TWENTY-THIRD ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

1979

Scoring Formula: 4R - W + 40

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This test was prepared from a list of Algebra I questions submitted by Cleveland State Community College.

DIRECTIONS:

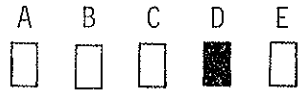
Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (a) 2/3 (b) 3 (c) 6

   (d) 3/2 (e) none of the above



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Simplify $\frac{a + 2/3}{5}$.

   (a) $5a + 2/15$

   (b) $\frac{3a + 2}{15}$

   (c) $\frac{15a + 10}{3}$

   (d) $5a + 2/3$

   (e) $\frac{3a + 2}{5}$

2. Factor $x^4 - y^4$ completely.

   (a) $(x^2 - y^2)(x^2 + y^2)$

   (b) $(x^2 - y^2)(x^2 - y^2)$

   (c) $(x^2 + y^2)(x - y)^2$

   (d) $(x^2 + y^2)(x - y)(x + y)$

   (e) $(x^2 - y^2)(x + y)^2$

3. The line with equation $2x + 3ky - 2 = 0$ is parallel to the line with equation $x + y = 4$. The value of k is

   (a) $-2/3$

   (b) $2/3$

   (c) $1/6$

   (d) -1

   (e) $-1/3$

4. Find the solution set to the system of equations $\begin{aligned} 2x + 6y &= 4 \\ 3x - y &= -4 \end{aligned}$.

   (a) $\{(1, 1)\}$

   (b) $\{(-1, -1)\}$

   (c) $\{(-1, 1)\}$

   (d) $\{(2, 1)\}$

   (e) $\{(1, -1)\}$

5. The factored form of $m(x - y) + (y - x)$ is

   (a) $(x - y)(m + 1)$

   (b) $(m + 1)(x + y)$

   (c) $(-m - 1)(x - y)$

   (d) $(m - 1)(x - y)$

   (e) can't be factored

6. When simplified, $(x^{-1} + y^{-1})^{-1}$ is equal to

   (a) $x + y$

   (b) $\frac{x + y}{xy}$

   (c) $\frac{xy}{x + y}$

   (d) $1/2$

   (e) none of the above

7. What is the value of $2x^2 - 4x$ when $x = 1/2$?

   (a) -1

   (b) -3

   (c) $3/2$

   (d) $-1/2$

   (e) $-3/2$

8. $\frac{a}{b} + \frac{c}{d} = \frac{a + c}{b + d}$ if and only if

   (a) $a + c = ad + bc$ and $b + d = bd$

   (b) $a^2d + b^2c = 0$

   (c) $b \neq 0$, $d \neq 0$, $b \neq -d$

   (d) $ad^2 + b^2c = 0$

   (e) $ad^2 + bc^2 = 0$

9. If the volume of a cube is 27 cubic inches, what is the total surface area in square inches?

   (a) 36

   (b) 54

   (c) 144

   (d) 18

   (e) 45

10. The area of a square is $(y^2 + 8y + 16)$ square units. The perimeter of the square is 24 units. The length of a diagonal is

   (a) 18y units

   (b) $y\sqrt{2}$ units

   (c) $y\sqrt{6}$ units

   (d) $3y\sqrt{2}$ units

   (e) 2y units

11. Solve the equation $t = \frac{x + a}{x - b}$ for a.

   (a) $a = 2tx - tb$

   (b) $a = t(x + b) - x$

   (c) $a = t - 2x - b$

   (d) $a = tx - tb - x$

   (e) none of the above

12. a, b, c represent positive real numbers and n represents a positive integer. Which of the following is an incorrect statement?

   (a) $a(b + c) = ab + ac$

   (b) $(ab)^n = a^n b^n$

   (c) $(\frac{a}{b})^n = \frac{a^n}{b^n}$

   (d) $(a + b)^n = a^n + b^n$

   (e) $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

13. Let $i = \sqrt{-1}$ and let a complex number z be defined as $z = a + bi$, where a and b are real. Define the conjugate of z, $\overline{z}$, as $\overline{z} = a - bi$. The conjugate of $\frac{1}{2 - 3i}$ is

(a) undefined

(b) $\frac{-1}{2 + 3i}$

(c) $\frac{2 + 3i}{13}$

(d) $\frac{2 - 3i}{13}$

(e) $2 + 3i$

14. The product of the roots of the quadratic equation $x^2 - 12x + 35 = 0$ is

(a) 12

(b) 35

(c) -12

(d) 2

(e) none of the above

15. If $\frac{x}{2} + \frac{x}{3} = 25$, the value of x is

(a) $x = 30$

(b) $x = 5$

(c) $x = 150$

(d) $x = 6$

(e) $x = 12$

16. If $\log_{10}2 = a$ and $\log_{10}3 = b$, what is the value of $\log_{10}4.5$?

(a) $2a - b$

(b) $2b - a$

(c) $\frac{2b}{a}$

(d) $\frac{2a^2 + 1}{a}$

(e) $3(b - a)$

17. Suppose that $a < b$ and $c > 0$. Which of the following is false?

(a) $a < (a + b)/2 < b$

(b) $a^2 < b^2$

(c) $a + c < b + c$

(d) $ac < bc$

(e) $b - a$ is a positive number

18. The quadratic equation whose roots are 4 and -3 is

(a) $x^2 - x - 12 = 0$

(b) $x^2 + x - 12 = 0$

(c) $x^2 - 7x + 12 = 0$

(d) $x^2 - 12 = 0$

(e) $x^2 - x + 12 = 0$

19. Solve the equation $a = \frac{1 + x}{1 - x}$ for x.

(a) $x = \frac{a - 1}{a}$

(b) $x = \frac{a + 1}{a - 1}$

(c) $x = \frac{a - 1}{a + 1}$

(d) $x = \frac{a + 1}{a}$

(e) none of the above

20. When simplified $\frac{8^{-\frac{2}{3}}}{16^{-\frac{3}{4}}}$ equals

(a) $\frac{1}{2}$

(b) 2

(c) 4

(d) 1

(e) none of the above

21. How many milliliters of 30% acid solution should be added to 20 milliliters of a 60% acid solution in order to obtain a solution which is 50% acid?

(a) 20

(b) 10

(c) $\frac{20}{3}$

(d) 60

(e) 30

22. A man has a pocketful of change which consists only of nickels, dimes, and pennies. He has 7 fewer nickels than dimes and 15 more pennies than dimes. He has exactly \$3.00 in change. The number of dimes he has is

(a) 13

(b) 35

(c) 10

(d) 20

(e) none of the above

23. The repeating decimal $.\overline{9}$ is the same number as which of the following?

(a) 9/10

(b) 99/100

(c) 1

(d) 999/1000

(e) all of the above

24. Solve $|4r - 2| \leq 8$ for r.

(a) $r \leq \frac{5}{2}$

(b) $r \geq -\frac{3}{2}$

(c) $r \geq \frac{5}{2}$

(d) $\frac{-3}{2} \leq r \leq \frac{5}{2}$

(e) $r \leq \frac{5}{2}$ or $r \geq \frac{-3}{2}$

25. Factor $x^3 - 2x^2 - 3x + 6$ completely.

(a) $(x^2 - 3)(x - 2)$

(b) $(x^2 - 3)(x + 2)$

(c) $(x - \sqrt{3})(x + \sqrt{3})(x - 2)$

(d) $(x - \sqrt{3})(x + \sqrt{3})(x + 2)$

(e) $(x - 3)(x + 3)(x - 2)$

26. When $x - 3$ is divided into $x^4 - 3x^3 + 2x + 7$, the remainder is

(a) 7

(b) -14

(c) 0

(d) -1/2

(e) none of the above

27. Given the following sets, which of the numbered statements are true?

$X = \{1, 3, 5, 7\}$, $Y = \{1, 7, 5, 3\}$, $Z = \{3, 5, 7, 9, 11\}$

1. $X = Y$
2. X is equivalent to Y
3. Y is equivalent to Z
4. $X \subseteq Z$
5. $\emptyset \subseteq Y$

(a) 1, 2, and 3 are true.

(b) 2 and 3 are true.

(c) 4 is true.

(d) 1, 4 and 5 are true.

(e) 1, 2 and 5 are true.

28. $(9)^{1/2} - (27)^{-2/3}$ equals

(a) 18

(b) 27/2

(c) 9/26

(d) 0

(e) 26/9

29. If Bill can paint a room in 8 hours and Jim can paint the same room in 4 hours, how many hours will it take both of them together to paint the room?

(a) 6 hours

(b) 2 2/3 hours

(c) 3 2/3 hours

(d) 3 hours

(e) 2 hours

30. Factor $3(x - y)^2 - 7(x - y) + 4$ completely.

(a) $(3x - y - 4)(x - y - 1)$

(b) $(3x - 3y - 2)(x - y - 2)$

(c) $(3x - 3y - 4)(x - y - 1)$

(d) $(3x - 3y + 2)(x - y + 2)$

(e) $(3x - 3y - 1)(x - y - 4)$

31. Solve for x : $\frac{2}{x - 3} - \frac{3}{x + 3} = \frac{12}{x^2 - 9}$.

(a) $x = 3$ and $x = -3$

(b) $x = 3$

(c) $x = -3$

(d) no solution for x

(e) $x = 0$

32. Which of the given equations does not have $\{1, -1\}$ as its solution set?

(a) $|x| = 1$

(b) $\left|\frac{1}{x}\right| = 1$

(c) $x^2 - 1 = 0$

(d) $x = \sqrt{1}$

(e) $\frac{1}{|x|} = 1$

33. When simplified, $\frac{2 + \sqrt{32}}{2}$ is equal to

(a) $1 + 4\sqrt{2}$

(b) $3\sqrt{2}$

(c) $1 + 2\sqrt{2}$

(d) $2 + 2\sqrt{2}$

(e) 10

34. The solution set for the inequality $\frac{2x - 3}{x + 5} > 0$ is

(a) $\left\{x \mid x < -5 \text{ and } x \geq \frac{3}{2}\right\}$

(b) $\left\{x \mid -5 \leq x \leq \frac{3}{2}\right\}$

(c) $\left\{x \mid -5 < x < \frac{3}{2}\right\}$

(d) $\left\{x \mid x < -5 \text{ and } x > \frac{3}{2}\right\}$

(e) none of the above

35. When simplified, $\frac{x + y}{x - y} - \frac{x - y}{x + y}$ is equal to

(a) $\frac{2x^2 + 2y^2}{(x - y)(x + y)}$

(b) $\frac{4xy}{(x + y)(x - y)}$

(c) $2y$

(d) -1

(e) 1

36. Factor the sum of $x - y$ and $x^2 - y^2$.

(a) $x^2 + x - y^2 - y$

(b) $x(x + 1) - y(y + 1)$

(c) $(x + y + 1)(x - y)$

(d) $(x - y)(x + 1)(y + 1)$

(e) $(x - y + 1)(x - y)$

37. Given that $b \neq r$ and $b \neq -r$, $\frac{c^2}{b^2 - r^2}$ can be written as

(a) $\frac{c^2}{2b(b - r)} + \frac{c^2}{2b(b + r)}$

(b) $\frac{c^2}{b - r} - \frac{c^2}{b + r}$

(c) $\frac{c}{b - r} - \frac{c}{b + r}$

(d) both (a) and (b)

(e) none of the above

38. Divide $\frac{a + b}{b}$ by $1 - \frac{a^2}{b^2}$.

(a) $-\frac{1}{a}$

(b) $\frac{b}{b - a}$

(c) $\frac{a}{b}$

(d) $\frac{b}{b + a}$

(e) $\frac{a}{a - b}$

39. V varies directly as the square of s. When $s = 2$, $V = 12$. Find the value of V when $s = 4$.

(a) 24

(b) 48

(c) 14

(d) 12

(e) none of these

40. The quadratic equation $3x^2 - kx + 3 = 0$ has two real, unequal roots. If k is a real number, then k must satisfy

(a) $k < 0$

(b) $k < -6$

(c) $|k| > 6$

(d) $k > 6$

(e) none of the above