TWENTY-SECOND ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

1978

Scoring Formula: 4R - W

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

(a) 2/3.   (b) 3.   (c) 6.

(d) 3/2.   (e) none of the above

The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a <u>heavy</u> black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is compiled.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Represent $\frac{3}{11}$ as a decimal fraction.

    (a) .273273 · · ·

    (b) .27

    (c) .28

    (d) $.\overline{26}$

    (e) $.\overline{27}$

2. $A \cap B = A \cup B$ will always be true in which one of the following cases?

    (a) $A \subset B$

    (b) $B \subset A$

    (c) $A = B$

    (d) always

    (e) never

3. Factor $x^4 - 81$ completely.

    (a) $(x + 3)(x - 3)(x^2 - 9)$

    (b) $(x + 3)^2(x - 3)^2$

    (c) $(x - 9)(x + 9)$

    (d) $(x^2 - 9)^2$

    (e) $(x + 3)(x - 3)(x^2 + 9)$

4. Which property of the real numbers is represented by $\frac{1}{3}(3r + 15) = \frac{1}{3}(3r) + \frac{1}{3}(15)$?

    (a) commutative property

    (b) associative property

    (c) reflexive property

    (d) distributive property

    (e) none of the above

5. A car radiator contains a 3 gallon mixture of alcohol and water which is 15% alcohol. What portion of the mixture must be replaced by pure alchohol in order to raise the alcohol content to 25%?

   (a) $\frac{3}{4}$ gal.

   (b) $\frac{9}{20}$ gal.

   (c) $\frac{6}{17}$ gal.

   (d) $\frac{3}{10}$ gal.

   (e) none of the above

6. Factor $x^3 - 8x^2 - 9x + 72$ completely.

   (a) $(x^2 + 9)(x - 8)$

   (b) $(x + 3)(x - 8)(x - 3)$

   (c) $(x + 3)^2(x - 8)$

   (d) $(x + 3)(x - 3)(x + 8)$

   (e) none of the above

7. If $\frac{x}{3} + \frac{x}{4} = 14$, solve for x

   (a) x = 20

   (b) x = 30

   (c) x = 12

   (d) x = 24

   (e) x = 28

8. Which of the following is false for a real number x?

   (a) x > 3 and x > 5 is equivalent to x > 5.

   (b) x < 3 and x < 5 is equivalent to x < 3.

   (c) x < 5 and x > 3 is equivalent to 3 < x < 5

   (d) x < 3 and x > 5 is equivalent to 5 < x < 3.

   (e) x > 3 or x < 5 is equivalent to x is any real number.

9. $(x^2 - 5x + 2)(3x - 2) =$

    (a) $3x^3 - 17x^2 + 16x - 4$

    (b) $x^2 - 2x$

    (c) $3x^2 - 2x + 4$

    (d) $3x^3 - 15x^2 + 6x - 4$

    (e) none of the above

10. Solve for t in the equation $p = at - prt$

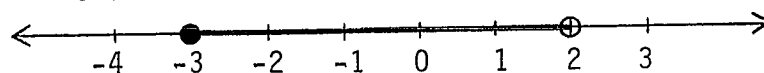   (a) $t = \dfrac{p - a}{pr}$

    (b) $t = \dfrac{p}{a - pr}$

    (c) $t = p - a + pr$

    (d) $t = p(a - pr)$

    (e) none of the above

11. The following graph represents the solution set of which inequality?

    (a) $-3 \geq x \geq 2$

    (b) $x \geq -3$ or $x < 2$

    (c) $x > -3$ or $x \leq 2$

    (d) $-3 \leq x < 2$

    (e) none of the above

12. The inequality $15(\frac{1}{5} - \frac{w}{3}) > -2w$ is equivalent to which of the following inequalities?

    (a) $w > \dfrac{3}{25}$

    (b) $w < 1$

    (c) $w > 0$

    (d) $w > 1$

    (e) none of the above

13. Solve the following equation for y : $x = \dfrac{y}{y + 2}$

    (a) $y = \dfrac{-2x}{1 - x}$, where $x \neq 1$

    (b) $y = \dfrac{x}{x + 2}$, , where $x \neq -2$

    (c) $y = \dfrac{2x}{1 - x}$, where $x \neq 1$

    (d) no solution

    (e) none of the above

14. Divide $\dfrac{a^2 - b^2}{3a}$ by $\dfrac{2a - 2b}{6}$

    (a) b

    (b) $\dfrac{a + b}{a - b}$

    (c) $\dfrac{1}{3} (a + b)$

    (d) $\dfrac{a + b}{a}$

    (e) $\dfrac{a - b}{a}$

15. Which of the following is the equation of a straight line which is perpendicular to the line whose equation is $y = \dfrac{3}{7}x + 2$?

    (a) $y = \dfrac{3}{7}x - 2$

    (b) $y = -\dfrac{7}{3}x$

    (c) $y = \dfrac{7}{3}x - 2$

    (d) $y = \dfrac{7}{3}x + 2$

    (e) $y = \dfrac{3}{7}x - \dfrac{1}{2}$

16. Find the slope of the perpendicular bisector of the line segment going through the points $(2, -3)$ and $(4, 1)$.

    (a) $-\frac{1}{2}$

    (b) $-\frac{1}{3}$

    (c) 2

    (d) $\frac{1}{3}$

    (e) $\frac{3}{2}$

17. $\sqrt{\frac{4}{3}\pi r^3}$, where $r \geq 0$, may be written as

    (a) $\frac{2r}{3}\sqrt{\pi}$

    (b) $2r\sqrt{\frac{\pi r}{3}}$

    (c) $2r\sqrt{\pi r}$

    (d) $\frac{2\pi r}{3}$

    (e) $\frac{2r\sqrt{\pi r}}{3}$

18. Simplify $\frac{2^{7x}}{4^{3x}}$.

    (a) 2^x

    (b) 4^{2x}

    (c) 4^x

    (d) 2^{4x}

    (e) cannot be simplified

19. All but one of the equations listed below have the same solution set. Which one has a different solution set from the others?

    (a) $y^2 = x^2$

    (b) $y^2 = |x^2|$

    (c) $|y|^2 = x^2$

    (d) $|y| = |x|$

    (e) $|y| = x$

20. If $f(x) = x^2 - 1$ and $g(x) = x - 7$, $f(g(x)) =$

    (a) $x^2 + x - 8$

    (b) $x^2 - 14x + 48$

    (c) $x^2 - 14x - 50$

    (d) $x^3 - 7x^2 - x + 17$

    (e) none of the above

21. Evaluate $-5\, m^4 \cdot \frac{1}{9}\, n^7$ when $m = -3$ and $n = -1$.

    (a) $\frac{17}{9}$

    (b) -45

    (c) $\frac{20}{3}$

    (d) -5625

    (e) none of the above

22. The lines with equations $x = y + 2$ and $x + y = 4$ intersect at the point

    (a) $(4, 2)$

    (b) $(3, 1)$

    (c) $(1, 3)$

    (d) $(-1, 3)$

    (e) $(3, -1)$

23. The sum and product of the roots of the equation $x^2 - x - 1 = 0$ are respectively

    (a) $1, 1$

    (b) $-1, 1$

    (c) $1, -1$

    (d) $-1, -1$

    (e) none of the above

24. Given $x \neq a$ and $x \neq -a$, $\dfrac{1}{x^2 - a^2}$ can be written as

 (a) $\dfrac{1}{(x + a)(x + a)}$

 (b) $\dfrac{1}{x - a} + \dfrac{1}{x - a}$

 (c) $\dfrac{1}{2a} \left(\dfrac{1}{x - a} - \dfrac{1}{x + a}\right)$

 (d) $ax \left(\dfrac{1}{x^2} - \dfrac{1}{a^2}\right)$

 (e) $\dfrac{1}{2a} \left(\dfrac{1}{x - a} + \dfrac{1}{x + a}\right)$

25. Evaluate $3i^6 - 4i^2 + i^4$ where $i = \sqrt{-1}$

 (a) -2

 (b) 8

 (c) 0

 (d) 2

 (e) 6

26. For what values of c does the equation $x^2 - 6x + c = 0$ have imaginary solutions?

 (a) $c < 9$

 (b) $c > 9$

 (c) $c \leq 9$

 (d) $c \geq 9$

 (e) none of the above

27. In a waterfront warehouse, suppose that 30 rats were captured and marked and set free again. Later 42 rats were captured and among these were 12 marked rats. Estimate the number of rats in the warehouse.
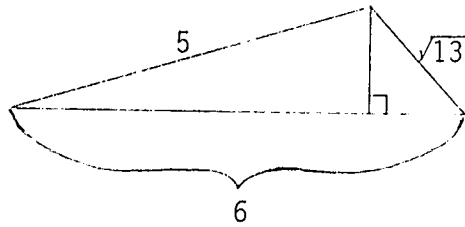
 (a) 100
 (b) 72
 (c) 105
 (d) 180
 (e) none of the above

28. The x-coordinate of the vertex of the parabola $y = 3x^2 - 4x + 6$ is

    (a) $\frac{1}{2}$

    (b) 1

    (c) $\frac{2}{3}$

    (d) $\frac{1}{3}$

    (e) $\frac{3}{4}$

29. The set of numbers x for which $|x - 3| < 2$ is true is

    (a) $\{x | 1 < x < 5\}$

    (b) $\{x | x > 0\}$

    (c) $\{x | 5 > x > -5\}$

    (d) $\{x | x < 2\}$

    (e) none of the above

30. An inlet pipe can fill a tank in 3 hours. A drain pipe can empty the tank in 4 hours. If both pipes are open, how long will it take to fill the tank?

    (a) 10 hours

    (b) 15 hours

    (c) 20 hours

    (d) 12 hours

    (e) 18 hours

31. What is the probability of rolling a sum of 7 on 2 dice?

    (a) $\frac{1}{7}$

    (b) $\frac{1}{6}$

    (c) $\frac{1}{12}$

    (d) $\frac{7}{36}$

    (e) $\frac{2}{3}$

32. Evaluate $\dfrac{2 + 5i}{4 - 3i}$ where $i = \sqrt{-1}$.

    (a) $\dfrac{-7 + 26i}{25}$

    (b) $\dfrac{1}{2} - \dfrac{5}{3}i$

    (c) $-\dfrac{7}{9}$

    (d) $\dfrac{23 + 26i}{7}$

    (e) none of the above

33. If we assume that (1) all a's are b's, and (2) no b's are c's, then a valid conclusion is

    (a) all a's are c's

    (b) all c's are a's

    (c) some a's are c's

    (d) no c's are a's

    (e) no valid conclusion

34. A truck and its load weigh 8 tons. If the load weighs one more ton than the truck, find the weight of the load.

    (a) $3\frac{1}{2}$ tons

    (b) 7 tons

    (c) $5\frac{1}{2}$ tons

    (d) $6\frac{1}{2}$ tons

    (e) $4\frac{1}{2}$ tons

35. $9^{\frac{3}{2}} + 8^{-\frac{2}{3}} =$

    (a) 31

    (b) 23

    (c) $\dfrac{109}{4}$

    (d) $17^{1/5}$

    (e) $\dfrac{86}{5}$

36. If $\log_{10}2 = .3010$, what is $\log_{10}40 - \log_{10}5$?

    (a) 8.3010

    (b) .6020

    (c) .3010

    (d) .9030

    (e) 2.408

37. How many ways are there of getting exactly 2 heads when 4 coins are flipped?

    (a) 2

    (b) 4

    (c) 6

    (d) 8

    (e) 12

38. Which of the following is an irrational number?

    (a) $\frac{3\pi}{2\pi}$.

    (b) 0

    (c) .313131 $\cdot$ $\cdot$ $\cdot$

    (d) .1010010001 $\cdot$ $\cdot$ $\cdot$

    (e) $\sqrt[5]{\frac{1}{32}}$

39. Two brothers, George and Randy, must mow the lawn once each week. It takes George three times as long as his older brother Randy to complete any task. The two are able to mow the entire lawn in 3 hours by agreeing that each will work singly for 1 hour and then be replaced by the other. How much of the lawn does Randy mow if he works the first shift?

    (a) $\frac{2}{3}$ (d) $\frac{6}{7}$

    (b) $\frac{3}{7}$ (e) none of the above

    (c) $\frac{1}{3}$

40. Find the height of the following triangle.

(a) 3.25

(b) 3

(c) 4

(d) 3.5

(e) 2.75