TWENTY-SEVENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

Scoring Formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. If $2(x + 3) - 3(x - 1)$ is simplified, the result is

   a) $-x + 9$

   b) $x - 9$

   c) $x + 9$

   d) $-x - 9$

   e) $x + 6$

2. The product $(x - 3y)(2x + z)$ is equal to

   a) $2x^2 + xz - 6xy - 3yz$

   b) $2x^2 - xz + 6xy - 3yz$

   c) $3x - 3y + z$

   d) $3x - 6xy + z$

   e) $x^3 - 4xy + x^2$

3. The expression $x^2 - 2xy - x(x - 2y(1 + y))$ when simplified is equal to

   a) $2xy^2$

   b) $-2xy^2$

   c) $2x^2 - 2xy^2$

   d) $x^2 - 2xy^2$

   e) $x^2 + 2xy + y^2$

4. The expression $(x^3y^{-2})^{-1}$ in simplified form is

   a) y^2/x^3

   b) x^3/y^2

   c) x^3y^2

   d) x^2y^3

   e) $x^3 - y^2$

5. When the expression $18x^2 + 9x - 20$ is factored $(ax + b)(cx - d)$ with a, b, c, d positive integers, then a =

   a) 1

   b) 2

   c) 3

   d) 6

   e) 9

6. The value of y which satisfies $\begin{aligned} 3x + 2y &= 7 \\ 3x - y &= 1 \end{aligned}$ is

   a) -2

   b) -1

   c) 0

   d) 1

   e) 2

7. If $f(x) = x^2 + 1$, then $f(f(x)) =$

   a) $x^4 + 2x^2 + 2$

   b) $x^2 + 1$

   c) $2x^2 + 2$

   d) $x^4 + 3x^3 + 6x^2 + 3x + 2$

   e) $x^4 + 2x^2 + 1$

8. The points (2,6) and (-1, -6) determine a straight line. What is the y-intercept of this line?

   a) -4

   b) -2

   c) 0

   d) 2

   e) 4

9. The slope of the line $2x + 3y + 4 = 0$ is

   a) 3/2

   b) 2/3

   c) -3/2

   d) -2/3

   e) -4

10. To receive \$1,000, how much must I borrow for one year if the bank charges a discount of 10% payable at once?

   a) \$1,000

   b) \$1,100

   c) \$ 900

   d) \$1,111.11

   e) \$ 909.10

11. If $a = 256$, then $\sqrt{a\sqrt{a\sqrt{a}}}$ equals

   a) 2

   b) 4

   c) 32

   d) 64

   e) 128

12. The domain of the function $\sqrt{\frac{t^2}{1 - t}}$ is

   a) $t < 1$

   b) $t \leq 1$

   c) $t > 1$

   d) $t \geq 1$

   e) $t < 0$

13. A pipe can fill a tub of water in 4 hours with the drain closed. The drain can empty the tank in 7 hours. How long will it take to fill the tank if the drain is open?

a) 2 2/13 hours

b) 3 hours

c) 9 1/3 hours

d) 11 hours

e) 28 hours

14. The system of equations $x + ay = 0$, $2x + 6y = 0$ has a unique solution unless a =

a) -3

b) 1

c) 3

d) 5

e) 6

15. If a, b and c are real numbers such that $a > b$, then which of the following must be true?

a) $ac > bc$

b) $ac \geq bc$

c) $ac \leq bc$

d) $ac < bc$

e) $a + c > b + c$

16. If John is 5 years younger than Tim is now, and John is 7 years younger than Tim will be when Tim is twice as old as John, how old is John now?

a) 2

b) 3

c) 5

d) 8

e) 10

17. A good guess for a real root of $x^3 + 4x - 9 = 0$ is

    a) -1.5

    b) -.5

    c) .5

    d) 1.5

    e) 2.5

18. A rectangle has area 48 square meters and perimeter 32 square meters. A possible value for the base of the rectangle is

    a) 2 meters

    b) 4 meters

    c) 8 meters

    d) 16 meters

    e) 24 meters

19. For what value of c does the equation $4x^2 - 12x + c$ have a double root?

    a) -144

    b) 0

    c) 9

    d) 81

    e) 144

20. When $x^3 + x + 1$ is divided by $x^2 + x + 1$ the remainder is

    a) $x + 2$

    b) x

    c) 0

    d) $-x$

    e) $-x - 2$

21. Simplify $\dfrac{\frac{1}{x-2} - \frac{2}{2x-1}}{\frac{1}{x-1} - \frac{2}{2x-1}}$

a) 3

b) -3

c) 1/3

d) $\frac{3(x-1)}{(x-2)}$

e) $\frac{3(x-2)}{(x-1)}$

22. What is the greatest common divisor of the polynomials $2x^2 - x - 6$ and $2x^2 + 11x + 12$?

a) $x - 2$

b) $x + 4$

c) $2x + 3$

d) $2x - 3$

e) $x^2 - 2x - 8$

23. What is the least common multiple of the polynomials $2x^2 - x - 6$ and $2x^2 + 11x + 12$?

a) $4x^2 + 10x + 6$

b) $x^2 + 12x + 18$

c) $2x^3 + 7x^2 - 10x - 24$

d) $4x^3 + 20x^2 + 8x - 72$

e) $4x^4 + 20x^3 + x^2 - 78x - 72$

24. If the operation $\odot$ is defined by $a \odot b = a^2 - b^2$ then $\odot$ is

a) both associative and commutative

b) associative but not commutative

c) commutative but not assciative

d) neither associative nor commutative

25. The solution set of $x^2 + x - 12 > 0$ is

a) $\{x \mid x > 12\}$

b) $\{x \mid -4 < x < 3\}$

c) $\{x \mid -3 < x < 4\}$

d) $\{x \mid x < -4\} \cup \{x \mid x > 3\}$

e) $\{x \mid x < -3\} \cup \{x \mid x > 4\}$

26. The product of $(a + b)$ and $(a^2 + 2b^2 - ab)$ is

a) $a^3 + ab^2 + 2b^3$

b) $a^3 + b^3$

c) $a^3 + 2ab^2 + b^3$

d) $a^3 + 2ab^2 + 2b^3$

e) $a^3 + 2a^2b + 2b + 3$

27. If $x = 1 - t$ and $y = 2t^2 + 3t + 1$, then

a) $y = 2x^2 - 7x + 6$

b) $y = 2x^2 + 7x - 6$

c) $y = 1 - x$

d) $y = 2x^2 + 3x + 1$

e) y cannot be determined from x.

28. Two trucks are carrying material to a road construction job. One truck can carry 4 tons more per trip than the other. If the smaller truck makes 5 trips and the larger truck makes seven trips, they can deliver a total of 112 tons of material. What is the capacity of the larger truck?

a) 5 tons

b) 7 tons

c) 9 tons

d) 11 tons

e) 13 tons

29. The solution set of the inequality $|x + 1| < |x - 1|$ is

a) $\{x \mid x > 0\}$

b) $\{x \mid x < 0\}$

c) $\{x \mid x > 3\} \cup \{x \mid x < -3\}$

d) $\{x \mid -1 < x < 1\}$

e) $\{x \mid -.5 < x < .5\}$

30. The base of a rectangle is three times its height. The area is 27 square meters. What is the perimeter?

a) 3 meters

b) 6 meters

c) 12 meters

d) 18 meters

e) 24 meters

31. Solve for x. $4/x + 3/x = 2/x + 1$

a) 0

b) 1

c) 5

d) 7

e) 9

32. Solve for the positive root $4/(x + 1) + 3/(x - 1) = 2.5$

a) 1

b) 2

c) 3

d) 4

e) 5

33. A man drives 120 miles in the same time that another man drives 80 miles. If the speed of the first driver is 20 miles per hour greater than the speed of the second driver, find the speed of the faster driver.

a) 60 mph
b) 90 mph
c) 120 mph
d) 180 mph
e) 240 mph

34. Solution A contains 40% water and 60% acid. Solution B contains 80% water and 20% acid. How many liters of solution A must be mixed with 5 liters of solution B to make a solution which is 50% water and 50% acid?

a) 1.67
b) 2.4
c) 3.6
d) 5
e) 15

35. Find a root of $(x - 4)(x - 1) = 10$

a) 14
b) 11
c) 6
d) 4
e) 2

36. The square of the positive integer x is 60 less than the square of another positive integer y. What is $x + y$?

a) 12
b) 18
c) 24
d) 30
e) 36

37. Let $x = 1 - | 2 - 3 |$ and $y = |1 - 2| - 3$. Which of the following is true?

a) $x = y$

b) $|x| = |y|$

c) $x < y$

d) $|x| < |y|$

e) more than one of the above

38. If $f(x) = x^2 + 1$, the $f(-x) =$

a) $-x^2 + 1$

b) $x^2 + 1$

c) $-(x^2 + 1)$

d) $1/(x^2 + 1)$

e) $x^2 + 1$

39. The fraction 12285/14553 reduced to lowest terms is

a) 5/7

b) 11/13

c) 29/37

d) 45/59

e) 65/77

40. Solve for x. $4(x + 3) + 5(x - 2) = 4(x + 8) - 3(x + 2)$

a) 1

b) 3

c) 5

d) 7

e) 9