THIRTIETH ANNUAL MATHEMATICS CONTEST

Sponsored by

THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

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Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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IBM Corporation, Nashville, Tennessee
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1. The expression $x^{-1} + y^{-1}$ can be simplified as

   (a) $\frac{1}{x + y}$ (b) $\frac{2}{xy}$ (c) $\frac{2}{x + y}$

   (d) $\frac{x + y}{xy}$ (e) $\frac{xy}{x + y}$

2. If x is replaced by 5 in the expression $\frac{x}{x - 5}$, the result is

   (a) 0 (b) 5 (c) undefined

   (d) $\frac{-1}{2}$ (e) -1

3. If John can paint a room in 6 hours and Bill can paint the same room in 9 hours, how long will it take them working together to paint the same room?

   (a) 4 hours (b) 7 1/2 hours (c) 4 3/4 hours

   (d) 3 3/5 hours (e) 3 hours

4. If point A has coordinates (3,2) and point B has coordinates (-2,-3), then the distance d and the slope m of the line segment joining A and B will be

   (a) d=10, m=1 (b) $d=5\sqrt{2}$, m=-1 (c) $d=\sqrt{2}$, m=1

   (d) d=10, m=-1 (e) $d=5\sqrt{2}$, m=1

5. The expression

$$\frac{x^2 + 2xy + y^2}{x + y + z} + \frac{-xz - yz}{-x - y - z}$$ may be

simplified and expressed as

(a) $\frac{x^2 + 2xy + y^2 + xz + yz}{x + y + z}$ (b) $2(x + y)$

(c) $x + y + z$ (d) $x + y$ (e) $(x + y)^2$

6. If $x = -3$, $y = 2$ and $z = 5$ then $\frac{x^2 + xz}{y + z^2}$ is equal to

(a) $\frac{-7}{4}$ (b) $\frac{4}{5}$ (c) $\frac{-6}{27}$

(d) $\frac{-8}{9}$ (e) $\frac{12}{5}$

7. If $a = \frac{b + c}{c}$, then the value of c is

a. $ca - b$ (b) $\frac{b}{a - 1}$ (c) $a - b$

(d) $\frac{a - b}{2}$ (e) No solution

8. The vertex of the parabola $y = x^2 - 2x - 3$ is at

a. (1,7) (b) (0,-3) (c) (1,-4)

(d) (3,0) (e) (3,-1)

9. The solution for $|2x - 3| \geq 11$ is

(a) $\emptyset$ (b) $-4 \leq x \leq 7$ (c) $x \leq 7$

(d) $x \leq -4$ or $x \geq 7$ (e) $x \leq -7$ or $x \geq 4$

10. When simplified, the expression $\frac{6 \pm \sqrt{12}}{6}$

(a) $1 \pm \sqrt{12}$ (b) $\frac{3 \pm \sqrt{3}}{3}$

(c) $\pm 2\sqrt{3}$ (d) $1 \pm \sqrt{2}$ (e) $1 \pm \sqrt{3}$

11. One root of $5x^2 = 10x - 4$ is between

(a) 3 and 4 (b) -4 and -3 (c) -1 and 0

(d) 1 and 2 (e) -2 and -1

12. The simplest form of $\frac{t^2 - 1}{1 - t}$ is

(a) $t + 1$ (b) $-t + 1$ (c) $-(t + 1)$

(d) t (e) -1

13. When simplified, the expression $\frac{1 + \frac{1}{x}}{1 - \frac{1}{x}}$ is equal to

(a) -1 (b) $\frac{x + 1}{x - 1}$ (c) $-\frac{x + 1}{x - 1}$

(d) 1 (e) x

14. A wire k meters long is formed into a square. The area of the square is

(a) $4k$ (b) k^2 (c) $\frac{k^2}{4}$

(d) $\frac{k^2}{16}$ (e) $\frac{k^2}{8}$

15. The expression $(2m + n)^2 - (2m - n)^2$ is equivalent to

(a) $2n^2$ (b) $8mn$ (c) $8mn + 2n^2$

(d) $4m^2 + 4mn + n^2$ (e) 0

16. If $15 - 2(3t + 6) < 15$, then

(a) $t < -2$ (b) $t > -2$ (c) $t > 2$

(d) $t < 2$ (e) $t \geq 0$

17. The solution to $x^{-2} + x^{-1} - 6 = 0$ is

(a) $\left\{-\frac{1}{3}, \frac{1}{2}\right\}$ (b) $\{-6\}$ (c) $\{3, -2\}$

(d) $\left\{-\frac{1}{6}\right\}$ (e) $\left\{\frac{1}{3}, -\frac{1}{2}\right\}$

18. $\frac{9x}{3x - 15} \cdot \frac{(x - 5)^2}{2(5 - x)}$ simplified is

(a) $\frac{x - 5}{10}$ (b) $\frac{3x(x-5)}{2(5 + x)}$ (c) $-\frac{3x}{2}$

(d) $\frac{9x(x - 5)}{2(3x - 5)}$ (e) $\frac{x}{2}$

19. The solution set for the system of equations

$$\begin{aligned} 0.3x + 0.2y &= 0 \\ x + 0.5y &= 0.5 \end{aligned}$$ is

(a) (0.2, 0.3) (b) (2,-3) (c) (-2,3)

(d) (2,3) (e) (0,0)

20. When the product of $\frac{x}{x + y}$ and $\frac{x^2 - y^2}{y}$ is divided by $(x-y)$, the result is

(a) $\frac{x + y}{y}$ (b) $\frac{x(x - y)}{y(x + y)}$ (c) $\frac{x + y}{x - y}$

(d) $\frac{x}{y}$ (e) 1

21. If $\frac{1}{1 - x^2} = \frac{a}{1 - x} + \frac{a}{1 + x}$ then

(a) $a = \frac{1}{2}$ (b) $a = 1$ (c) $a = -1$

(d) $a = -\frac{1}{2}$ (e) a cannot be found.

22. In a chess match, a win of 1 game counts 1 point, a draw counts as 1/2 point, and a loss as 0. After 14 games during a championship match, the leader was 3 points ahead of his opponent. How many points did the leader have?

(a) 5 1/2 (b) 5 (c) 8

(d) 8 1/2 (e) 6

23. Factor completely $y^2 - 0.2y - 0.08$

(a) $(y + 0.4)(y - 0.2)$ (b) $(y - 4)(y + 2)$ (c) $(y - 0.4)(y + 0.2)$

(d) $(y - 0.4)(y - 0.2)$ (e) $(y - 0.8)(y + 1)$

24. The graph of the equation $y^2 = x^2 - 1$ is

(a) a straight line (b) a circle (c) ellipse

(d) hyperbola (e) parabola

25. The completely factored form of $(y^2 + y - 9)^2 - 9$ is

(a) $(y - 3)(y + 3)(y - 2)^2$ (b) $(y - 3)^4$

(c) $(y^2 - 6y - 3)(y^2 - 6y + 3)$ (d) $(y^2 + y - 18)^2$

(e) $(y + 3)(y - 2)(y + 4)(y - 3)$

26. The simplified form of $\dfrac{1 + \frac{1}{3a}}{1 - \frac{1}{9a^2}}$ is

(a) $\frac{1}{3a-1}$ (b) -1 (c) $\frac{3a + 1}{9a^2 - 1}$

(d) $a - 3$ (e) $\frac{3a}{3a - 1}$

27. A rectangle and triangle have equal bases. The altitude of the rectangle is 25 feet and the altitude of the triangle is 20 feet. The combined area of the rectangle and triangle is 280 square feet. Find the measure of the base.

(a) $\frac{59}{5}$ feet (b) 10 feet (c) 8 feet

(d) 12 feet (e) $\frac{56}{5}$ feet.

28. The solution set of

$$\frac{x}{x^2 - 1} - \frac{1}{x^2 - 1} + \frac{2}{x + 1} = 0 \text{ is:}$$

(a) $\{0\}$ (b) $\{-1, 1\}$ (c) $\{1\}$

(d) $\varnothing$ (e) $\{0, 2\}$

29. Change to simplest radical form:

$$\frac{\sqrt{a + b} + \sqrt{a - b}}{\sqrt{a + b} - \sqrt{a - b}}$$

(a) $\frac{3a - b}{2b}$ (b) $\frac{a + 2\sqrt{a^2 - b^2}}{b}$

(c) $\frac{2a + 2b\sqrt{a^2 - b^2}}{2b}$ (d) $\frac{2a + a^2 - b^2}{b}$ (e) $\frac{a + \sqrt{a^2 - b^2}}{b}$

30. The solution for $\frac{x+1}{x-1} \leq 0$ is

(a) $x \leq -1$ or $x \geq 1$ (b) $-1 \leq x \leq 1$

(c) $-1 \leq x < 1$ (d) $-1 < x \leq 1$

(e) $x \leq -1$ or $x > 1$

31. Factor completely $x^2y^2 + 7xy + 12$.

(a) $(xy - 3)(xy - 4)$ (b) $(xy + 3)(xy + 4)$ (c) $(x + 3)(y + 4)$

(d) $(x - 3)(y - 4)$ (e) $(xy + 6)(xy + 2)$

32. If $\frac{4}{\sqrt{x^2 - 9}} = 1$, then x is

(a) 0 (b) 4 (c) ± 4

(d) ± 5 (e) 25

33. The expression $x^4 - 1$ completely factored is

(a) $(x^2 - 1)^2$ (b) $(x + 1)(x - 1)(x + 1)^2$

(c) $(x^2 + 1)(x + 1)(x - 1)$ (d) $(x^2 + 1)^2$

(e) $(x - 1)^2(x + 1)^2$

34. $\frac{x^2 + 13x + 36}{x^2 - 16}$ reduces to

(a) $\frac{-9}{4}$ (b) $\frac{9}{4}$ (c) $\frac{x + 9}{x - 4}$

(d) $\frac{x + 9}{x + 4}$ (e) $\frac{x + 4}{x - 4}$

35. If $3^{a + b} = \frac{1}{9}$ and $3^{a - b} = 9$, then b is

(a) 2 (b) -2 (c) 0

(d) 1 (e) -1

36. The quotient of $(8x^3 - 27) \div (2x - 3)$ is:

(a) $4x^2 - 9$ (b) $4x^2 + 6x - 9$ (c) $4x^2 + 6x + 9$

(d) $4x^2 - 6x + 9$ (e) $4x^2 + 9$.

37. One of the intercepts of $2x - 4y = 8$ is

(a) (-4,0) (b) (2,0) (c) (0,4)

(d) (0,-2) (e) (0,2)

38. If each dimension of a rectangle were increased by 5 feet, the area would be increased by 95 square feet, and one dimension would become twice the other. Find the original dimensions:

(a) $\frac{285}{14}$ and $\frac{14}{3}$ (b) 3 and 11 (c) $\frac{95}{3}$ and $\frac{190}{3}$

(d) -3 and -11 (e) 9 and 7.

39. In a survey of the musical tastes of a group of high school students, it was found that :
22 liked Twisted Sister
25 liked Madonna
39 liked Bruce Springsteen
9 liked both Madonna and Twisted Sister
17 liked Twisted Sister and Bruce Springsteen
20 liked Madonna and Bruce Springsteen
6 liked all three
4 liked none of these.
How many students were surveyed?

(a) 142 (b) 50 (c) 100 (d) 46 (e)86

40. The contrapositive of the statement, "If wishes were horses, beggars would ride" is
(a). If beggars ride, then wishes are horses.
(b) If wishes are not horses, then beggars do not ride.
(c) Wishes are horses and beggars do ride.
(d) If beggars do not ride, then wishes are not horses.
(e) Wishes are horses, but beggars do not ride.

The following problem(s) has/have been omitted from your test due to some type of error (mathematical or typographical). Please locate your division and observe that on those particular questions in your division an OMIT has been written through that problem on your test. DO NOT ANSWER these, as they have been omitted statewide and the grading formula reflects that fact.

If you have other questions relative to this matter, please raise your hand and someone will come to you.

Division	Problem(s)
Algebra I	5
Algebra II	36

Correct the following problem in the Geometry Division.

36. d. $-a + b + c > -a' + b' + c'$

e. $a + b + c > a' + b' + c'$
