TWENTY-FIRST ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

1977

Scoring Formula: 4R - W

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This test was prepared from a list of Algebra I questions submitted by Volunteer State Community College.

DIRECTIONS:

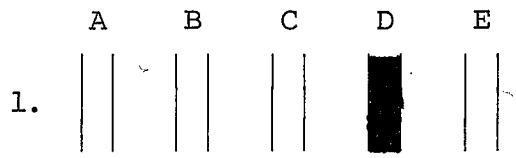
Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

(a). 2/3. (b). 3. (c). 6.

(d). 3/2 (e). none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Factor the expression $4x^2 - 16x + 2xy + 15 - 3y$.

   (a). $(2x + y)x + (5 - y)3$

   (b). $(2x + y)(3x - y + 4)$

   (c). $(2x - 3)(2x - 5 + y)$

   (d). $(2x + 2)(2x + 2 - y)$

   (e). none of these

2. The graph of $y = x^2 - 3x - 10$ crosses the x-axis at what points?

   (a). (5,0), (2,0)

   (b). (0,-2), (0,5)

   (c). (3,0), (10,0)

   (d). (5,0), (-2,0)

   (e). (0,5), (0,2)

3. If $\frac{2}{x+1} - 3 = \frac{4x+6}{x+1}$, then which of the following is true?

   (a). $x = 1/7$

   (b). $x \neq -1$

   (c). $x = 1$

   (d). $x > 0$

   (e). none of these

4. The distance between the points P (-4, -6) and Q (-3, 7) is

   (a). $2\sqrt{2}$

   (b). $\sqrt{218}$

   (c). $\sqrt{170}$

   (d). $5\sqrt{2}$

   (e). none of these

Identification Number: _____
(Copy from Answer Sheet)

IDENTIFICATION FORM
Tennessee Mathematics Teachers' Association
High School Mathematics Contest

Date: _____ Weighted Score: _____

Contest Subject (check one):

Algebra I _____

Algebra II _____

Geometry _____

Comprehensive_____

Advanced Topics_____

Regional Center (check one):

East _____

Middle_____

West _____

Please PRINT All Information

Name: _____
(First Name) (Middle Initial) (Last Name) (Age) (Sex)

Home Address: Street _____ City _____

County _____ State _____

Phone: AC _____ Number _____

Name of Your School _____

School Address: City _____ State _____

Official Class Status: Freshman ___ Sophomore ___ Junior ___ Senior ___

List all Mathematics Courses in which you are now enrolled or in which you have been enrolled at some time during this school year: _____

_____

_____

List all Mathematics Courses you have completed in high school: _____

_____

Teacher who is teaching you mathematics this year (GIVE FULL NAME):

Mr. _____
Mrs._____
Miss_____ _____
(First Name) (Last Name)

5. If x is a real number and $-|x| = x$, then which of the following is <u>always</u> true?

(a). $x \leq 0$

(b). $x > 0$

(c). $x \neq 0$

(d). b and c are both true

(e). none of these

6. The product of the real values of the solution to the equation $|x + 1| = 3\,|x - 3|$ is

(a). 20

(b). 10

(c). 10/3

(d). 25/2

(e). 5

7. Solve for x and y : $5x + 2y = 3$
$7x - 6y = 2$

(a). $x = \frac{1}{2}$, $y = \frac{1}{4}$

(b). $x = \frac{1}{4}$, $y = \frac{1}{2}$

(c). $x = -\frac{1}{4}$, $y = \frac{1}{2}$

(d). $x = \frac{1}{2}$, $y = 1$

(e). $x = -\frac{1}{2}$, $y = \frac{11}{4}$

8. The set $\{0, 1\}$ is closed with respect to which of the following?

I. addition
II. subtraction
III. multiplication

(a). I, II, and III

(b). only I and II

(c). only I

(d). only III

(e). none of these

9. One of the roots of $4(x - 2)^2 - 4(x - 2) + 1 = 0$ is

   (a). -5

   (b). 2/5

   (c). 3/5

   (d). 5

   (e). 5/2

10. The cost of labor varies jointly as the number of workers and the number of days they work. If 8 men working 9 days each earn \$576.00 then to earn \$624.00, 6 men must work:

   (a). 11 days

   (b). 13 days

   (c). 14 days

   (d). 15 days

   (e). none of these

11. The factors of $\frac{1}{4}x^{\frac{1}{2}} - \frac{1}{2}x^{\frac{1}{4}}y^{\frac{1}{4}} + \frac{1}{4}y^{\frac{1}{2}}$ are

   (a). $(\frac{1}{2}x^{\frac{1}{2}} - \frac{1}{2}y^{\frac{1}{2}})^2$

   (b). $(\frac{1}{2}x^{\frac{1}{4}} - \frac{1}{2}y^{\frac{1}{4}})(\frac{1}{2}x^{\frac{1}{4}} + \frac{1}{2}y^{\frac{1}{4}})$

   (c). $(x^{\frac{1}{4}} - y^{\frac{1}{4}})^2$

   (d). $(\frac{1}{2}x^{\frac{1}{4}} - \frac{1}{2}y^{\frac{1}{4}})^2$

   (e). $(x^{\frac{1}{2}} - y^{\frac{1}{2}})^2$

12. $2^3 \cdot 4^{-\frac{3}{2}} + 2y^0$ equals

   (a). 3

   (b). -3

   (c). 2

   (d). -2

   (e). 1

13. Reduce the following to lowest terms: $\dfrac{-6x^2 + 9x + 6}{2 - 5x + 2x^2}$

(a). -3

(b). $\dfrac{-3(2x - 1)}{1 + 2x}$

(c). $\dfrac{3(2x + 1)}{1 - 2x}$

(d). $\dfrac{-3(2x + 1)}{(1 - 2x)(2 - x)}$

(e). none of these

14. The solution set of the equation $x-5 = \sqrt{x+7}$ is

(a). {2}

(b). {2, 9}

(c). {9}

(d). {-2, 9}

(e). {}

15. A quadratic equation whose roots have a sum of 3 and a product of 5/4 is

(a). $x^2 - \dfrac{5x}{4} + 3 = 0$

(b). $4x^2 - 12x + 5 = 0$

(c). $2x^2 - 6x + 5 = 0$

(d). $x^2 - 5x + 4 = 0$

(e). none of these

16. If the reciprocal of $y - 1$ is $y + 1$ then y equals

(a). 4

(b). 0

(c). ± 1

(d). ± 2

(e). $\pm\sqrt{2}$

17. If 100 chickens eat 100 bushels of corn in 100 days, how many bushels will 10 chickens eat in 10 days?

(a). 1

(b). 5

(c). 10

(d) 100

(e). none of these

18. $\sqrt[2]{3} \cdot \sqrt[3]{2}$ is equal to

(a). $\sqrt[6]{108}$

(b). $\sqrt[5]{6}$

(c). $\sqrt[6]{6}$

(d). $\sqrt[5]{108}$

(e). $\sqrt[5]{5}$

19. For the inequality $\left|\frac{4 - 2x}{2}\right| > 8$, the solution set is

(a). $\{x \mid x > -6\} \cup \{x \mid x < 6\}$

(b). $\{x \mid -6 < x < 10\}$

(c). $\{x \mid x < -6\} \cup \{x \mid x > 10\}$

(d). $\{x \mid x > -6\} \cup \{x \mid x < 10\}$

(e). none of these

20. If $\frac{2}{x + 4} - \frac{3}{x + 6} = \frac{5}{x^2 + 10x + 24}$, then the solution set is

(a). $\{5\}$

(b). $\{4\}$

(c). ϕ

(d). $\{6\}$

(e). none of these

21. If ϕ represents the empty set, how many of the following four statements are true?

I. $\phi = 0$
II. $\phi = \{0\}$
III. $\phi = \{\phi\}$
IV. $\phi \in \phi$

(a). all of the four are true

(b). three of the four are true

(c). two of the four are true

(d). one of the four is true

(e). none of the four are true

22. A function which has the property that $f(-x) = -f(x)$ is

(a). $f(x) = 1$

(b). $f(x) = \sqrt{x}$

(c). $f(x) = x^2$

(d). $f(x) = x^3$

(e). $f(x) = |x|$

23. If $a * b$ is a binary operation on the set of integers and $a * b = a + b + 1$, then the identity element of $*$ is

(a). 0

(b). -1

(c). 1

(d). b-1

(e). none of these

24. If a varies inversely as the square of b and if $a = 12$ when $b = 2$, find b when $a = 3$.

(a). $\sqrt{8}$

(b). 1

(c). $\frac{1}{4}$

(d). $\frac{1}{2}$

(e). none of these

25. $3^a \cdot 9^{2a}$ is equivalent to which of the following?

I. 27^{3a}

II. 3^{5a}

III. 243^a

(a). only I

(b). only II

(c). only III

(d). only II and III

(e). none of these

26. If Grandma drives 20 kilometers at a constant rate of 40 kmph and an additional 20 kilometers at a constant rate of 60 kmph, her average rate for the entire 40 kilometers is

(a). 44 kmph

(b). 46 kmph

(c). 48 kmph

(d). 50 kmph

(3). none of these

27. A rope 18 feet long is cut into two pieces whose lengths are in the ratio 3:4. The length of the <u>longest</u> piece of rope is

(a). 12 4/7 feet

(b). 10 4/7 feet

(c). 10 2/7 feet

(d). 7 5/7 feet

(e). none of these

28. If a and b are integers, which of the following is <u>always</u> true?

(a). $|a-b| = |a| - |b|$

(b). $|a-b| \geq |a| + |b|$

(c). $|a-b| = |a| + |b|$

(d). $|a-b| \leq |a| - |b|$

(e). $|a-b| \geq |a| - |b|$

29. $-32^{\frac{3}{5}} + 16^{\frac{3}{4}}$ is equal to

(a). -16

(b). 72

(c). 16

(d). 0

(e). 56

30. If A and B are sets such that $A \cup B = A \cap B$, then

(a). A is the null set

(b). A and B are disjoint sets

(c). A is the complement of B

(d). A is a proper subset of B

(e). A is equal to B

31. The solution set of the equation $x = 2$ is

(a). $\{\frac{1}{2}\}$

(b). $\{-\frac{1}{2}\}$

(c). $\{-2\}$

(d). $\{2\}$

(e). cannot be determined

32. Ben is twice as old as Ken. In two years Ben will be $1\frac{1}{2}$ times as old as Ken. How old is Ben now?

(a). 2

(b). $1\frac{1}{2}$

(c). 6

(d). 4

(e). none of these

33. Find the solution set of the following equation: $(q - 4)(q + 3) = -10$

(a). $\{-2, 1\}$

(b). $\{-6, -13\}$

(c). $\{-2, -1\}$

(d). $\{2, -1\}$

(e). none of these

34. $\sqrt[4]{\sqrt[3]{\sqrt{2}}}$ is equal to

(a). $2^{\frac{1}{7}}$

(b). $2^{\frac{1}{8}}$

(c). $2^{\frac{1}{24}}$

(d). $2^{\frac{1}{12}}$

(e). none of these

35. A man invests a part of \$32,000 at 5% and the remaining part at 3%. If the income from the first is equal to the income from the second, how much does he have invested at each rate?

(a). \$16,000 at 5%, \$16,000 at 3%

(b). \$15,000 at 5%, \$17,000 at 3%

(c). \$12,000 at 5%, \$20,000 at 3%

(d). \$12,000 at 3%, \$20,000 at 5%

(e). none of these

36. Evaluate $(2-i^2)(4-i^4)$ where $i = \sqrt{-1}$.

(a). 9

(b). 15

(c). 11

(d). $2 + i$

(e). $2 - i$

37. The coordinates of the midpoint of the line segment joining the points (-2, 1) and (4, 9) are

(a). (5, 1)

(b). (-3, -4)

(c). (1, 5)

(d). (4, 3)

(e). (3, 10)

38. Solve the formula $s = \frac{n}{2}\left[2a + (n-1)d\right]$ for d.

(a). $d = \frac{2ns - 2a}{n^3 - n^2}$

(b). $d = \frac{2s - 2na}{n^2 - n}$

(c). $d = \frac{2s + 2na}{n^2 + n}$

(d). $d = \frac{2na - 2s}{n + n^2}$

(e). none of these

39. The expression $2y^3 - 6y^2 - y + 3$ can be factored as

(a). $(2y - 3)(y - 1)$

(b). $(2y^2 - 1)(y - 3)$

(c). $(2y - 1)(y - 3)$

(d). $(2y^2 + 1)(y - 3)$

(e). $(y^2 - 3)(2y - 1)$

40. Which of the following statements are <u>always</u> true?

I. If a whole number is divisible by 2 and by 3, it is divisible by 6.

II. If a whole number is divisible by 4 and by 6, it is divisible by 24.

III. If a whole number is divisible by 3, by 5 and by 7, it is divisible by 105.

(a). only I

(b). only II

(c). only III

(d). only I & II

(e). only I & III