FORTY-SIXTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra I   2002

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Scoring formula: 4R − W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

1) $5x - 3\left[2 - 3(2 - x)\right] =$

   a) $15x^2 - 29x + 12$

   b) $-4x + 12$

   c) $5x^2 - 23x + 12$

   d) $2x + 12$

   e) $2x + 6$

2) In baseball the slugging average of a player is found by dividing his or her total bases (**1** for a single, **2** for a double, **3** for a triple, and **4** for a homerun) by his or her official number of times at bat. Swinging Pete has three homeruns, one triple, two doubles, and eleven singles. If his slugging average is **1.2**, how many official times has he been at bat?

   a) **12**

   b) **20**

   c) **24**

   d) **25**

   e) **36**

3) Choose the solution set of the inequality $\dfrac{1}{3}\left(\dfrac{1}{3}x - 1\right) \geq x - 2$.

   a) $\left[\dfrac{15}{8}, \infty\right)$

   b) $\left(-\infty, \dfrac{15}{8}\right]$

   c) $\left[\dfrac{3}{2}, \infty\right)$

   d) $\left(-\infty, \dfrac{3}{2}\right]$

   e) $\left(-\infty, \dfrac{9}{8}\right]$

4) Which statement is *true* regarding the equation $3 - 2(3 - x) = 5x - 3(x + 1)$?

   a) The solution set is $\{0\}$.

   b) The solution set is $\varnothing$.

   c) The solution set is $\{4\}$.

   d) The solution set is $\{\pm 3\}$.

   e) All of the above statements are false.

5) $\dfrac{24-6x}{x^2-5x+4} \div \dfrac{12x^2-3}{2x^2-x-1} =$

    a) $\dfrac{2}{1-2x}$ b) $\dfrac{-3}{2x-1}$ c) $\dfrac{2x-1}{-2}$

    d) $\dfrac{6}{1-2x}$ e) $\dfrac{4-x}{2(2x-1)}$

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6) To complete the square in x, what number must be added to x^2-3x ?

    a) 3 b) $\dfrac{3}{2}$ c) $\dfrac{9}{2}$

    d) $\dfrac{3}{4}$ e) $\dfrac{9}{4}$

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7) An automobile costs **$18,600** when new and depreciates at a rate of **$1200** per year. If **y** is the value of the vehicle after **x** years, choose the equation which expresses the relationship between **x** and **y**.

    a) $x+1200y=18600$ b) $1200x+y=18600$

    c) $x-1200y=18600$ d) $y-1200x=18600$

    e) $1200y-x=18600$

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8) At Taco Heaven, one burrito and two tacos cost a total of **$2.48**, while three burritos and one taco cost a total of **$3.99**. What would be the total cost of six burritos and five tacos?

    a) **$8.27**   b) **$8.89**   c) **$9.15**

    d) **$9.90**   e) **$10.05**

9) For $x \geq 0$, $x^{3/4} \cdot x^{1/2} =$

a) $\sqrt[8]{x^3}$

b) $\sqrt[3]{x^8}$

c) $\sqrt[4]{x}$

d) $\sqrt[5]{x^4}$

e) $\sqrt[4]{x^5}$

10) The ace of diamonds is drawn from a standard **52**-card deck and not replaced. What is the probability that a second card drawn from the reduced deck will be a diamond? A standard deck has 13 diamonds.

a) $\dfrac{4}{17}$

b) $\dfrac{1}{17}$

c) $\dfrac{1}{16}$

d) $\dfrac{3}{13}$

e) $\dfrac{2}{13}$

11) $\dfrac{x-3}{x-4} - \dfrac{x^2-11}{x^2-3x-4} =$

a) $\dfrac{2}{x-4}$

b) $\dfrac{-2(x+4)}{(x-4)(x+1)}$

c) $\dfrac{-14}{(x-4)(x+1)}$

d) $\dfrac{-2}{x+1}$

e) $\dfrac{-x^2+x-14}{(x-4)(x+1)}$

12) The domain of $f(x) = \sqrt{4-3x}$ is

a) $x \leq \dfrac{3}{4}$

b) $x \geq \dfrac{3}{4}$

c) $x \leq \dfrac{4}{3}$

d) $x \geq \dfrac{4}{3}$

e) $x \geq -\dfrac{4}{3}$

13) If $f(x) = -x^2 + 4x - 5$, then $f(x+2) =$

    a) $-x^2 + 4x + 7$ b) $-x^2 + 4x - 1$

    c) $-x^2 - 1$ d) $-x^3 + 2x^2 + 3x - 10$

    e) $-x^2 + 8x + 7$

14) A freight train two miles long is traveling at a steady speed of **40** mph. If the train enters a tunnel two miles long at 2:00 p.m., at what time will the rear of the train emerge from the tunnel?

    a)   **2:03** p.m.   b)   **2:05** p.m.   c)   **2:06** p.m.

    d)   **2:10** p.m.   e)   **2:12** p.m.

15) A store owner discounted all merchandise **30%**. After sixty days any merchandise left was discounted an additional **20%** off the sale price. After the second discount became effective, Mrs. Save-a-Buck bought a camera for **$504**. What was the original price of the camera?

    a)   **$756**   b)   **$810**   c)   **$840**

    d)   **$900**   e)   **$1008**

16) When $x^3 + k$ is divided by $x + 2$, the remainder is known to be **-15.** Find the numerical value of k.

    a)   -7   b)   4   c)   5

    d)   8   e)   23

17) $\left[\left(x^{1/2}+y^{1/2}\right)\left(x^{1/2}-y^{1/2}\right)\right]^2 =$

    a) $x^2 - y^2$ b) $x^{1/2} - y^{1/2}$ c) $x^2 + y^2$

    d) $x^2 - 2xy + y^2$ e) $x^{1/16} - y^{1/16}$

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18) If $x = 3\sqrt{5}$ and $y = \sqrt{5}$ in the given triangle, find **z**.
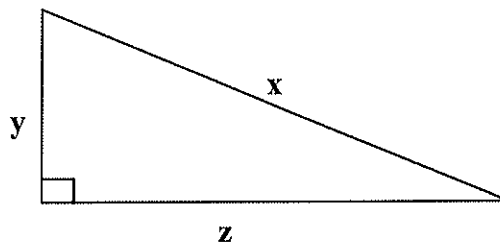
    a) $2\sqrt{5}$ b) $4\sqrt{5}$ c) $4\sqrt{10}$

    d) $2\sqrt{10}$ e) $5\sqrt{2}$

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19) If $\dfrac{2}{x} - \dfrac{3}{a} = \dfrac{1}{3}$, then $x =$

    a) 2 b) $-2a$ c) $a - 3$

    d) $\dfrac{2a}{a+3}$ e) $\dfrac{6a}{a+9}$

20) Suppose that x varies directly as the square of y and inversely as z and that x = 6 when y = -3 and z = 12. Find x when y = 9 and z = 36.

a) -72

b) 18

c) $-\dfrac{4}{9}$

d) $\dfrac{7}{18}$

e) $-\dfrac{1}{3}$

21) If $\dfrac{2x+2}{x} = \dfrac{x+1}{x-3}$, then the *sum* of the solutions is

a) 5

b) -2

c) -7

d) $\dfrac{5}{2}$

e) 2

22) Solve the equation $x^3 - 3x^2 - 4x + 12 = 0$ and find the *sum* of the largest and the smallest solutions.

a) 1

b) 2

c) -1

d) 5

e) -3

23) The arithmetic mean of a set of 50 numbers is 32. The arithmetic mean of a second set of 70 numbers is 53. Find the arithmetic mean of all the numbers.

a) 26.5

b) 85

c) 44.25

d) 42.5

e) 70.8

24) If $|x - 3| = k$ for $k \geq 0$, which of the following is *true*?

a) $x = k - 3$

b) $x = k + 3$

c) $x = k \pm 3$

d) $x = 3 \pm k$

e) $x = -3 \pm k$

25) The given chart shows building codes safety standards for T-Town.

Construction Type	Maximum Slope
Wheelchair Ramps	0.125
Walking Ramps	0.300
Driveway/Street Parking	0.220

A wheelchair ramp connecting the street to the ticket office at T-Town Stadium ends 8 feet above street level. What is the shortest horizontal distance that this ramp can span and meet T-Town safety standards?

a) 80 feet

b) 64 feet

c) 40 feet

d) 32 feet

e) 15.625 feet

26) $\dfrac{\left(2x^{-3}y^4\right)^{-3}}{4x^{-4}y^{-6}} =$

a) $\dfrac{-2x^5}{y^6}$

b) $\dfrac{3x^{13}y^{18}}{2}$

c) $\dfrac{x^{10}}{24y}$

d) $\dfrac{x^{10}}{24y^{13}}$

e) $\dfrac{x^{13}}{32y^6}$

27) The length of a rectangle is one foot shorter than twice the width. If the area of the rectangle is **36** sq ft, find the rectangle's perimeter.

a) **20** feet

b) **24** feet

c) **25** feet

d) **26** feet

e) **30** feet

28) $\left(\sqrt{8} - \sqrt{2} + \sqrt{x} \right)^2 =$

a) $6 + x$

b) $10 + x$

c) $2 + 2\sqrt{2x} + x$

d) $6 + 2\sqrt{2x} + x$
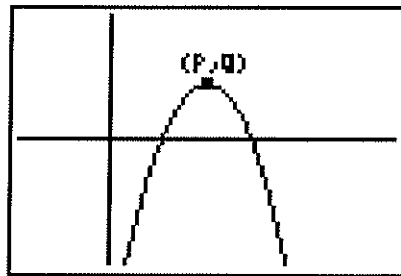
e) $10 + 2\sqrt{2x} + x$

29) $\dfrac{2x^{-1} - 1}{4x^{-2} - 1} =$

a) $\dfrac{x}{2} + 1$

b) $\dfrac{1}{2x} + 1$

c) $\dfrac{x}{x + 2}$

d) $\dfrac{2x}{2x + 1}$

e) $2x + 1$

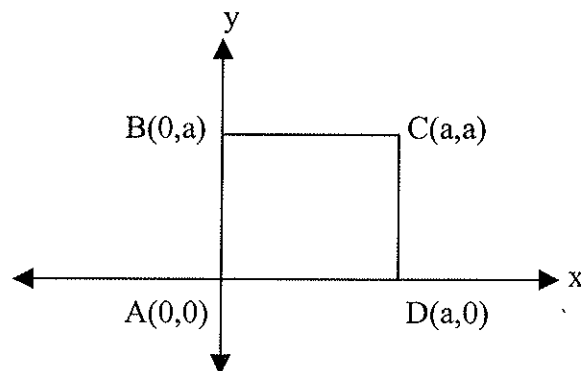30) If w is the *positive* solution of $x^2 - 4x = 1$, then $w^2 + 5w =$

a) $17 + 5\sqrt{3}$

b) $17 + 9\sqrt{3}$

c) $-1 + 5\sqrt{5}$

d) $19 + 5\sqrt{5}$

e) $19 + 9\sqrt{5}$

31) A graphics calculator shows the graph of $y = ax^2 + bx + c$ with vertex **(P,Q)** as given below. Which of the following statements is *false*?

a) The curve is a function.

b) The curve is decreasing for all real numbers **x**.

c) **a**, the coefficient of x^2, is a negative number.

d) $b^2 - 4ac > 0$

e) $P = -\dfrac{b}{2a}$

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32) In the given figure, if **P** is the midpoint of $\overline{CD}$ and **Q** is the point one-fourth of the distance from **A** to **D**, find the area of $\triangle$ **PQD**.

a) $\dfrac{a^2}{8}$

b) $\dfrac{a^2}{16}$

c) $\dfrac{3a^2}{8}$

d) $\dfrac{3a^2}{16}$

e) $\dfrac{3a^2}{2}$

33) Two years ago Hank was nine years older than twice Polly's age then. Eighteen years from now the sum of their ages will be **97**. Find the *positive difference* of their present ages.

   a)   **8**   b)   **19**   c)   **25**

   d)   **29**   e)   **32**

34) If $\sqrt{x} - \sqrt{3} = 2$, then $2 - x =$

   a) -5 b) -3 c) $-5 - 2\sqrt{6}$

   d) $-15 - 2\sqrt{6}$ e) $-5 - 4\sqrt{3}$

35) One number is three less than a second number. If twice the reciprocal of the larger number is added to three times the reciprocal of the smaller number, the result is five times the reciprocal of their product. Find the *smaller* number.

   a) $-\dfrac{4}{5}$ b) $1\dfrac{1}{3}$ c) $-3\dfrac{2}{3}$

   d) $\dfrac{4}{9}$ e) $2\dfrac{3}{4}$

36) If $f(1) = 1$ and $f(n) = n + f(n - 1)$ for all natural numbers $n \geq 2$, find the value of $f(6)$.

   a)   **6**   b)   **9**   c)   **11**

   d)   **21**   e)   **25**

37) A sweatshirt costs five times the price of a hat. A jacket costs two-thirds the price of a sweatshirt. A tee-shirt costs three dollars less than a hat. Find the price of a jacket if all four items cost a total of **$121**.

a) $12

b) $18

c) $40

d) $45

e) $90

38) When Sally left for college, her savings totaled **$2000**. Her father agreed to fully reimburse every dollar less than or equal to **$200** which she spent during any month and to reimburse one-fifth of her monthly spending in excess of **$200**. If Sally spends **$300** each month, how many months will it take to exhaust her savings?

a)   **20** months

b)   **25** months

c)   **30** months

d)   **45** months

e)   **50** months

39) If $2x + 5y = -k$ and $x - y = 3k$, express y as a function of x.

a) $y = 3x$

b) $y = 2x - 1$

c) $y = -\dfrac{1}{4}x + 2$

d) $y = -5 - x$

e) $y = -\dfrac{1}{2}x$

40) If $\dfrac{8}{3 + \sqrt{5}} - \dfrac{5}{\sqrt{5}} = a\sqrt{5} + b$, where a and b are integers, then $a =$

a) -3

b) 0

c) 1

d) $-\dfrac{1}{2}$

e) $\dfrac{9}{20}$