FORTY-FOURTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2000

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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ALGEBRA II TEST 2000

1. Simplify $\dfrac{a^{-2}b^{-3}}{a^{-3}+b^{-2}}$.

(a) $\dfrac{a}{b^6}$ (b) $\dfrac{a^5}{b}$ (c) $\dfrac{a}{b}$ (d) ab (e) $\dfrac{a}{a^3b+b^3}$

2. Completely factor $128x^6 - 2y^6$.

(a) $2(2x-y)(2x+y)(16x^4+4x^2y^2+y^4)$
(b) $2(2x-y)(2x+y)(4x^2-2xy+y^2)(4x^2+2xy+y^2)$
(c) $2(2x-y)(2x+y)(16x^4-4x^2y^2+y^4)$
(d) $2(2x-y)(2x+y)(4x^2+y^2)(4x^2+y^2)$
(e) $2(8x^3-y^3)(8x^3+y^3)$

3. Solve $x^2-(\sqrt{3}+\sqrt{5})x+\sqrt{15}=0$.

(a) $\{3,5\}$ (b) $\{-3,5\}$ (c) $\{\sqrt{3},\sqrt{5}\}$ (d) $\{\sqrt{3},-5\}$ (e) $\{9,25\}$

4. A certain debt will be repaid after n months where $416=\dfrac{n}{2}[2(11)+(n-1)2]$.

Determine how many months repayment will take.

(a) 26 (b) 18 (c) 31 (d) 16 (e) 28

5. Which expression below is equivalent to $\sqrt{\dfrac{4}{27}}\cdot\sqrt[5]{\dfrac{27}{8}}$?

(a) $\dfrac{\sqrt[5]{9}}{\sqrt[3]{8}}$ (b) $\dfrac{\sqrt[5]{2^2}}{\sqrt[10]{3^9}}$ (c) $\dfrac{1}{\sqrt[10]{2}}$ (d) $\dfrac{\sqrt[4]{2}}{2}$ (e) $-\sqrt[10]{2}$

6. Simplify $\frac{1}{x}\left(x+\frac{1}{x}\right)^{-1}$

(a) $\frac{1}{x^2+1}$

(b) 0

(c) $\frac{x^2+1}{x^2}$

(d) $x+1$

(e) $1+x^2$

7. Which of the following is a factor of $a^{4m}-16$?

(a) a^m-2

(b) a^m+2

(c) $a^{2m}+4$

(d) all of the above

(e) none of the above

8. One positive number is three less than twice another number. The sum of their squares is 41. Find the sum of the two numbers.

(a) 9

(b) 41

(c) $\frac{36}{5}$

(d) 32

(e) 12

9. If $\log_a 3 = 0.08$, $\log_a 7 = 0.17$, and $\log_a 11 = 0.24$, find $\log_a \frac{11}{63}$.

(a) 0.23 (b) -0.09 (c) 1.5 (d) -1.75 (e) -0.76

10. Find the expression for the inverse of the function $f(x)=\dfrac{x+1}{x-1}$ if the inverse exists as a function.

(a) The inverse does not exist.
(b) $f^{-1}(x)=\dfrac{x-1}{x+1}$
(c) $f^{-1}(x)=\dfrac{x+1}{x-1}$
(d) $f^{-1}(x)=\dfrac{1+x}{1-x}$
(e) $f^{-1}(x)=\dfrac{1-x}{1+x}$

11. Find a value of k so that $x-2$ will be a factor of x^3-4x^2+kx+2.

(a) -5 (b) 3 (c) -3 (d) 5 (e) 0

12. Factor the expression $\left(x^2+6\right)^{7/2}+2x\left(x^2+6\right)^{5/2}+x^2\left(x^2+6\right)^{3/2}$.

(a) $\left(x^2+6\right)^{3/2}(x+2)^2(x+3)^2$
(b) $\left(x^2+6\right)^{3/2}\left(x^2+x+6\right)^2$
(c) $\left(x^2+6\right)^{1/2}(x+2)^2(x+3)^2$
(d) $\left(x^2+6\right)^{1/2}\left(x^2+x+6\right)^2$
(e) $\left(x^2+6\right)^{3/2}(x-2)^2(x+3)^2$

13. A linear equation of the graph of a line having slope $=\dfrac{-3}{2}$ and x - intercept $=8$ is:

(a) $6x-9y=72$
(b) $9x+6y=72$
(c) $2x-3y=24$
(d) $6x+9y=72$
(e) $2x+3y=8$

14. What is the second term in the binomial expansion of $\left(x^2-3\right)^{18}$?

(a) $-54x^{34}$ (b) $3x^{17}$ (c) $18x^{34}$ (d) $-3x^{18}$ (e) x^{36}

15. Which of the following intervals contains a solution of $2^{2n}2^{n^2}=256$.

(a) [-2, 0] (b) [0, 1] (c) [3, 5] (d) [6, ∞) (e) (-∞, -3]

16. A nursery is offering 2 package deals on shrubbery. They have 2 sizes of shrubs classified as small and large. Deal #1 offers one small shrub and two large shrubs for $22. Deal #2 offers 3 small shrubs and one large shrub for $28. How much is the nursery actually charging for each size of shrub?

(a) $6 and $8
(b) $6.60 and $7.80
(c) $6.80 and $7.60
(d) $4 and $7
(e) $3.60 and $8.35

17. If a is a positive number, then the number of distinct real solutions of the equation $x^4+x^3=a$ must be:

(a) 0 (b) 1 (c) 2 (d) 3 (e) 4

18. Which interval below contains the solution(s) of the equation $3|2x-3|=x-1$?

(a) $\left(-2,\frac{9}{7}\right]$ (b) $\left(\frac{-3}{2},\frac{3}{2}\right)$ (c) $\left(\frac{7}{5},\frac{8}{5}\right]$ (d) $\left[\frac{9}{7},\frac{7}{5}\right]$ (e) $(0,1)$

19. Solve the inequality $\frac{2x+7}{8x-5} > -3$.

(a) $x < \frac{-7}{2}$ or $x > \frac{5}{8}$

(b) $x < \frac{4}{13}$ or $x > \frac{5}{8}$

(c) $-7 < x < 5$

(d) $x < \frac{-8}{26}$ or $x > \frac{8}{26}$

(e) $\frac{4}{13} < x < \frac{5}{8}$

20. Write the equation of the circle whose endpoints of a diameter are (2, -5) and (8, -1).

(a) $(x+3)^2 + (y-5)^2 = 13$

(b) $(x-5)^2 + (y+3)^2 = 5$

(c) $(x-5)^2 + (y-3)^2 = 52$

(d) $(x-3)^2 + (y+2)^2 = 10$

(e) $(x-5)^2 + (y+3)^2 = 13$

21. If y varies jointly as x and t^2 and if $y = 48$ when $x = 2$ and $t = 1$, determine the value of y when $x = 1$ and $t = 2$.

(a) 64 (b) 104 (c) 96 (d) 42 (e) 36

22. The solution set of the equation $\log_2 x + \log_2 (x-4) = 5$ is:

(a) $\{-4\}$ (b) $\{8\}$ (c) $\{18\}$ (d) $\{-1,5\}$ (e) $\{-4,8\}$

23. Attempts to solve the equation $w = 1 - \sqrt{2 - \frac{w}{2}}$ introduce an extraneous solution. Identify the extraneous solution from the choices below.

(a) $\frac{-1}{2}$ (b) 4 (c) 1 (d) 2 (e) 0

24. Suppose $f(x)=\sqrt{x}$. Determine the value of $f\big(f(64)\big)$.

(a) $2\sqrt{2}$ (b) 64 (c) $\pm 8i$ (d) $\pm 2\sqrt{2}$ (e) ± 64

25. If x is an odd integer greater than one, which of the following must be an odd integer?

(a) x^3-2x (b) x^3-1 (c) $\dfrac{3x-3}{x-1}$ (d) a and c (e) a, b, and c

26. The difference between the lengths of the sides of a rectangle is 7 meters, and the length of its diagonal is 13 meters. The area of the rectangle in square meters is:

(a) 30 (b) 21 (c) 8 (d) 60 (e) 42

27. Find a function whose graph is a parabola with vertex at (6, -1) that passes through the point (3, 2).

(a) $f(x)=-\dfrac{1}{3}(x-3)^2+2$

(b) $f(x)=\dfrac{1}{3}(x-6)^2+1$

(c) $f(x)=\dfrac{1}{3}(x-6)^2-1$

(d) $f(x)=-(x-1)^2+6$

(e) $f(x)=\dfrac{1}{9}(x+6)^2-1$

28. Express the quotient $\dfrac{2+3i}{1+i}$ of complex numbers in the form $a+bi$.

(a) $\dfrac{5}{2}+\dfrac{1}{2}i$ (b) $-\dfrac{1}{2}+\dfrac{1}{2}i$ (c) $\dfrac{1}{2}+\dfrac{5}{2}i$ (d) $2+5i$ (e) $-2+i$

29. Determine the maximum value of $6-3x-2x^2$.

(a) 13 (b) 6 (c) 8 (d) $\dfrac{48}{9}$ (e) $\dfrac{57}{8}$

30. Joseph Walker can paint a house in 12 hours. His friend Simon Clay can paint the same house in 18 hours. How long will it take Joseph and Simon to paint the house if they work together?

(a) 36 hours
(b) 7 hours, 0.20 minutes
(c) 15 hours
(d) 8 hours, 15 minutes
(e) 7 hours, 12 minutes

31. If $\log_x a = 3$, $\log_x b = 5$, and $\log_x c = 2$, determine the value of $\log_c\left(a^3 b\right)$.

(a) 135 (b) 2^{14} (c) 30 (d) 7 (e) 28

32. Factor $x^2 + (x-y)^2 - 2x(x-y)$.

(a) $x(x-2)$
(b) $x(x-y)(x-2)$
(c) y^2
(d) $(x-y)(1-2xy)$
(e) $(x-y)(x-2y)$

33. Which of the following polynomials has zeros of $3+2i$ and $2+i$?

(a) $p(x) = x^4 - 2x^3 - 6x^2 + 22x + 65$
(b) $p(x) = x^4 + 2x^3 - 6x^2 - 22x + 65$
(c) $p(x) = x^4 - 10x^3 - 42x^2 - 82x + 65$
(d) $p(x) = x^4 + 10x^3 + 42x^2 + 82x + 65$
(e) $p(x) = x^4 - 10x^3 + 42x^2 - 82x + 65$

34. How many 7-letter patterns can be formed from the letters of the word *avarice* ?

(a) 5040 (b) 2520 (c) 1260 (d) 420 (e) 14

35. The hyperbola whose equation is $2x^2 - y^2 + 6y = 5$ has foci:

(a) (-6, 3) and (6, 3)
(b) $(\pm\sqrt{6}, 3)$
(c) (0, -3) and (0, 9)
(d) $(0, 3 \pm \sqrt{6})$
(e) $(0, \pm\sqrt{6})$

36. Solve the system of equations $\begin{cases} ax - 9y = 102 \\ -x + 6y = -61 \end{cases}$ for x and y if the determinant of the coefficient matrix is equal to 9.

(a) $\left(\frac{149}{3}, \frac{-34}{3}\right)$
(b) $\left(-21, \frac{-41}{3}\right)$
(c) $(7, 9)$
(d) $(7, -9)$
(e) $\left(21, \frac{20}{3}\right)$

37. Find all real and imaginary solutions of the equation, $3ix^2 + 7x - 4i = 0$.

(a) $x = i, x = \frac{-4i}{3}$
(b) $x = -i, x = \frac{-4i}{3}$
(c) $x = -i, x = \frac{4i}{3}$
(d) $x = i, x = \frac{4i}{3}$
(e) $x = -6i, x = \frac{8i}{3}$

38. A customer in a coffee shop wishes to purchase a blend of two coffees: Kenyan, costing \$5.60 per pound, and Colombian, costing \$3.50 per pound. The customer buys 3 pounds of such a blend at a cost of \$11.55. How many pounds of Colombian coffee went into the blend?

(a) 1.75 (b) 0.58 (c) 2 (d) 2.5 (e) 0.5

39. Determine the domain of the function, $f(x)=\dfrac{\sqrt[4]{2x+1}}{\sqrt[3]{2x}-2}$.

(a) $(-0.5,+\infty)$
(b) $[-0.5,+\infty)$
(c) $[-0.5,4)\cup(4,+\infty)$
(d) $(-0.5,+\infty)\cup(4,+\infty)$
(e) $(4,+\infty)$

40. Determine a and b so that $x(x+1)(x+2)(x+3)+1=\left(x^2+ax+b\right)^2$.

(a) $a=7,\ b=4$
(b) $a=6,\ b=2$
(c) $a=1,\ b=7$
(d) $a=3,\ b=1$
(e) $a=0,\ b=7$