THIRTY–FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I 1987

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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1. The cost, C dollars, of K apples is 12¢ less than the cost of K-5 pears. The cost of three pears is S dollars. S =

    (a) $\dfrac{3(C-12)}{K}$

    (b) $\dfrac{3\ (C-12)}{K-5}$

    (c) $\dfrac{3(C+12)}{K}$

    (d) $\dfrac{3\ (C+.12)}{K-5}$

    (e) $\dfrac{K(C-.12)}{3}$

2. One factor of $X^7 - 53X^3 + 27X^2 + 25$ is:

    (a) $(X - 1)$

    (b) $(X - 5)$

    (c) $(X + 5)$

    (d) $(X + 1)$

    (e) $X + 2$

3. If $|X + 2| < 5$, then the solution set for X is $X > a$ where a =

    (a) -8

    (b) -7

    (c) -10

    (d) -15

    (e) -20

4. Farmer Brown wishes to expand his rectangular garden so that the ratio of the new area to the old area is 7 to 5. If only the length of the garden can be changed, what will be the ratio of the new length to the old length?

    (a) $\dfrac{5}{7}$

    (b) $\dfrac{25}{49}$

    (c) $\dfrac{7}{5}$

    (d) $\dfrac{49}{25}$

    (e) $\dfrac{7 - 5}{5}$

5. $\dfrac{X - \dfrac{1}{X}}{\dfrac{X^2 - 1}{X}}$ =   (where $X \neq 0, 1, -1$)

   (a) $\dfrac{X^2 - X}{X^2 - 1}$ (b) X^2

   (c) $\dfrac{1}{X^2}$ (d) 1 (e) $\dfrac{X^2 - 1}{X^3 - 1}$

6. One factor of $54X^2 + 3X - 40$ is:

   (a) $(9X + 8)$ (b) $(9X - 8)$

   (c) $(6X + 5)$ (d) $(27X + 2)$ (e) $(7X + 5)$

7. The line through the origin and the point with coordinates $(5, -3)$ has slope:

   (a) -3 (b) 5

   (c) $\dfrac{-3}{5}$ (d) $\dfrac{-5}{3}$ (e) -5

8. $\dfrac{X^2 - 4}{X^2 + 5} < 0$ for

   (a) $X < -2$ or $X > 2$ (b) $-2 < X < 2$

   (c) $X < -5$ and $-2 < X < 2$ (d) $X < 2$ (e) $X > 2$

9. $1 + \dfrac{1}{3 + \dfrac{1}{3 + \dfrac{1}{3}}}$ =

   (a) $\dfrac{10}{33}$ (b) $\dfrac{28}{27}$

   (c) $\dfrac{43}{3}$ (d) $\dfrac{43}{33}$ (e) 27

10. If $\dfrac{a + 2b}{b} = a$ then

   (a) $(a - 2)(b - 1) = 2$ (b) $a + 2 = ab$

   (c) $a = \dfrac{2b}{b + 1}$ (d) $b = \dfrac{a - 2}{a}$ (e) $a + 2 = a$

11. The length L of a rectangle exceeds three times the width w by 5 units. Therefore

   (a) $L = 3W - 5$ (b)   Area $= \dfrac{1}{3}(L^2 - 5L)$

   (c) $3L - 5 = w$ (d) $3w = 5L$

   (e) $w = \dfrac{5}{3}L$

12. $F(X) = 7 - 3X - 2X^3$ Which of the following is true?

   (a) $F(-1) = 8$ (b) $F(-1) = 2$

   (c) $F(\frac{1}{2}) = \dfrac{21}{4}$ (d) $F(\frac{1}{2}) = F(-\frac{1}{2})$

   (e)   The graph of $F(X)$ is a straight line.

13. $5\sqrt{50} - 7\sqrt{8} + 2\sqrt{32} =$

   (a) $19\sqrt{2}$ (b) $-2\sqrt{2}$

   (c) $-4\sqrt{2}$ (d) $4\sqrt{2}$

   (e) $\sqrt{90}$

14. The solution to $9^{(1-2X)} = 81^{X-1}$ is $X =$

   (a) $3/4$ (b) $\dfrac{10}{11}$

   (c) $3/5$ (d) $1/3$

   (e) $\dfrac{10}{19}$

Algebra I

15. $-\left\{-2 - \left[X + (3 - X) -3\right] + 3X\right\} =$

    (a) $3x - 2$ (b) $-3X - 2$

    (c) $3X + 2$ (d) $-3X + 2$

    (e) $-3X - 8$

16. $(2X^2)(3X)^3 =$

    (a) $6X^3$ (b) $54X^6$

    (c) $108X^5$ (d) $18X^5$

    (e) $54X^5$

17. If $\dfrac{X + 2}{X^2 - X - 12} = 0$ then

    (a) $X = 2$ (b) $X = -2$

    (c) $X = 4$ (d) $X = 3$

    (e) $X = -3$

18. For the System of linear equations $\begin{array}{l} ax + 2by = 4 \\ bx + 2ay = 1 \end{array}$ to have the solution

    $x = 1$ and $y = -1$, the values for a and b need to be

    (a) $a = -1$, $b = 1$

    (b) $a = 0$, $b = -2$

    (c) $a = 2$, $b = -1$

    (d) $a = -2$, $b = -3$

    (e) $a = 3$, $b = -2$

Algebra I

19. $\dfrac{5-\sqrt{2}}{3-\sqrt{2}} =$

    (a) $\dfrac{13 + 2\sqrt{2}}{7}$ (b) $\dfrac{17 + 2\sqrt{2}}{7}$

    (c) $\dfrac{17 - 2\sqrt{2}}{7}$ (d) $\dfrac{13 + 2\sqrt{2}}{11}$

    (e) $13 + 2\sqrt{2}$

20. $\dfrac{X^2 - 9}{2X^2 - 6X} \div \dfrac{2X^2 + 5X - 3}{4X^2 - 1}$ simplifies to:

    (a) 1   (b) 2

    (c) $\dfrac{2X + 1}{2X}$ (d) $\dfrac{2X - 1}{2X}$

    (e) $2X + 1$

21. The least common multiple of $2^3 \cdot 5$, $2 \cdot 3^2 \cdot 5^2$, $2^2 \cdot 3 \cdot 5$ is

    (a) $(2 \cdot 3 \cdot 5)^2$ (b) $(2 \cdot 3 \cdot 5)^3$

    (c) $2^3 \cdot 3 \cdot 5^2$ (d) $2^2 \cdot 3^2 \cdot 5^2$

    (e) $2^3 \cdot 3^2 \cdot 5^2$

22. $\dfrac{\dfrac{3}{a} - \dfrac{3}{b}}{\dfrac{1}{a} + \dfrac{1}{b}} =$

    (a) $\dfrac{3(a - b)}{(a + b)}$ (b) $\dfrac{(a - b)}{3(a + b)}$

    (c) $\dfrac{b - a}{3(b + a)}$ (d) $\dfrac{3(b - a)}{b + a}$

    (e) $\dfrac{a - b}{b + a}$

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23. $\dfrac{2}{X-2} = \dfrac{3}{X+2} + \dfrac{X}{X^2-4}$ has solution set:

    (a) $\{5\}$ (b) $\{-5\}$

    (c) $\{-1\}$ (d) $\{2\}$

    (e) $\{3\}$

24. Two-thirds of a number exceeds one half of that number by 4. The number is:

    (a) 6   (b) 12

    (c) 24   (d) 48

    (e) 96

25. Two trains leave point A at 7 a.m. and 8:30 a.m. The late train overtakes the second train at 12:00 noon. The speed of the faster train is 70 m.p.h. What is the speed of the other train?

    (a) 47   (b) 48

    (c) 49   (d) 50

    (e) 51

26. The diagonal of a rectangle is 13 inches. The width of the rectangle is 5 inches. What is the length of the rectangle?

    (a) 12   (b) 11

    (c) 10   (d) 9

    (e) 8

27. Which one of the quadratic equations below has non-real roots?

    (a) $2X^2 - 17X + 31 = 0$ (b) $5X^2 - X + 1 = 0$

    (c) $5X^2 - X - 1 = 0$ (d) $2X^2 + 17X + 21 = 0$

    (e) $X^2 - X - 13 = 0$

28. If the digits of a two-digit integer are reversed the new integer exceeds the original by 20 less than the old integer. The sum of the digits is 11. The original integer is:

    (a) 29   (b) 38

    (c) 47   (d) 56

    (e) 65

29. Ed can do a job in 8 hours. Bob can do the same job in 12 hours. Ed and Bob worked together for 3 hours and Bob completes the job alone. How long does Bob work alone?

    (a) 1.8 hours   (b) 7.2 hours

    (c) 1.0 hours   (d) 4.5 hours

    (e) 6 hours

30. Which property below is not valid for the set of all integers?

    (a) Associative law for addition
    (b) Commutative law for multiplication
    (c) Identity law for multiplication
    (d) Inverse law for multiplication
    (e) Distributive law of multiplication over addition.

31. Which function does not have an X-intercept?

    (a) $F(X) = X^2 + 1$ (b) $F(X) = X$

    (c) $F(X) = X^3 + 1$ (d) $F(X) = X^{77} - X + 1$

    (e) $F(X) = X \left(X^{77} - X + 1 \right)$

32. If the smaller of two numbers is increased by 2/3 the larger, the resulting number is 4/3 the larger. s and L denotes smaller and larger. Therefore

    (a) $3s = L$ (b) $3s = 2L$

    (c) $2/3L = 3s$ (d) $4/3s = 2/3L$

    (e) $L = 4/3s$

Algebra I

33. A vessel contains V cc of acid and water with A% acid. Add M cc of water and N cc of acid (100%). What percentage of acid is now in the vessel?

(a) $A + N$

(b) $A + \dfrac{N}{100}$

(c) $\dfrac{VA + N}{V}$

(d) $\dfrac{VA + N}{V + N + M}$

(e) $\dfrac{VA + 100N}{V + M + N}$

34. $\left| 5X - 2 \right| > 12$ is valid for:

(a) $X < -2$ or $X > 2$

(b) $-2 > X > 2.8$

(c) $X < -2$ or $X > 2.8$

(d) $-10 < X < 14$

(e) $-2 < X < 2$

35. The quotient $\left(X^5 - 32 \right) \div \left(X - 2 \right) =$

(a) $X^4 - 16$

(b) $X^4 + 16$

(c) $X^4 - 2X^3 + 4X^2 - 8X + 16$

(d) $X^4 + 2X^3 + 4X^2 + 8X + 16$

(e) $X^6 - 2X^5 - 32X + 64$

36. The conditional "If P, then Q" is logically equivalent to:

(a) If not Q, then not P

(b) If Q, then P

(c) If not P, then not Q

(d) P and not Q

(e) P or not Q

37. The discriminant of the equation $X^2 - CX + 2a = 0$ is:

(a) $b^2 - 4ac$

(b) $B^2 - 4AC$

(c) $\dfrac{C}{2a}$

(d) $c^2 - 8a$

(e) $-c^2 + 8a$

Algebra I

38. If $(2 - 5\sqrt{2})X = 1$ and $X = a + b\sqrt{2}$ then

(a) $a = \dfrac{1}{23}$

(b) $b = \dfrac{5}{46}$

(c) $a = -\dfrac{1}{10}$

(d) $b = -\dfrac{1}{23}$

(e) $b = -\dfrac{5}{46}$

39. For non-empty sets A, B, and C, which of the following is <u>not</u> always true?

(a) $A \cap (B \cup C) = A \cap (C \cup B)$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) $A \cap (B \cup C) = (A \cap B) \cup C$

(d) $A \cap (B \cup C) = (B \cup C) \cap A$

(e) $A \cap (B \cup C) = (C \cap A) \cup (B \cap A)$

40. $\left(2^x \cdot 3^y\right)^2 =$

(a) $(2^{x + 2})(3^{y + 2})$

(b) $(2^{2x} + 3^{2y})$

(c) $6^{2(x + y)}$

(d) $2^{x^2} \cdot 3^{y^2}$

(e) $2^{2x} \cdot 3^{2y}$