a. $2x - 3y = 6$
    $3x + 2y = 2$

b. $2x + 3y = 6$
    $3x + 2y = 6$

c. $y = x + 2$
    $y = x - 2$

d. $y = \dfrac{1}{2}x + 4$
    $y = 2x - 7$

e. $x = 1$
    $x + y = 1$

2.   Solve the following equation for x: $\dfrac{2}{3}(2x - 4) = \dfrac{1}{2}(x + 3) - 2$.

a. $\dfrac{5}{13}$ b. $\dfrac{23}{5}$ c. $\dfrac{13}{5}$ d. $\dfrac{11}{13}$ e. $\dfrac{13}{11}$

3.   If $A = \{0, 1, 2\}$ and $B = \{3, 4, 5, 6\}$, which of the following sets of ordered pairs represent a function from A to B?

a. $\{(3,0), (4,0), (5,0), (6,0)\}$ b. $\{(0,3), (0,4), (0,5), (0,6)\}$

c. $\{(0,6), (1,6), (2,6)\}$ d. $\{(1,3), (2,4)\}$

e. $\{(0,0), (1,1), (2,2)\}$

4.   For which of the following is y a function of x?

a. $y = x^2$ b. $y^2 = x$ c. $x^2 + y^2 = 4$ d. $x = 1$ e. $x^2 - y^2 = 4$

5.   Find the solution to the following inequality: $|2 - x| \geq 3$.

a. $-1 \leq x \leq 5$ b. $x \geq 1$ c. $x \geq 5$ or $x \leq -1$ d. $x \leq -1$ e. $x \geq 5$

6. Assuming no divisors of zero, simplify as much as possible: $\dfrac{\left(16 - \dfrac{1}{x^2}\right)}{\left(\dfrac{1}{4x^2} - 4\right)}$.

a. -4 b. 4 c. $\dfrac{64x^2 - 4}{1 - 16x^2}$ d. $\dfrac{\dfrac{16x^2 - 1}{x^2}}{\dfrac{1 - 16x^2}{4x^2}}$ e. $\dfrac{4(x^2 - 1)}{1 - x^2}$

7. You are driving from Memphis to visit a friend in Florida, a trip of 1200 miles, one-way. You begin the trip with a full tank of gas, and after traveling 410 miles, you refill the tank for $18.50. What is the best approximation for the total cost of gasoline for the round trip?

a. $54.15   b. $108.29   c. $216.60   d. $27.08   e. $270.75

8. If the line $y = mx + b$ is parallel to $y + 3x = 7$ and passes through $(-2,4)$, then $m + b$ is equal to

a. 3   b. 13   c. 7   d. -13 e. -5

9. Solve $\left|\dfrac{x-3}{2}\right| \le |x + 5|$ for x.

a. $(-\infty, -13] \cup \left[\dfrac{7}{3}, +\infty\right)$ b. $\left(-13, \dfrac{7}{3}\right] \cup [5, +\infty)$ c. $(-\infty, -13] \cup \left[\dfrac{-7}{3}, +\infty\right)$

d. $\left(-\infty, \dfrac{-7}{3}\right] \cup [5, \infty)$ e. $\left[\dfrac{7}{3}, 5\right]$

10. Simplify the following: $(2x^2 - 9x + 2) - (4x^2 + 7x - 3) + (-x^2 + 4x + 5)$.

a. $-x^2 - 12x + 10$ b. $-3x^2 - 12x$ c. $-3x^2 - 20x + 10$

d. $3x^2 + 12x - 10$ e. $-3x^2 - 12x + 10$

11. Assuming no divisors of zero, simplify the expression:

$$\left(\frac{x-6}{x^2-9}\right)^{-1}\left[x(x+3)^{-1}-2(x-3)^{-1}\right]$$

a. $x-6$ b. $x-1$ c. $x+1$ d. $1-x$ e. $x-3$

12. Find the vertex of the parabola $y=2x^2-4x+5$.

a. (3,1) b. (-1,3) c. (1,-3) d. (1,3) e. (0,5)

13. Solve the following proportion, correct to two decimals: $\quad \dfrac{1.20}{x}=\dfrac{2.34}{5.69}$.

a. 0.49 b. 2.92 c. 9.15 d. 11.10 e. 2.04

14. Simplify $\dfrac{\sqrt[3]{-32}}{\sqrt{8}}$.

a. $\sqrt[3]{-2}$ b. $\dfrac{\sqrt[3]{-4}}{\sqrt{2}}$ c. $\sqrt{-2}$ d. $\sqrt{2}$ e. $\sqrt[3]{-4}$

15. Simplify. $\left(x^2y^{3^2}\right)^5\left(x^3y^4\right)$

a. $x^{10}y^{14}$ b. $x^{13}y^{29}$ c. $x^{10}y^{15}$ d. $x^{13}y^{34}$ e. $x^{13}y^{49}$

16. Suppose f is an even function. If $g(x)=f(f(x))$, then g is:

a. symmetric with respect to the origin.

b. symmetric with respect to the x-axis.

c. even.   d. odd.   e. neither even nor odd.

17. Solve $\sqrt{7x+4}+x=8$ for x. Add all the solutions. The sum is

a. -7 b. 5 c. 3 d. 23 e. 20

18. Solve $x^4 + x^3 - 6x^2 < 0$ for x.

    a. $(-3, 2)$ b. $(-\infty, -3) \cup (0, 2)$ c. $(-\infty, -3) \cup (2, \infty)$ d. $(-3, 0) \cup (0, 2)$

    e. $(-3, 0) \cup (2, \infty)$

19. Which of the following equations/inequalities have a solution set that is empty?

    I. $|3x - 1| = -6$ II. $3x + 2 > 3x - 1$ III. $|2x + 7| \leq 0$

    a. I only   b. II only   c. III only   d. I and II only   e. I, II and III

20. If (4,6) is a point on the graph of $y = f(x)$, then which of the following must be a point

    on the graph of $y = \dfrac{1}{2} f(x - 2)$?

    a. $(2, 3)$ b. $(2, 12)$ c. $(6, 3)$ d. $(6, 12)$ e. $(4, 10)$

21. Mrs. Browning drove from City A to City B, a distance of 240 miles. For the first 120
    miles, her average speed was 40 mph. For the last 120 miles, her average speed was 60 mph.
    What was her average speed for the entire trip?

    a. 46 mph   b. 48 mph   c. 50 mph   d. 52 mph   e. 54 mph

22. When driving down a mountain road, you see a warning sign that says "13% downgrade".
    As you proceed down a straight stretch of road, your elevation drops 1000 feet. What has
    been the horizontal change in your position?

    a. 130 ft.   b. 13,000 ft.   c. $\dfrac{100,000}{13} ft.$ d. $\dfrac{1000}{13} ft.$ e. $\dfrac{10,000}{13} ft.$

23. Two wires are connected in parallel. The resistance in the first wire is 0.03 ohm, and the

    resistance in the second wire is R_2. The formula $R = \dfrac{0.03 R_2}{R_2 + 0.03}$, where R is the total

    resistance, expresses the relationship between the resistances. If the total resistance must
    be less than 0.01 ohm, what must the resistance in the second wire be? Assume resistance
    is a non-negative quantity.

    a. between 0.1 and 0.2 ohm   b. between 0.015 and 0.025 ohm

    c. between 0.01 and 0.02 ohm   d. between 0 and 0.015 ohm

    e. between 0.02 and 0.025 ohm

24. Find the slope of the line passing through the center of the circle

$(x-3)^2 + (y+2)^2 = 25$ and the vertex of the parabola $y = 3(x+1)^2 + 8$.

a. -5 b. $-\dfrac{5}{2}$ c. $-\dfrac{2}{5}$ d. $\dfrac{2}{3}$ e. $\dfrac{3}{2}$

25. Assume P varies directly as Q and inversely as the square of T. If P = 2 when Q = 12 and T = 4, what is the value of Q when P = 8 and T = 1?

a. 3   b. 12   c. 24   d. 768   e. $\dfrac{64}{3}$

26. A man has 20 coins consisting of dimes and quarters. If the dimes were quarters and the quarters were dimes, he would have \$0.90 more than he does now. How much money does he currently have?

a. \$2.90   b. \$3.50   c. \$2.75   d. \$3.05   e. \$3.20

27. Mike's T-shirt Company sells t-shirts in bulk. An order of 100 t-shirts costs \$360, and an order of 150 shirts costs \$510. Assuming that a linear relationship exists between the number of t-shirts purchased and the cost for the order, how many t-shirts can be purchased for \$1224?

a. 306   b. 340   c. 360   d. 388   e. 408

28. Simplify $\dfrac{\left(3ab^3\right)^2}{6a^2b^{-3}}$.

a. $\dfrac{b^9}{2}$ b. $\dfrac{b^8}{2a}$ c. $\dfrac{3b^8}{2}$ d. $\dfrac{3b^9}{2}$ e. b^9

29. Which of the following are true for all real numbers?

I. $|a+b| = |a| + |b|$ II. $|a \cdot b| = |a||b|$ III. $|a-b| \le |a| - |b|$

a. I only   b. II only   c. III only   d. I and II   e. II and III

30. Simplify. $\dfrac{3x^3 - 9x + 6}{x^3 + 2x^2 - x - 2}$

    a. $\dfrac{x-1}{x+1}$
    b. $\dfrac{3x-3}{x+1}$
    c. 3
    d. $\dfrac{3x^2 - 2x + 1}{x^2 - 1}$
    e. $\dfrac{3x-1}{x+1}$

31. Solve for x. $-4|x - 3| - 8 \le 4$

    a. $(-\infty, \infty)$
    b. no solution
    c. $[0, 6]$
    d. $[0, \infty)$
    e. $[6, \infty)$

32. Which circle has its center at (2,3)?

    a. $x^2 - 6x + y^2 - 4y + 12 = 0$
    b. $x^2 - 4x + y^2 - 6y - 3 = 0$

    c. $x^2 - 6x + y^2 - 4y + 4 = 0$
    d. $x^2 + 4x + y^2 + 6y + 29 = 0$

    e. $x^2 + y^2 - 13 = 0$

33. The solution set for the following equation is. $\sqrt{3t + 1} = 2 - \sqrt{3t}$

    a. the empty set
    b. $\left\{\dfrac{1}{2}\right\}$
    c. $\left\{\dfrac{3}{16}\right\}$
    d. $\left\{\dfrac{16}{3}\right\}$
    e. $\{2\}$

34. What is the value of -2^{-4} ?

    a. 16
    b. -16
    c. $\dfrac{1}{16}$
    d. $-\dfrac{1}{16}$
    e. 8

35. Solve the following equation for x. $a^2 x^{-2} - 2ax^{-1} - 1 = 0$. Then x =

    a. $2a \pm |a|\sqrt{2}$
    b. $a \pm |a|\sqrt{3}$
    c. $\dfrac{a}{2} \pm 3a$
    d. $-2a \pm |a|\sqrt{3}$

    e. $-a \pm |a|\sqrt{2}$

36. Suppose that x and y are two real numbers such that $x - y = 2$ and $x^2 + y^2 = 8$.
Find $x^3 - y^3$.

a. -20 b. $6\sqrt{3}$ c. 12 d. $-6\sqrt{3}$ e. 20

37. Carol Todd leaves Norwich University in her car on an important business trip. She drives 200 miles to Laconia, New Hampshire. On the return trip, she slows her average speed by 10 miles per hour in order to enjoy the scenery. If the return trip takes 1 hour more than the trip to Laconia, what was her average speed from Norwich to Laconia?

a. 55 mph   b. 45 mph   c. 50 mph   d. 57 mph   e. 60 mph

38. At a Parke Davis experimental lab, there is a chemical reagent vat with two input pipes, one marked A and the other B. If both pipes are opened, the vat will fill in 3 hours. How long does it take pipe B working alone to fill the vat if pipe A takes 2 hours longer to fill it than pipe B? Approximate your answer to the nearest minute.

a. 7 hours   10 minutes   b. 3 hours   20 minutes   c. 5 hours   10 minutes

d. 3 hours   30 minutes   e. 4 hours   25 minutes

39. Solve the following inequality for x: $\dfrac{2x}{x-3} \le \dfrac{x-1}{x+1}$

a. $\left[-3 - 2\sqrt{3}, -1\right) \cup \left[-3 + 2\sqrt{3}, 3\right)$ b. $\left(-2 - \sqrt{3}, -1\right) \cup \left(-3 + 2\sqrt{3}, 3\right)$

c. $\left(-2 - 2\sqrt{3}, -1\right) \cup \left(-3 + 3\sqrt{3}, 3\right)$ d. $\left[-2 - 3\sqrt{3}, -1\right) \cup \left[-3 + 2\sqrt{3}, 3\right)$

e. $\left[-2 - 2\sqrt{3}, -1\right) \cup \left(-1, 3\right)$

40. Use the tables of values below to find f(g(2)).

x	1	2	3	4
f(x)	3	4	2	1

x	1	2	3	4
g(x)	2	3	4	1

a. 1   b. 2   c. 3   d. 4   e. 6