

FORTIETH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra I 1996

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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1. Simplify:

$$-|-4| - \frac{16 - (-2)^2}{-3^3 - (-3)(5)} \cdot -9$$

- a. 36
- b. -13
- c. -45
- d. 14
- e. 4

2. Simplify:

$$-2[8 - 2(4 - 3x) - (x + 3)]$$

- a. $-19x + 54$
- b. $-38x + 54$
- c. $-10x - 3$
- d. $-19x + 38$
- e. $-10x + 6$

3. Simplify:

$$\frac{(-2x^4y^5)^3}{(8x^3y^2)^2}$$

- a. $-\frac{x^2y^5}{4}$
- b. $-\frac{x^6y^{11}}{8}$
- c. $-\frac{x^{18}y^{19}}{4}$
- d. $\frac{x^2y^5}{512}$
- e. $-\frac{x^{14}y^{12}}{16}$

4. Which of the following is a factor of $6x^2 - 34x + 20$?

- a. $(2x - 4)$
- b. $(3x + 4)$
- c. $(3x + 2)$
- d. $(3x - 2)$
- e. $(x - 2)$

5. Solve:

$$|5 - 2x| + 3 < 10$$

a. $(-1, 9)$

b. $\left(-\infty, -\frac{15}{2}\right) \cap \left(-\frac{5}{2}, \infty\right)$

c. $(-1, 6)$

d. $\left(-\frac{15}{2}, -\frac{5}{2}\right)$

e. $\left(-\frac{5}{2}, \infty\right)$

6. A bag contains 3 red gumballs, 5 green gumballs, and 4 yellow gumballs. If one gumball is chosen from the bag, then what is the probability that it will be a yellow gumball?

a. $\frac{1}{3}$

b. $\frac{1}{4}$

c. $\frac{1}{12}$

d. $\frac{1}{5}$

e. $\frac{1}{2}$

7. Solve for x:

$$3 - \sqrt{3x + 1} = x$$

a. $x = 0$ and $x = 3$

b. $x = 0$ only

c. $x = 8$ and $x = 1$

d. $x = 1$ only

e. $x = 3$ and $x = -3$

8. Simplify:

$$\frac{5 + \frac{5}{x-3}}{4 + \frac{4}{x-3}}$$

a. $\frac{5}{4}$

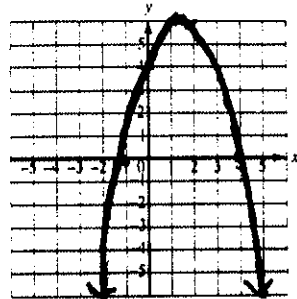
b. $x + 9$

c. $\frac{5x+2}{4x+1}$

d. $\frac{4}{5}$

e. $x - 3$

9. Which equation represents the graph of:



a. $y = x^2 + 4$

b. $y = -x^2 + 4$

c. $y = -x^2 + 3x + 4$

d. $y = x^2 - 3x - 4$

e. $y = -x^2 - 4$

10. What is the value for y in the system of equations below?

$$4x - 3y = 7$$

$$y = 2x - 5$$

a. -3

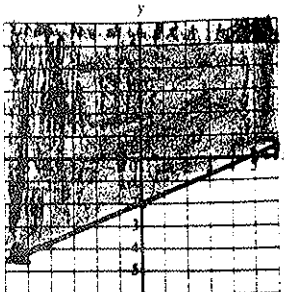
b. -1

c. -4

d. 3

e. 4

11. The shaded area of the graph below represents which of the following?



- a. $2x - 5y \geq 10$
 b. $2x + 5y \geq 10$
 c. $5x - 2y \leq 10$
 d. $5x + 2y \geq 10$
 e. $2x - 5y \leq 10$
12. Factor Completely:
- $2x^3 - x - 4x^2 + 2$
- a. $(2x - 1)(x^2 - 2)$
 b. $(2x^2 - 1)(x - 2)$
 c. $2x(x - 1)(x - 2)$
 d. $(2x + 1)(x - 1)(x - 2)$
 e. $(x + 2)(2x^2 - 4x - 1)$
13. A piggy bank contains dimes and nickels. There are 18 more nickels than dimes. The total value of the coins in the bank is \$2.70. Find the number of nickels in the banks.
- a. 28
 b. 30
 c. 12
 d. 42
 e. 24
14. Solve for p:
 $3p - 4q = ap + q$
- a. $p = -\frac{5q}{3a}$
 b. $p = \frac{5q + ap}{3}$
 c. $p = \frac{3p - 5p}{a}$
 d. $p = \frac{5q}{3 - a}$
 e. $p = -\frac{3a}{5q}$

15. Solve for x:

$$\frac{2x}{4x - 3} = \frac{x + 2}{2x + 3}$$

- a. 2
- b. -3
- c. -6
- d. 0
- e. 4

16. Find the equation of the line passing through the point $(-3, 2)$ with the slope $-\frac{4}{3}$.

- a. $4x + 3y = -18$
- b. $3x + 4y = 6$
- c. $4x + 3y = -6$
- d. $3x - 4y = 18$
- e. $4x - 3y = 6$

17. Simplify:

$$\frac{3}{2x - 1} - \frac{2}{3x + 5}$$

- a. $\frac{-1}{5x + 4}$
- b. $\frac{13x + 13}{(x - 5)(2x + 1)}$
- c. $\frac{5x^2 + 13}{(x - 5)(2x + 1)}$
- d. $\frac{-1}{(x - 5)(2x + 1)}$
- e. $\frac{5x + 17}{(2x - 1)(3x + 5)}$

18. A ball is thrown down from the top of a building that is 240 feet high. The initial speed of the ball is 32 ft/sec. Use the equation $d = vt + 16t^2$ to find how many seconds later the ball will hit the ground (d is the distance in feet, v is the initial speed of the ball, and t is the time in seconds.)

- a. 7.5 seconds
- b. 5 seconds
- c. 5 seconds
- d. 3 seconds
- e. 2 seconds

19. Find the equation of the line passing through the point $(-3, -5)$ and perpendicular to the line $3x - 5y = 7$.

a. $y = 3x + 9$

b. $y = \frac{1}{3}x - 4$

c. $y = -\frac{5}{3}x - 10$

d. $y = \frac{3}{5}x - \frac{6}{5}$

e. $y = -5$

20. Factor Completely:

$$4x^2 + 20xy + 25y^2 - 100$$

a. $(2x + 5y + 10)(2x + 5y - 10)$

b. $(4x + 5y)^2(x + 5)(x - 5)$

c. $(4x + 5y + 10)(x + 5)(x - 5)$

d. $(4x + 25)(x + 5y)(y + 2)(y - 2)$

e. $(2x + 5)^2(x + 5y)(y + 2)(y - 2)$

21. If $a * b = a^2 + b^2$, then find $(x - 2) * (-2x)$

a. $x^2 - 4x - 4$

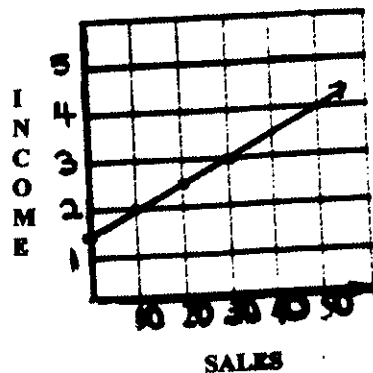
b. $5x^2 - 4x + 4$

c. $x^2 - 6x + 4$

d. $5x^2 + 4x + 4$

e. $x^2 + 4$

22. Jason is a salesperson for a large manufacturing firm. The graph below depicts the relationship between Jasons' monthly income (in thousands of dollars) and his sales (in thousands of dollars). Find an equation of the line representing Jasons' income.



- a. $I = .5s + 1.5$
 b. $I = .05s + 1500$
 c. $I = .2s + 1000$
 d. $I = .02s + 1500$
 e. $I = 2s + 1000$

23. Simplify:

$$\frac{\sqrt{2}}{\sqrt{5} - 2}$$

a. $\frac{2}{3}$

b. $\sqrt{10} + 2$

c. 2

d. $\sqrt{10} + 2\sqrt{2}$

e. $\sqrt{10} + 2$

24. Simplify:

$$(2x^{-2}y^5)^{-3}$$

a. $\frac{-8y^2}{x^5}$

b. $\frac{y^2}{8x^5}$

c. $-\frac{y^2}{8x^5}$

d. $\frac{-8x^6}{y^{15}}$

e. $\frac{x^6}{8y^{15}}$

25. Solve for x:

$$-3[2(3x - 5) - 2(5x - 3)] - 8 = 4(2x - 5)$$

a. $x = -6$

b. $x = 6$

c. $x = -2$

d. $x = 2$

e. $x = 8$

26. Find the next term in the sequence.
625, 125, 25, 5, 1, _____

a. -5

b. $\frac{1}{5}$

c. 5

d. $-\frac{1}{5}$

e. 25

27. Find the solutions to the systems of equations.

$$x^2 - y^2 = 12$$

$$y = \frac{1}{2}x$$

a. (6, 4), (-6, -4)

b. (± 4 , ± 6)

c. (4, 2), (-4, -2)

d. (± 6 , ± 4)

e. (2, 4), (-2, -4)

28. Find the roots:

$$x^2 - 6x + 6 = 0$$

a. $-3 \pm \sqrt{3}$

b. $3 \pm \sqrt{15}$

c. $-3 \pm \sqrt{15}$

d. $3 \pm 2\sqrt{3}$

e. $3 \pm \sqrt{3}$

29. Which of the following equations must be true for the equation:
- $$|a + b| = 0$$
- a. $|a| + |b| = 0$
- b. $|a| = -|b|$
- c. $|a| - |b| = 0$
- d. $a = b$
- e. $|a + b| - |a - b| = 0$
30. If R varies inversely as T and R = 36 when T = 3.5, then find R when T = 3.
- a. 54
- b. 33
- c. 30
- d. 42
- e. 18
31. In a triangle, the measure of one angle is 12° more than the measure of the second angle. The third angle is 4° less than the sum of the two other angles. Find the measure of the third angle.
- a. 52°
- b. 97°
- c. 45°
- d. 88°
- e. 76°
32. Simplify:
- $$3\sqrt{2} + \sqrt{8}$$
- a. $4\sqrt{10}$
- b. $5\sqrt{10}$
- c. $5\sqrt{4}$
- d. 12
- e. $5\sqrt{2}$

33. Find the domain of the following function.

$$f(x) = \frac{x - 3}{x - 7}$$

- a. all real numbers
- b. $x \neq 3$
- c. $x \neq -3$
- d. $x \neq 7$
- e. $x \neq -7$

34. Find the equation of the line containing the points $(-2, 3)$ and $(4, -2)$.

a. $y = \frac{5}{6}x - \frac{4}{3}$

b. $y = -\frac{5}{6}x + \frac{4}{3}$

c. $y = \frac{6}{5}x + \frac{27}{5}$

d. $y = -\frac{6}{5}x + \frac{3}{5}$

e. $y = -2$

35. What is the remainder when

$$\frac{x^3 + 2x^2 - 5x + 30}{x + 3}$$

- a. 0
- b. -60
- c. 60
- d. -36
- e. 36

36.

Let $f(x) = x^2 - \frac{x}{2} + \frac{1}{4}$ with $g(x) = x^{-2}$,

then find $f(g(-2))$.

a. $\frac{3}{16}$

b. $-\frac{3}{16}$

c. $\frac{71}{4}$

d. $-\frac{71}{4}$

e. $\frac{57}{4}$

37. An experienced painter can paint a large room in 4 hours. With his assistant's help the room can be painted in 3 hours. How long would it take the assistant working alone to paint the room?

- a. 2 hours
- b. 6 hours
- c. 12 hours
- d. 16 hours
- e. 20 hours

38. Solve for z

$$\frac{z + 6}{7} \geq \frac{z + 7}{6} + 2$$

- a. $z \leq -15$
- b. $z \geq 15$
- c. $z \leq -97$
- d. $z \geq 97$
- e. $z \leq 3$

39. If $x = 2 + 3\sqrt{5}$, then $x^2 - 5x - 15 =$

a. -51

b. $24 - 15\sqrt{5}$

c. $24 + 15\sqrt{5}$

d. $24 - 3\sqrt{5}$

e. $24 + 3\sqrt{5}$

40. If a , b , and c are natural numbers such that $a < b$ and $b < c$, then which of the following is **false**.

a. $c > a$

b. $ab < bc$

c. $-a > -c$

d. $-\frac{a^2}{b} > -\frac{c^2}{b}$

e. $\left(-\frac{a}{b}\right)^2 > \left(-\frac{c}{b}\right)^2$

