THIRTY-SEVENTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I 1993

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Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.
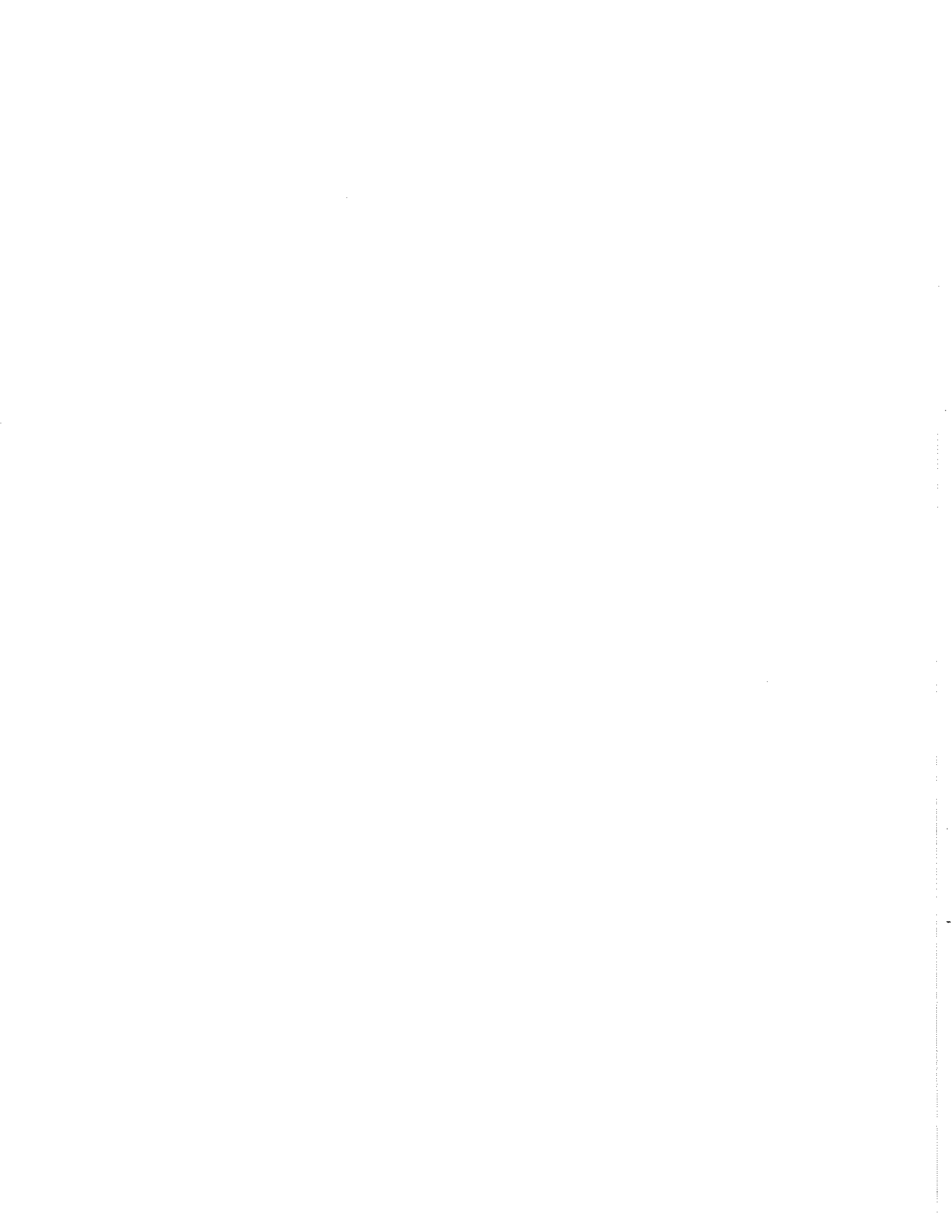

Contributors to TMTA for Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee

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NOTE: 1994 CONTEST DATE--APRIL 12

1. If $a > 0$ and $b < 0$,   then $|a - b|$ = ?

    A. $-a - b$

    B. $a + b$

    C. $b - a$

    D. $a - b$

    E. $2a$

2. Which one of the following events is *commutative*?

    A. Getting out of bed and taking a shower

    B. Studying for an algebra test and taking an algebra test

    C. Putting on your right shoe and putting on your left shoe

    D. Putting on your shoe and putting on your sock

    E. Turning on the television and watching television

3. Chris owed his brother $12. He repaid $9 and later borrowed $13. What positive or negative number represents his financial status?

    A. 16

    B. 34

    C. 8

    D. -16

    E. -34

4. A plane ticket costs x dollars for an adult and y dollars for a child. Represent the total cost for 3 adults and 2 children.

    A. $5x + 5y$

    B. $2x + 3y$

    C. $3x - 2y$

    D. $3x + 2y$

    E. $x - y$

5. If $p < q$ and $r < 0$, which one of the following statements is <u>false</u>?

    A. pr < qr

    B. pr > qr

    C. p + r < q + r

    D. p - r < q - r

    E. p - q < 0

6. Suppose the binary operation, *, on the set, Z, of integers is defined by a * b = a + b + 1. What is the identity element for the operation, * ?

    A. 0

    B. 1

    C. -1

    D. no identity element

    E. -a

7. Find the missing numerator for

$$\frac{5}{x^2 - x} = \frac{?}{x^3 - x}$$

    A. $5x(x + 1)$
    B. $5x(x + 1)(x - 1)^2$
    C. $5x + 1$
    D. $5(x + 1)$
    E. $x(x - 1)$

8. Which statement is <u>always</u> true?

    A. A polynomial is a trinomial.

    B. A trinomial is a binomial.

    C. A binomial is a polynomial.

    D. A polynomial of degree 4 has 4 terms.

    E. A monomial has no coefficient.

9. What rational number <u>cannot</u> be represented by

$$\frac{3x}{x(x-1)} \ ?$$

    A. $\frac{3}{2}$

    B. $-\frac{3}{2}$

    C. 1

    D. $\frac{1}{2}$

    E. 0

10. Solve for P: $A - P = Prt$

    A. $P = Art - 1$

    B. $P = A - rt$

    C. $P = \dfrac{A}{1 + rt}$

    D. $P = \dfrac{A}{1 - rt}$

    E. $P = A + Prt$

11. A printing press can complete a job in 1 hour 15 minutes. A newer press can complete the same job in 50 minutes. How long would it take for the two presses to complete the job if they worked together?

    A. 30 minutes

    B. 62.5 minutes

    C. 20 minutes

    D. 25 minutes

    E. 2 hours 5 minutes

12. In how many ways can 3 white balls and 2 black ones be arranged in a row such that the 2 black balls are <u>not</u> adjacent to each other?

    A. 120

    B. 10

    C. 48

    D. 72

    E. 5

13. Suppose an urn contains 3 white balls and 2 black balls. Two balls are drawn at random without replacement. What is the probability that both are white?

    A. $\dfrac{1}{5}$

    B. $\dfrac{3}{10}$

    C. $\dfrac{3}{5}$

    D. $\dfrac{5}{6}$

    E. $\dfrac{6}{25}$

14. *Divide 2^6 by $\dfrac{1}{2}$*

    A. 2^3

    B. 4^6

    C. 2^5

    D. 2^7

    E. 1^6

15. Evaluate $3^{-1} + 3^{-2}$

    A. $\dfrac{4}{9}$

    B. 1

    C. $\dfrac{1}{27}$

    D. $\dfrac{1}{12}$

    E. 9

16. *If $x^3 = 10$, what does $x^6 = ?$*

    A. 20

    B. 60

    C. 100

    D. 1000

    E. 30

17. Assume <u>a</u> is a positive number greater than one. Arrange the following terms in order from smallest to largest:

$-(-a)^3, \quad -a^3, \quad (-a)^4, \quad -a^4$

A. $-a^4, \quad -a^3, \quad -(-a)^3, \quad (-a)^4$

B. $(-a)^4, \quad -(-a)^3, \quad -a^3, \quad -a^4$

C. $-(-a)^3, \quad -a^3, \quad (-a)^4, \quad -a^4$

D. $-a^3, \quad -a^4, \quad -(-a)^3, \quad (-a)^4$

E. $(-a)^4, \quad -a^4, \quad -(-a)^3, \quad -a^3$

18. What is the multiplicative inverse of a^{-3} ?

A. $\dfrac{1}{a^3}$

B. a^3

C. $-\dfrac{1}{a^3}$

D. $-a^3$

E. 1

19. The lines with equations $\dfrac{x}{2} + \dfrac{y}{3} = 1$ and $6x + 2y = 13$ will intersect in which quadrant?

A. I

B. II

C. III

D. IV

E. On a coordinate axis

20. Which of the following is a factor of $12z^2 - 50z - 28$?
A. $2z - 7$

B. $3z + 14$

C. $2z + 1$

D. $3z + 2$

E. $6z - 7$

21. When an object is dropped, the distance that it falls in t seconds varies directly as the square of t. If an object falls 400 feet in 5 seconds, how far will it fall in 8 seconds?

    A.   128 feet

    B.   256 feet

    C.   640 feet

    D.   1024 feet

    E.   16,384 feet

22. The <u>complete</u> factorization, over the rational numbers, for $8x^3 + 20x^2 - 18x - 45$ is _____?_____

    A. $(4x^2 + 9)(2x - 5)$

    B. $(2x + 3)(2x - 3)(2x + 5)$

    C. $(2x + 3)(2x - 3)(2x - 5)$

    D. $(2x - 3)(2x - 3)(2x + 5)$

    E. Will not factor.

23. Which one of the following statements is true about the line, $4x - 3y - 12 = 0$?

    A.   The x-intercept is 4.

    B.   The y-intercept is -4.

    C.   The slope is -4/3.

    D.   The point, (3,-4), in on the graph of the line.

    E.   The line is perpendicular to the line $3x - 4y = 25$.

24. Find the width of a rectangle with a perimeter of 54 meters if its length is 3 meters less than twice its width.

    A.   17 meters

    B.   15 meters

    C.   14 meters

    D.   10 meters

    E.   8 meters

25. Determine all integers $\underline{k}$ so that $2x^2 + kx + 5$ may be factored on the integers.

    A. 7, -7

    B. 11, -11

    C. 7, -7, 11, -11

    D. 7, 11

    E. 7, -7, 11, -11, 2, 5

26. An equation of the line passing through $(-2,-8)$ and parallel to the line $5x + y = 11$ is

    A. $5x + y = -18$

    B. $5x + y = 18$

    C. $5x + y = 2$

    D. $2x + 8y = 11$

    E. $-2x - 8y = 11$

27. Determine $\underline{k}$ so that the points $(k,3)$ and $(-1,k)$ are on a line perpendicular to the line, $y = 3x + 5$.

    A. 5/2

    B. 5

    C. -2

    D. 0

    E. 2

28. The shaded area represents which of the following?

    A. $3x + 4y \geq 6$

    B. $3x - 4y \leq -18$

    C. $4x - 3y \leq -17$

    D. $3x - 4y \geq -18$

    E. $4x - 3y \geq -17$
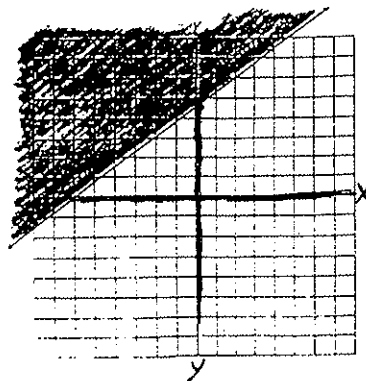
29. A shoe store sells a pair of tennis shoes that costs $40 for $52 and a pair of basketball shoes that costs $50 for $77. If the markup policy of the store for items that cost more than $30 is linear and is reflected in the pricing of these two items, find the selling price of a pair of running shoes that costs $60.

   A.   85

   B.   87

   C.   92

   D.   97

   E.   102

30. $$\dfrac{\dfrac{2x}{x-1}}{\dfrac{2x}{x-1}-2} = \ ?$$

   A. x

   B. $x(x-2)$

   C. -2

   D. $\dfrac{2x}{x-1}$

   E. $\dfrac{3x-4}{2}$

31. The solution set for the inequality, $2|5-2x| > 6$, is ___?___

   A. $\{x:\ x < 1\}\ \cup\ \{x:\ x > 4\}$

   B. $\{x:\ x < -\dfrac{1}{2}\}\ \cup\ \{x:\ x > \dfrac{9}{2}\}$

   C. $\{x:\ 1 < x < 4\}$

   D. $\{x:\ -\dfrac{1}{2} < x < \dfrac{9}{2}\}$

   E. $\{x:\ x < 2\}$

32. Find the radius of a circle if $(-6,1)$ and $(2,-3)$ are the endpoints of a diameter of the circle.

    A. 6

    B. 12

    C. $\sqrt{5}$

    D. $2\sqrt{5}$

    E. $4\sqrt{5}$

33. Solve for x: $1 - \sqrt{2x + 1} = x$

    A. $x = 4$

    B. $x = 0$

    C. $x = 2$ and $x = -2$

    D. $x = 4$ and $x = 0$

    E. $x = 0$ and $x = 2$

34. Solve for q: $\dfrac{q}{7} - 3 > \dfrac{q - 4}{3} + 1$

    A. $q < -14$

    B. $q > -14$

    C. $q < -7$

    D. $q > -7$

    E. $q < 3$

35. The trajectory of a circus performer, Marvin the Magnificent, shot from a cannon is given by the graph of the function

$$f(x) = x - \frac{1}{100} x^2.$$

What is the maximum height of Marvin above the muzzle of the cannon?

    A. 15
    B. 25
    C. 50
    D. 75
    E. 100

36. Given that $a^u = a^v$ *if and only if* $u = v$. What is the sum of the solutions to the equation

$$2^{x^2} \cdot 4^{-2x} = \frac{1}{8} ?$$

    A.   −4

    B.   −2

    C.   2

    D.   3

    E.   4

37. An exercise machine is equipped with an odometer that measures the distance ridden in miles and a timer that measures the elapsed time in minutes. The graph below represents John's last session on the machine. What was John's average rate in miles per minute?

ODOMETER READING

ELAPSED TIME IN MINUTES
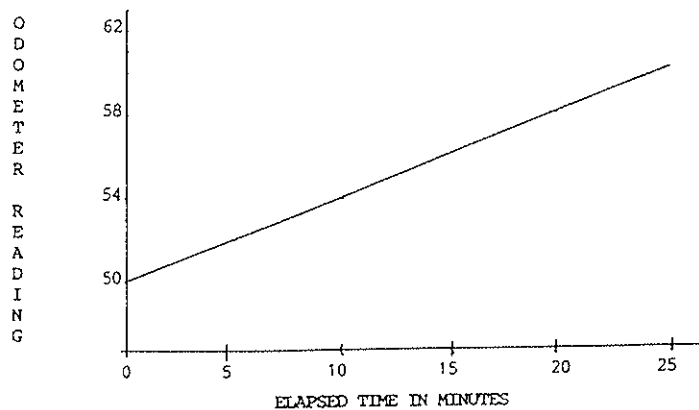
    A.   0.4 miles per minute

    B.   0.8 miles per minute

    C.   2.8 miles per minute

    D.   4.0 miles per minute

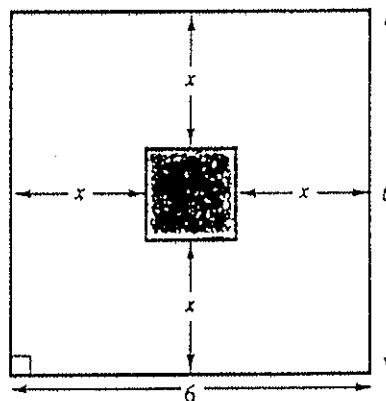   E. 10.4 miles per minute

38. Which of the following is equivalent to the statement "twice $\underline{c}$ is no greater than 7 units from -3?"

    A. $|2c - 3| < 7$

    B. $|2c - 3| \leq 7$

    C. $|2c + 3| < 7$

    D. $|2c + 3| \leq 7$

    E. $|2c - 7| \leq 3$

39. Solve for x: $5x^2 - 4x - 2 = 2x^2 + 2x - 1$

    A. $\dfrac{3 \pm 2\sqrt{3}}{3}$

    B. $\dfrac{3 \pm \sqrt{6}}{3}$

    C. $1 \pm 4\sqrt{3}$

    D. $1 \pm \sqrt{6}$

    E. $\dfrac{3 \pm \sqrt{2}}{3}$

40. Find a polynomial representing the area of the shaded region.

    A. $36 - x^2$

    B. $36 - 4x^2$

    C. $36 - 24x + 4x^2$

    D. $36 - 12x + 4x^2$

    E. $36 - 12x - 4x^2$