THIRTY-FIFTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I   1991

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.
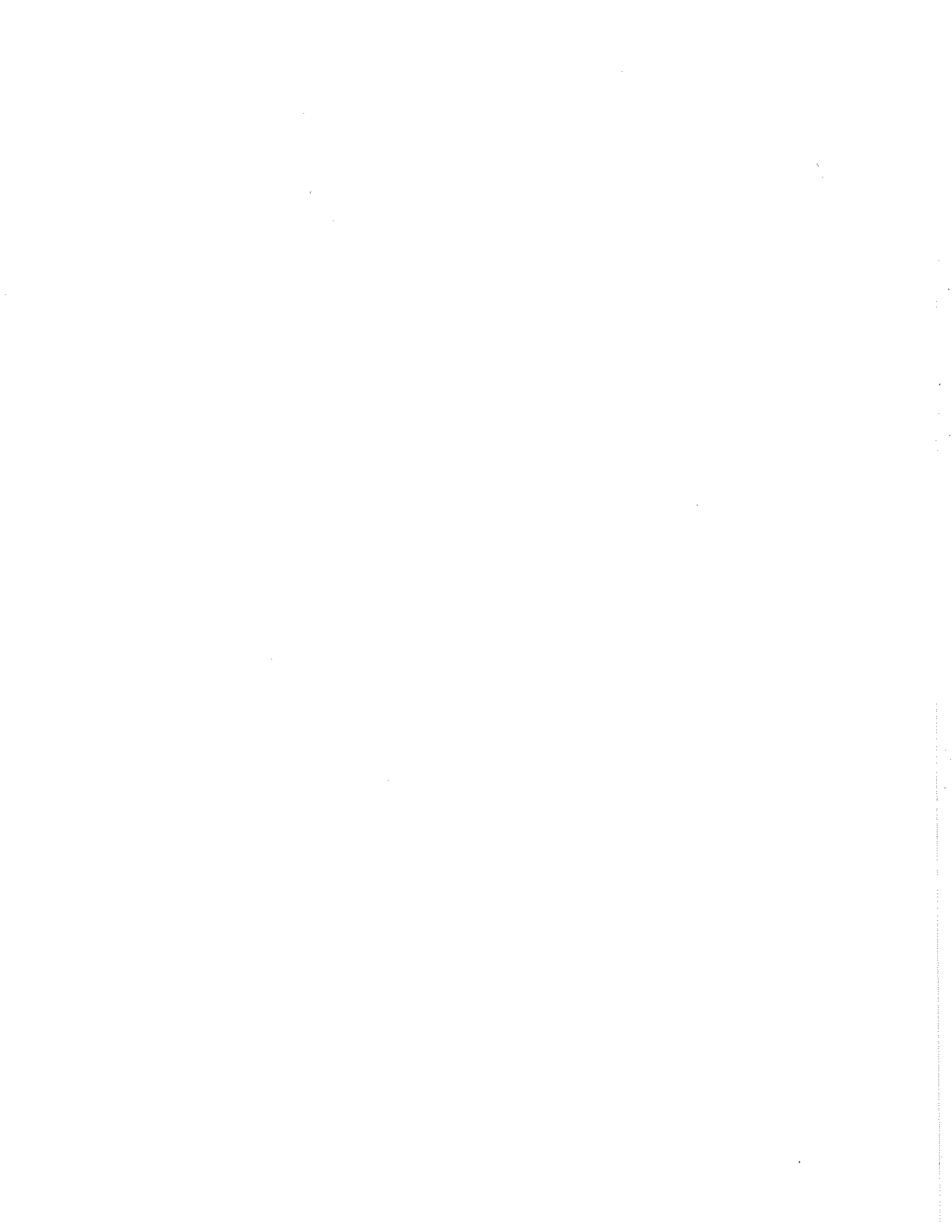
When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

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1.   If a is an integer, which of the following <u>must</u> be an integer?

    (a) $\sqrt{a}$

    (b) $(1/2)^a$

    (c) $\dfrac{\sqrt{a}}{a}$

    (d) $(-1)^a$

    (e) $\dfrac{1}{\sqrt{a}}$

2.   Solve for x over the Real Numbers: $x^4 + 2x^2 - 8 = 0$

    (a) ± 2

    (b) $2 \pm \sqrt{2}$

    (c) $\pm 2, \pm \sqrt{2}$

    (d)   2, -4

    (e) $\pm \sqrt{2}$

3.   The y-intercept of the graph of $3x^2 - 2x - 1 = y$ is:

    (a)   1

    (b)   0

    (c)   -1/3

    (d)   -1

    (e)   1/3

4. Simplify $$\frac{by + cy - br - cr}{b^2 + 6bc + 5c^2}$$

    (a) $$\frac{(y-r)\,(b+c)}{(b+3c)\,(b+2c)}$$

    (b) $$\frac{2y-2r}{b+11}$$

    (c) $$\frac{y-r}{b+5c}$$

    (d) $$\frac{y+r}{b+5c}$$

    (e) cannot be simplified

5. If the expression $15x^2-7x-2$ is factored into $(ax+b)\,(cx-d)$ where a, b, c, and d are positive integers, then a =

    (a) 1

    (b) 3

    (c) 5

    (d) 2

    (e) No such factorization is possible

6. Simplify

$$x + \cfrac{1}{x + \cfrac{1}{x + \cfrac{1}{x}}}$$

    (a) $$\frac{x^4 + 3x^2 + 1}{x^3 + 2x}$$

    (b) $x + 1$

    (c) $$\frac{3x^2 + 3}{x}$$

    (d) $4x + 3$

    (e) $$\frac{3x^2 + 1}{2x}$$

7.   Which relation represents a function?

A = { (0,0) (4,1) (5,0) }
B = { (-3,5) (-1,6) (-1,0) }
C = { (2,1) (3,1) (4,1) }

    (a)   A only

    (b)   B only

    (c)   C only

    (d)   A and B

    (e)   A and C

8.   Which property justifies the following expression: $[a(b+c)]d = a[(b+c)d]$?

    (a)   associative property

    (b)   distributive property

    (c)   closure property

    (d)   identity property

    (e)   commutative property

9.   Find k such that the equation $kx^2 + x + k = 0$ has a repeated real solution.

    (a) $k = [1,-1]$

    (b) $k = 0$

    (c) $k = [1/2,-1/2]$

    (d) $k = [4,-4]$

    (e) $k = [-2,2]$

10. Charles scored 22 points in one basketball game and 19 points in another. What is the least number of points he must score in the next game in order to have an average greater than 20 for the 3 games?

    (a)   18

    (b)   19

    (c)   20

    (d)   21

    (e)   22

11. For what value(s) of x is the rational expression undefined?
$$\frac{x}{6x^2 + x - 2}$$

    (a)   0

    (b)   1/2

    (c)   1/2 and -2/3

    (d)   -2/3

    (e)   0, 1/2 and -2/3

12. The least common multiple of the polynomials $2x^2 - 11x + 5$ and $4x^2 - 8x + 3$ is

    (a) $2x - 1$

    (b) $4x^2 - 4x + 1$

    (c) $4x^3 - 28x^2 + 43x - 15$

    (d) $2x^2 - 13x + 15$

    (e) $8x^4 - 60x^3 + 114x^2 - 73x + 15$

13. What is the solution of the equation $\sqrt{7y - 5} - 6 = 4$

    (a)   9/7

    (b)   15

    (c)   15/7

    (d)   95/7

    (e)   no real solution

14. When simplified, $x^n(x^2 + 2x + 1)$ becomes

    (a) $x^{2n} + 2x^n + x^n$

    (b) $xn^2 + 2x^{n+2} + x^n$

    (c) $x^{n+2} + 2x^{2n} + x^n$

    (d) $x^{n+2} + 2x^{n+1} + x^n$

    (e) $x^{2n} + 2x^{n+1} + 1$

15. What is the length of the diagonal of a rectangle whose length is 12m and whose width is 5m?

    (a)   17m

    (b)   13m

    (c)   7m

    (d) $\sqrt{13}$m

    (e) $\sqrt{17}$m

16. $\left(\dfrac{64}{125}\right)^{2/3}\left(\dfrac{x^{-2}y^5}{x^2y^{-5}}\right)^{1/2} =$

    (a)   16/25

    (b) $\dfrac{4\ x}{5\ y}$

    (c) $\dfrac{16}{25xy}$

    (d) $\dfrac{16y^5}{25x^2}$

    (e) $\dfrac{16x^2}{25y^2}$

17. The sum of $(x - 1)^2 + (x - 2)^2 =$

    (a) $2x - 3$

    (b) $2x^2 - 6x + 5$

    (c) $2x^2 - 5$

    (d) $2x^2 + 5$

    (e) $2x^2 - 6x - 5$

18. Katie, who lives in the country, works in the city which is located 36 miles from her home. Every morning she leaves her home at 6:30 a.m. and drives to the city. At the end of her working day she returns home on the same route that she had followed that morning. Which one of the following mathematical properties best describes her daily routine?

    (a)   associative property

    (b)   distributive property

    (c)   closure property

    (d)   identity property

    (e)   commutative property

19. Write $(a^2 - 9)^{-2/5}$ in radical form

    (a) $\sqrt[5]{a^2-9}$

    (b) $\sqrt[5]{(a^2-9)^2}$

    (c) $\dfrac{1}{\sqrt[5]{a^2-9}}$

    (d) $\dfrac{1}{\sqrt[5]{(a^2-9)^2}}$

    (e) $\dfrac{a+3}{\sqrt[5]{a-3}}$

20. For nonzero numbers u, v, w:

$$3u = \frac{v}{2} \quad \text{and} \quad 2v = \frac{w}{3}$$

    Then $\dfrac{u}{w} = ?$

    (a)   1/9

    (b)   1/1

    (c)   1/36

    (d)   4/1

    (e)   1/4

21.   If $m + n = 6$ and $m - n = 2$, find $m^2 - n^2$.

      (a)   20

      (b)   32

      (c)   16

      (d)   12

      (e)   2

22.   The expression $\dfrac{1/a^2 - 1/b^2}{a^2 - b^2}$ simplifies to

      (a)   -1

      (b) $1/a^2b^2$

      (c)   -ab

      (d)   1

      (e) $-1/a^2b^2$

23.   In order for the multiplicative inverse property to be true for the expression $(a+b)$, which of the following must be true?

      (a) $a+b \neq 0$

      (b) $a = b$

      (c) $a = -b$

      (d) $a = 2b$

      (e) $2a = b$

24.   A survey of 200 workers at a company revealed that 115 of the workers were union members, 53 of the workers were satisfied with working conditions and 27 of the workers were union members who were satisfied with working conditions. How many non-union workers were not satisfied with the working conditions?

      (a)   32

      (b)   59

      (c)   61

      (d)   58

      (e)   26

25. The areas of a square and of a rectangle are the same. What is the measurement of the side of the square if the width of the rectangle is 3 cm and its length is 60 cm longer than the side of the square?

  (a) 12 cm

  (b) 15 cm

  (c) 18 cm

  (d) 10 cm

  (e) 6 cm

26. The complex number $(-1 + i)^5$ is equivalent to

  (a) $-16 - 4i$

  (b) $4 - 4i$

  (c) $14 - 4i$

  (d) $-5 + 5i$

  (e) 0

27. The sum of the roots of the equation $4x^3 - 4x^2 - 7x = 0$ is:

  (a) $4\sqrt{2}$

  (b) $2\sqrt{2}$

  (c) $2 + \sqrt{2}$

  (d) 1

  (e) 2

28. If the expression $16x^4 - 81y^4$ can be factored completely,

  (a) There will be four first degree factors

  (b) There will be two first degree factors and one second degree factor

  (c) There will be two second degree factors

  (d) There will be one first degree factor and one third degree factor

  (e) The expression cannot be factored.

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29.   What equation is graphed at right?

    (a) $2y - x = 10$

    (b) $2x + y = -5$

    (c) $2y + x = -5$

    (d) $2x - y = 10$

    (e) $2x - y = 5$
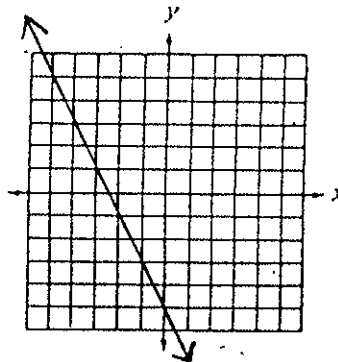
30.   What is the value of "r" so that the line through $(8, r)$ and $(4, 5)$ has a slope of -4?

    (a)   11

    (b)   -11

    (c)   -21

    (d)   4

    (e)   -4

31.

Consider this set of 25 test scores where the tally system has been used to record them:

99–1	77–1
95–1	75–11
91–1	74–11
90–1	72–11
88–1	71–11
87–1	70–11
82–1	65–1
81–1	60–1
80–111	
79–1	

The score of 80 would be considered the

(a)   mean

(b)   mode

(c)   median

(d)   standard deviation

(e)   dispersion

32. Which of the following is an equation in slope - intercept form for the line that has an x-intercept of -9 and a y - intercept of 9?

      (a) $y = -x - 9$

      (b) $y = 9x - 1$

      (c) $y = x + 9$

      (d) $y = x - 9$

      (e) $y = 9x - 1$

33. Solve: $x - 2y = 1$
$2x + 4y = -3$

      (a)   (-5/8, -1/4)

      (b)   (-1, -1)

      (c)   (3, 1)

      (d)   (1, 3)

      (e)   (-1/4, -5/8)

34. Solve for <u>x</u> in terms of the other variables.

$$\frac{x + y}{2} - \frac{y - z}{3} = 2x + z$$

      (a) $x = -y - z$

      (b) $x = \dfrac{y - 6z + 4}{9}$

      (c) $x = \dfrac{y - 4z}{9}$

      (d) $x = \dfrac{y}{2} - \dfrac{y\text{-}z}{3} - z$

      (e) $\emptyset$

35. What is the slope of a line that is perpendicular to:
    $2kx - 3k^2y = 4a$?

    (a) $-4a/3k^2$

    (b) $2/3k$

    (c) $-3k/2$

    (d) $-3k^2/4a$

    (e) $-(4a - 2k)/3k^2$

36. The complete factorization of $12x^3y + 26x^2y^2 + 10xy^3$ is:

    (a) $2 (6xy + 5y^2) (x^2 + xy)$

    (b) $2 (3xy + 5y^2) (2x^2 + xy)$

    (c) $2xy (2x + 5y) (3x + y)$

    (d) $2xy (6x + 5y) (x + y)$

    (e) $2xy (3x + 5y) (2x + y)$

37. Mike and Danny, working together, can paint the exterior of a house in 6 days. Mike, by himself, can complete this job in 5 days less than Dan. How long will it take Mike to complete the job by himself?

    (a)   15 days

    (b)   10 days

    (c)   2 days

    (d)   2 3/11 days

    (e)   7 3/11 days

38. An office worker takes 55 minutes to return from the job each day. This person rides a bus that averages 30 mph and then walks the rest of the way at 4 mph. If the total distance is 21 miles from office to home, what is the distance walked each day?

    (a)   .75 miles

    (b)   .25 miles

    (c)   1.0 miles

    (d)   1.5 miles

    (e)   .5 miles

39. The remainder when $(s^3 + 4)$ is divided by $(s - 2)$ is

(a) $s^2 - 2$

(b) $s^2 + 2s + 4$

(c) $\quad -4$

(d) $s^3 - s + 2$

(e) 12

40. A young lady from St. Louis wishes to visit a friend in Sewanee, TN. She's already decided she'll fly from St. Louis to Nashville via one of four major airlines. When she gets to Nashville, she can either rent a car, take the bus, fly in a small plane to the Sewanee airport, or have her friend meet her in Nashville and drive her to Sewanee. In how many different ways can she travel round trip, assuming she doesn't use the same mode (or company) of transportation twice?

(a) 16

(b) 8

(c) 25

(d) 14

(e) None, since once you are at Sewanee, you never want to go home again.