

FORTY-THIRD ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra I 1999

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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ALGEBRA I

1. What is an equation of the line through the points (3, 2) and (6, 7)?
 - a) $y = -x - 1$
 - b) $3x + 5y = 19$
 - c) $3x + 5y = 1$
 - d) $y = \frac{5}{3}x - 3$
 - e) $3x - 5y = -17$

2. What is the solution of $\frac{1}{x+2} + \frac{2}{x-3} = \frac{3}{x^2 - x - 6}$?
 - a) $\frac{1}{2}$
 - b) $\frac{1}{3}$
 - c) $\frac{2}{3}$
 - d) $-\frac{1}{2}$
 - e) no solution

3. A rectangle has a perimeter of length p. The length of the rectangle is twice as long as the width. What is the area of the rectangle?
 - a) $\frac{p^2}{4}$
 - b) $\frac{p^2}{6}$
 - c) $\frac{p^2}{12}$
 - d) $\frac{p^2}{16}$
 - e) $\frac{p^2}{18}$

4. What is the remainder when $(5y^3 - 2y^2 + 10)$ is divided by $(y + 3)$?
 - a) -121
 - b) -143
 - c) 0
 - d) 97
 - e) 127

5. If $4^{2a+b} = \frac{1}{16}$ and $4^{a-b} = 64$, then b equals :
 - a) $\frac{8}{3}$
 - b) $-\frac{8}{3}$
 - c) 0
 - d) $\frac{2}{3}$
 - e) $-\frac{2}{3}$

6. The solution set of $16x^3 = 36x$ is the same as the solution set of which of the following?

- a) $x(2x-3)(2x+3) = 0$ d) $16(x^2-9) = 0$
b) $2x(8x+1)(x-9) = 0$ e) none of the above
c) $4(4x^2-9) = 0$

7. Given $x \neq -3, 5, -4$, perform the operation: $\frac{x^2-10x+25}{x+3} \cdot \frac{x^2+8x+15}{x^2-x-20}$

- a) $\frac{x^2-2x+5}{x+3}$ c) $\frac{x^2-25}{x+4}$ e) $\frac{x-5}{x+4}$
b) $\frac{x+5}{x^2-x}$ d) $\frac{x-5}{x+3}$

8. $3 \begin{bmatrix} 4 & 2 & 1 \\ 3 & -1 & 0 \end{bmatrix} - 2 \begin{bmatrix} 1 & 4 & -2 \\ -1 & 2 & 1 \end{bmatrix} =$

- a) $\begin{bmatrix} 12 & 6 & 3 \\ 9 & -3 & 0 \end{bmatrix}$ c) $\begin{bmatrix} 14 & 14 & -1 \\ 7 & 1 & 2 \end{bmatrix}$ e) $\begin{bmatrix} 10 & -2 & -1 \\ 7 & -7 & -2 \end{bmatrix}$
b) $\begin{bmatrix} 10 & -2 & 7 \\ 11 & -7 & -2 \end{bmatrix}$ d) $\begin{bmatrix} -2 & -8 & 4 \\ 2 & -4 & -2 \end{bmatrix}$

9. Kathy plans to run 6 miles in one hour. If she averages 4 m.p.h. for the first 3 miles, what speed must she average during the last 3 miles?

- a) 6 mph b) 8 mph c) 10 mph d) 12 mph e) 14 mph

10. Use absolute value to express the statement: k is within 0.01 of 8.

a) $|k - 0.01| \leq 8$ c) $|k - 0.01| \geq 8$ e) $|k| \leq 8.01$

b) $|k - 8| \leq 0.01$ d) $|k - 8| \geq 0.01$

11. The domain of the function $g(x) = \frac{x+1}{\sqrt{x+4}}$ is:

a) $(-\infty, -1)$ c) $(-4, \infty)$ e) all real numbers except 0

b) $(0, \infty)$ d) $(-3, 3)$

12. $5\sqrt{24} - \sqrt{150} =$

a) $3\sqrt{7}$ c) $4\sqrt{10}$ e) $5\sqrt{6}$

b) $-2\sqrt{5}$ d) $-7\sqrt{26}$

13. A bag contains 3 red marbles and 7 blue marbles. A marble is drawn out, then returned to the bag. Again, a marble is drawn out. What is the probability that both marbles drawn out were red?

a) $\frac{9}{100}$ b) $\frac{1}{5}$ c) $\frac{3}{10}$ d) $\frac{49}{100}$ e) $\frac{7}{10}$

14. $(2x + 3)(x - 2)(x + 5) =$

a) $2x^3 + 9x^2 - 11x - 30$

b) $2x^3 - 4x^2 + 5x - 30$

c) $2x^3 + 6x^2 - 10x - 30$

d) $2x^3 - 8x^2 + 7x - 30$

e) $2x^3 - 30$

15. What is the solution set of $|3x + 4| > 8$?
- a) $(-\infty, -6) \cup (2, \infty)$
- b) $(-\infty, -4) \cup (\frac{4}{3}, \infty)$
- c) $(-\infty, -\frac{5}{4})$
- d) $(3, \infty)$
- e) $(-5, 3)$
16. The line through the points $(r, 3)$ and $(8, r)$ has a slope of -2 . What is the value of r ?
- a) -5 b) -10 c) 9 d) 10 e) 13
17. Jessica wants to expand her rectangular patio so that the ratio of the new area to the old area is 5 to 3 . If she can only change the length of the patio, what is the ratio of the new length to the old length?
- a) $\frac{5}{3}$ c) $\frac{8}{5}$ e) more information is needed
- b) $\frac{25}{9}$ d) $\frac{64}{25}$
18. For $y \neq 0, \frac{4}{3}$ the expression $\frac{2 + \frac{1}{y}}{3 - \frac{4}{y}}$ is equivalent to:
- a) -3 b) $\frac{6y^2 - y - 4}{y^2}$ c) $\frac{2y - 3}{3y}$ d) $\frac{2y + 1}{3y - 4}$ e) 2

19. If $f(t) = 2t^2 - t + 3$, then $f(4t + 1) =$

a) $8t^2 - t + 3$

c) $8t^2 - t + 4$

e) undefined

b) $32t^2 + 12t + 4$

d) $32t^2 - 4t + 2$

20. Solve the system: $3x + 7y = 1$
 $5x - 3y = 9$

a) $x = -4, y = \frac{13}{7}$

c) $x = \frac{3}{2}, y = -\frac{1}{2}$

e) no solution

b) $x = 5, y = -2$

d) $x = -3, y = -8$

21. Simplify: $(3x - 1)^{\frac{1}{2}} + \frac{3}{4}(3x - 1)^{-\frac{1}{2}}$

a) $\frac{1}{4}(3x - 1)^{-\frac{1}{2}}$

c) $\frac{7}{4}(3x - 1)^{\frac{1}{2}}$

e) $\frac{7}{4(3x - 1)^{\frac{1}{2}}}$

b) $\frac{12x - 1}{4(3x - 1)^{\frac{1}{2}}}$

d) $\frac{9x + 7}{4(3x - 1)^{\frac{1}{2}}}$

22. What is the least common multiple of the polynomials $3x^2 - 7x + 2$ and $x^2 + x - 6$?

a) $x - 2$

c) $3x^3 + 2x^2 - 19x + 6$

e) $3x^4 - 4x^3 - 13x^2 + 44x - 12$

b) $4x^2 - 6x - 4$

d) $4x^2 - 7x - 6$

23. What is the solution of $\frac{2x + 3}{x + 2} = \frac{5}{x + 1}$?

a) $\pm \frac{\sqrt{14}}{2}$

b) $\pm \frac{\sqrt{47}}{4}$

c) $5 \pm \sqrt{3}$

d) $\frac{4}{3} \pm \frac{\sqrt{2}}{3}$

e) $\pm \sqrt{3}$

24. Which of the following is a factor of $x^3 - 4x^2 - 7x + 10$?
- a) $x - 10$ b) $x - 4$ c) $x + 1$ d) $x + 2$ e) $x + 5$
25. In a class of 30 students, 11 students have taken chemistry, 4 students have taken chemistry and biology, and 9 students have not taken either course. How many students have taken biology?
- a) 17 b) 14 c) 11 d) 6 e) 4
26. If $G = \frac{4}{7}F + 4H$, then $F =$
- a) $\frac{4}{7}G + 4H$ c) $\frac{7}{4}G - 4H$ e) $\frac{7}{4}G - 7H$
- b) $7G - 4H$ d) $\frac{4}{7}G + 7H$
27. The profit in dollars for a business in June was 30% higher than it was in May. Combined profit for the two months was \$6,900. What was the profit in June?
- a) \$3,550 b) \$3,780 c) \$3,900 d) \$4,060 e) \$4,200
28. Solve: $4x^2 + 8x - 10 = (2x - 3)(2x + 7)$
- a) $-7 \pm \sqrt{11}$ c) $4 \pm \sqrt{10}$ e) no solution
- b) $\frac{5}{2} \pm \frac{\sqrt{7}}{2}$ d) $-\frac{3}{4} \pm \frac{\sqrt{3}}{4}$

29. $\frac{3+x}{x^2} + \frac{6}{x} + x + 1 =$

a) $\frac{x^3 + x^2 + 7x + 3}{x^2}$

c) $\frac{2x + 10}{x^2}$

e) $\frac{x^3 + 2}{x^3}$

b) $\frac{x^2 + 2x + 10}{x^2}$

d) $\frac{x^3 + 2x + 4}{x^3}$

30. The equation $(x + y)^2 = x^2 + y^2$ is true whenever:

a) $xy = 0$

c) $y = \frac{1}{x}$

e) x and y are real numbers

b) $x = -y$

d) $x^2 = y^2$

31. Allentown is 30 miles north of Oakdale. Smithville is 20 miles east of Allentown. How far is Smithville from Oakdale?

a) $\sqrt{600}$ miles

c) $\sqrt{1500}$ miles

e) $\sqrt{2500}$ miles

b) $\sqrt{1300}$ miles

d) $\sqrt{1800}$ miles

32. $\begin{bmatrix} 4 & 2 & 1 \\ 3 & -1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 4 & -2 \\ -1 & 2 & 1 \end{bmatrix} =$

a. $\begin{bmatrix} 2 & 20 & -6 \\ 4 & 10 & -7 \end{bmatrix}$

c) $\begin{bmatrix} 5 & 6 & -1 \\ 2 & 1 & 1 \end{bmatrix}$

e) Does not exist.

b) $\begin{bmatrix} 4 & 8 & -2 \\ -3 & -2 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 10 & 1 \\ -1 & -5 \end{bmatrix}$

33. Find the least common multiple of the greatest common divisor of 36 and 48 and the greatest common divisor of 45 and 150.

a) 90

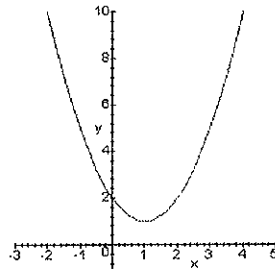
b) 180

c) 30

d) 60

e) 36

34. Which function best represents the given graph?



- a) $y = (x - 1)^2 + 1$ c) $y = (x + 1)^2 - 1$ e) $y = (x - 1)^2$
 b) $y = x^2 + 1$ d) $y = (x - 1)^2 - 1$
35. In solving the system $3x - 7y = 31$ using matrix algebra, you would do which of the following?
 $2x - 4y = 18$

a) $\begin{bmatrix} 3 & -7 \\ 2 & -4 \end{bmatrix} \begin{bmatrix} 31 \\ 18 \end{bmatrix}$ c) $\begin{bmatrix} -3 & 7 \\ -2 & 4 \end{bmatrix} \begin{bmatrix} 31 \\ 18 \end{bmatrix}$ e) $\begin{bmatrix} -3 & -2 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} 31 \\ 18 \end{bmatrix}$

b) $\begin{bmatrix} -2 & 7 \\ -1 & 3 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} 31 \\ 18 \end{bmatrix}$ d) $\begin{bmatrix} 3 & 2 \\ -7 & -4 \end{bmatrix} \begin{bmatrix} 31 \\ 18 \end{bmatrix}$

36. Of the following which is not a subset of the set of rational numbers?

- a) natural numbers c) whole numbers e) counting numbers
 b) real numbers d) integers

37. The volume of a sphere varies directly as the cube of its radius. To double the volume of a sphere, what do you have to multiply the old radius by?

- a) $\sqrt[3]{2}$ b) 2 c) $\frac{3}{2}$ d) $\sqrt{3}$ e) 8

38. If A' is the complement of set A , then $(A' \cap B)' =$

a) $A \cap B'$ c) $A \cup B'$ e) $(A' \cap B) \cup (A \cap B)'$

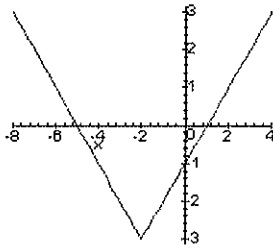
b) $B \cap A'$ d) $B \cup A'$

39. $\frac{840,000 \times 0.03}{0.00021 \times 600} =$

a) 7.2×10^{10} c) 2.0×10^{-4} e) 3.1752×10^3

b) 2.0×10^5 d) 1.4×10^3

40. Which function best describes this graph?



a) $f(x) = |x+2| - 3$ c) $f(x) = (x+2)^2 - 3$ e) $f(x) = \frac{|x|}{2} - 3$

b) $f(x) = \begin{cases} -x, & x \leq 2 \\ x, & x > 2 \end{cases}$ d) $f(x) = |x-2| - 3$

