FORTY-FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra I 1997

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:
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ALGEBRA I

1. The equation of the line through $(-2, 1)$ perpendicular to $3x + 2y = 7$ is

   a) $y = \frac{2}{3}x + \frac{7}{3}$

   b) $y = -\frac{3}{2}x - 2$

   c) $y = \frac{2}{3}x - \frac{1}{3}$

   d) $y = \frac{3}{2}x + 4$

   e) $y = -\frac{2}{3}x - \frac{1}{3}$

2. The solutions of $18x^3 = 8x$ are

   a) $0, \pm\frac{2}{3}$

   b) $\pm\frac{3}{2}, 0$

   c) $\pm\frac{2}{3}$

   d) $\pm\frac{3}{2}$

   e) $\sqrt[3]{\frac{4}{9}}$

3. Solve for x where $2(x+3) - (4x+8) = -5(x-2) + 6$.

   a) $x = -10$

   b) $x = -6$

   c) $x = 6$

   d) no solution

   e) $x = \frac{30}{-7}$

4. Solve for x where $\frac{3}{x-3} - \frac{1}{x+3} = \frac{18}{x^2-9}$.

   a) $x = 3$

   b) $x = 6$

   c) $\pm 3\sqrt{2}$

   d) no solution

   e) $x = -3$

5. All values of x which solve the inequality $3(x+4)-(1+5x) \geq 3x-4$ are given by

a) $-3 \geq x$
b) $x \geq -\frac{7}{5}$
c) $x \leq 3$
d) $3 \leq x$
e) $x \leq -\frac{7}{5}$

6. Intensity of sound (I) varies inversely to the square of distance (D) from the source. In order for the intensity to double it would be necessary to:

a) Multiply D by 2.
b) Divide D by 4.
c) Multiply D by $\sqrt{2}$.
d) Divide D by $\sqrt{2}$.
e) Divide D by 2.

7. Balls numbered 1 through 10 are placed into a container. Two balls are drawn out at random (one at a time with replacement). The probability that the sum of the two numbers drawn is less than or equal to 10 is

a) .50
b) .45
c) .55
d) < .45
e) < .55

8. Solve for x :

$$\begin{bmatrix} x & 3 \\ 5 & y \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 2 & 9 \end{bmatrix} - \begin{bmatrix} y & 1 \\ -3 & 5 \end{bmatrix}$$

a) $x = 3$
b) $x = 1$
c) $x = -1$
d) $x = -3$
e) $x = 4$

9. $\dfrac{(3.0 \times 10^4)^3}{6.0 \times 10^2} =$

a) 4.5×10^9
b) 5.0×10^9
c) 4.5×10^{10}
d) 5.0×10^{11}
e) 5.0×10^4

10. The operation " ♦ " as defined over the real numbers by $a \blacklozenge b = a^2 + b^2$ is

a) associative and commutative
b) associative but not commutative
c) commutative but not associative
d) neither associative nor commutative
e) not a binary operation on the real numbers

11. Written in complex form: $\frac{4-3i}{2+i} =$

a) $1-2i$
b) $2-3i$
c) 2
d) $\frac{11}{3} - \frac{10}{3}i$
e) $\frac{10}{3} - \frac{11}{3}i$

12. If x and y are real numbers satisfying $2x+3y=13$ and $3x-4y=45$, then $4x+5y=$

a) 16
b) 51
c) 0
d) -11
e) 29

13. $\dfrac{5}{x - \dfrac{5}{x + \dfrac{5}{x}}} =$

a) $\frac{5}{x^3}$
b) $\frac{5(x^2+5)}{x^3}$
c) $\frac{5}{x^3-25}$
d) $\frac{5}{x^3+5x-5}$
e) $\frac{x^2+5}{5x^3}$

14. If $U = \{ a, b, c, d, e \}$
$A = \{ a, c, e \}$
$B = \{ a, b, e \}$
$C = \{ b, c, d \}$

then $(A' \cup B) \cap C$ = _____.

a) U
b) $\{ b, e \}$
c) $\{ a, b, d, e \}$
d) $\varnothing$
e) $\{ b, d \}$

15. Which function's graph is shown in figure 1?

a) $f(x) = 5x^5 - 3x^3 + 3x^2 - 5$
b) $f(x) = 2x^4 - x^3 + 8x^2 - 4x - 5$
c) $f(x) = -x^4 - 5x^3 + 9x^2 + 2x - 5$
d) $f(x) = -4x^3 + 12x^2 - 3x - 5$
e) $f(x) = 3x^3 + 9x^3 - 7x^2 - 5$

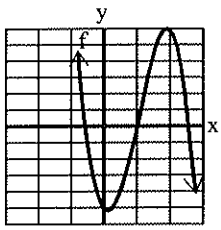

Figure 1.

16. Written in scientific notation, (0.000123)(0.000000001) gives a result of

a) 0.00000000000123
b) 1.23×10^{-11}
c) 1.23×10^{13}
d) 1.23×10^{-12}
e) 1.23×10^{-13}

17. The bouncing pin in the SPRINT© commercial goes up only one third of the height of its previous jump. How high will it jump on the third bounce if the first time it goes up 18 mm?

a) 2 mm
b) 4 mm
c) 6 mm
d) 4.5 mm
e) 8 mm

18. Given set $A = \{ \%, @, \& \}$ and operations * and # defined by the tables

*	%	@	&
%	%	@	&
@	@	&	%
&	&	%	@

#	%	@	&
%	%	%	%
@	%	@	&
&	%	&	@

(% * &) # (@ * @) = _____.

a) & # @
b) *
c) &
d) @
e) % # @

19. Which of the following sets of ordered pairs does <u>not</u> define a function?

a) $\{(-1,2),(4,3),(-2,1),(2,0)\}$
b) $\{(1,-3),(3,3),(-1,-3),(2,0)\}$
c) $\{(1,-2),(4,-4),(-2,3),(1,0)\}$
d) $\{(-2,2),(2,2),(-1,2),(1,2)\}$
e) $\{(0,4),(1,3),(2,2),(3,1)\}$

20. The graph of the polynomial $P(x) = 5(x-1)^2 - 4$ is a parabola with axis of symmetry and vertex:

a) $x = 5$; $(-1, -4)$
b) $x = -5$; $(-1, -4)$
c) $x = 1$; $(1, -4)$
d) $x = -1$; $(1, -4)$
e) $x = 1$; $(1, 4)$

21. The center C and radius r of the circle $x^2 + y^2 - 18x - 16y + 125 = 16$ are:

a) $C = (9, 8)$; $r = 6$
b) $C = (-8, -9)$; $r = 36$
c) $C = (8, 9)$; $r = 6$
d) $C = (9, 8)$; $r = 36$
e) $C = (-9, -8)$; $r = 6$

22. $(1-2x^2)^3 =$

a) $-(4x^3-4x^2+1)^3$
b) $-8x^6-12x^4-6x^2-1$
c) $1-8x^6$
d) $-16x^8+8x^6-12x^4+6x^2+1$
e) $-8x^6+12x^4-6x^2+1$

23. The solution to the equation $\left(\frac{1}{2}\right)^x = 16$ is

a) $\frac{1}{4}$
b) -4
c) -8
d) $-\frac{1}{4}$
e) 4

24. The inequality $\frac{x-4}{x+4} < 0$ has as its solution set

a) $(-\infty, 0)$
b) $(-\infty, 4)$
c) $(-\infty, -4)$
d) $(-4, 4)$
e) $(-\infty, -4) \cup (-4, 4)$

25. Solve $|3x+6| = -3$

a) -3
b) -1
c) 1
d) 3
e) no solution

26. The additive inverse of $\frac{x-y}{xy}$ is

a) $\frac{xy}{x-y}$
b) $\frac{y-x}{xy}$
c) $-\frac{xy}{x-y}$
d) $\frac{x-y}{yx}$
e) $\frac{xy}{y-x}$

27. Given the function $f(x) = 2x^2 + 4$ evaluate the difference quotient $\frac{f(x+h)-f(x)}{h} =$

a) $\frac{4x^2+4hx+h^2+8}{h}$

b) $4x+2h$

c) $\frac{2h^2+4}{h}$

d) $\frac{-x^2+2hx+h^2+8}{h}$

e) 1

28. Solve for x in $|3x-4|+20=24$.

a) $0, -\frac{40}{3}$

b) $0, -\frac{8}{3}$

c) $\frac{8}{3}, -\frac{40}{3}$

d) $-\frac{8}{3}, \frac{40}{3}$

e) $0, \frac{8}{3}$

29. Simplify the following expression.

$$\frac{2xy-y}{8x^2-12xy} \div \frac{3xy-6y}{12x^2-24x}$$

a) $\frac{1}{3y}$

b) $\frac{2x-1}{2x-3y}$

c) $\frac{y^2(2x-1)}{16x^2(2x-3y)}$

d) $\frac{y^2}{48x^2y}$

e) $\frac{y-1}{-8x}$

30. $x^2+4=-5x$ has <u>no</u> solution if the replacement set is

a) positive integers
b) rational numbers
c) complex numbers
d) real numbers
e) negative integers

31. Simplify the following expression.

$$\left(\frac{16a^{-8}b^{5}}{a^{4}b^{7}}\right)^{\frac{1}{4}}$$

a) $\dfrac{16}{a^{3}\sqrt{b}}$

b) $\dfrac{4}{a\sqrt{b}}$

c) $\dfrac{2}{a\sqrt{b}}$

d) $\dfrac{2}{a^{3}\sqrt{b}}$

e) $\dfrac{2}{\sqrt{ab}}$

32. The domain of $f(x) = \dfrac{\sqrt{3x-12}}{x-4}$ is

a) $\{x \mid x \in \mathbb{R}\}$

b) $\{x \mid x \in \mathbb{R},\ x > 4\}$

c) $\{x \mid x \in \mathbb{R},\ x \neq 4\}$

d) $\{x \mid x \in \mathbb{R},\ x \geq 4\}$

e) $\{x \mid x \in \mathbb{R},\ x > 0\}$

33. If $F(x) = 3x^{2} + 2x + 1$, then $F(x^{2} - 1) =$?

a) $9x^{4} + 12x^{3} + 10x^{2} + 4x$

b) $3x^{4} + 2x^{3} - 3x^{2} - 2x + 1$

c) $9x^{4} - 16x^{2} + 8$

d) $3x^{4} + 2x^{3} - 2x^{2} - 2x + 1$

e) $3x^{4} - 4x^{2} + 2$

34. In math class, John has taken 3 tests, each worth 20% of his total grade, and has made grades of 92%, 83%, and 85%, respectively. He has a 100% average on his homework, worth 10% of his final grade. All that remains is a final exam worth 30% of his total grade. To the nearest percent, what must he make on the final to have a 90% average in the class?

a) 88%

b) 90%

c) 93%

d) 70%

e) 95%

35. Simplify: $\dfrac{by - cy + cr - br}{y^2 - r^2}$

a) $\dfrac{b-c}{y-r}$

b) $\dfrac{(b-c)(c-b)}{y-r}$

c) $\dfrac{b-c}{y+r}$

d) $\dfrac{-(b-c)}{y+r}$

e) $\dfrac{-1}{y-r}$

36. Find the solution set of this equation

$$2x^3 + 4x^2 - 8x - 7 = 9$$

a) $\{0, -4, 2\}$

b) $\{4, -4, -2\}$

c) $\{-2, 2, 2\}$

d) $\{2, -2, -2\}$

e) $\{0, 2, -2\}$

37. When Harold and Kay work together to do a job it takes 6 days. If Harold works alone it takes him 10 days. How long would it take Kay to do the job alone?

Which of the following equations should you use to solve this problem?

a) $x = 10 - 6$

b) $\dfrac{1}{x} + \dfrac{1}{10} = \dfrac{1}{6}$

c) $\dfrac{1}{x} = \dfrac{1}{10} - \dfrac{1}{6}$

d) $x = \dfrac{1}{10} - \dfrac{1}{6}$

e) $\dfrac{1}{x} = \dfrac{1}{10} + \dfrac{1}{6}$

38. Which of the following rational numbers is closest to zero on the number line?

a) -0.107

b) $\frac{-3}{7}$

c) $\frac{1}{18}$

d) -0.113

e) $\frac{2}{51}$

39. A new long-distance telephone service promises a 59% savings on international calls. If your current long-distance carrier charged \$9.75 for a call to London, what would you expect the same call to cost, to the nearest penny, using the new carrier?

a) \$4.00
b) \$9.16
c) \$3.99
d) \$9.17
e) \$5.75

40. Which of the following is the equation of a line parallel to the line whose equation is $y = \frac{3}{5}x + 1$?

a) $6x + 10y = 13$

b) $5x - 3y = 6$

c) $3x - 5y = 6$

d) $y = \frac{5}{3}x - 6$

e) $\frac{5}{6}x - \frac{1}{2}y = 1$