NINETEENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

1975

Scoring Formula: 4R - W

EDITED BY:

Billy Edwards
and
Joe E. Kirk, Jr.
University of Tennessee at Chattanooga
Chattanooga, Tennessee

This test was prepared from a list of Algebra I questions submitted by University of the South.

DIRECTIONS:

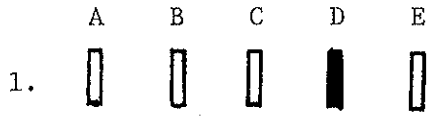
Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (a). 2/3. (b). 3. (c) 6.

   (d). 3/2. (e). none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. What is the remainder when $2x^3 + 11x^2 - 31x + 20$ is divided by $2x - 3$?

   (a) 0

   (b) 5

   (c) 10

   (d) -3

   (e) 11

2. Find the distance between the points $P(-2,3)$ and $Q(5,4)$.

   (a) $7\sqrt{2}$

   (b) 5

   (c) $5\sqrt{2}$

   (d) $\sqrt{8}$

   (e) 1

3. Which of the following sets of ordered pairs does not represent a function?

   (a) $\{(1,1), (2,2), (3,3), (4,4)\}$

   (b) $\{(1,-1), (2,0), (3,-1), (4,0)\}$

   (c) $\{(1,5), (2,5), (3,5), (4,5)\}$

   (d) $\{(1,1), (2,4), (3,9), (4,17)\}$

   (e) $\{(1,2), (3,4), (4,1), (1,3)\}$

4. The greatest common divisor of 640 and 1032 is

   (a) 2

   (b) 4

   (c) 16

   (d) 8

   (e) 64

5. If x is a real number and $x^2 < x$, then

   (a) $x < 0$

   (b) $x = 0$

   (c) $x > 1$

   (d) $0 < x < 1$

   (e) $x < -1$

6. The lines with equations $3x + 2y = 5$ and $2x - y = 1$ have

   (a) no common point

   (b) exactly one common point

   (c) exactly two common points

   (d) exactly three common points

   (e) infinitely many common points

7. The solution set of $\frac{x+1}{2} - \frac{x-1}{2} + \frac{x}{3} = 8$ is

   (a) $\{21\}$

   (b) $\{12, -12\}$

   (c) $\{8\}$

   (d) $\{84, 96\}$

   (e) $\{1\}$

8. Last year George's salary was \$100 per week. In January he received a 20% pay increase, but in June his pay was reduced by 20%. How much is George earning per week now?

   (a) \$110

   (b) \$104

   (c) \$100

   (d) \$96

   (e) \$90

9. The equation $\frac{1}{x+3} + \frac{1}{x-5} = 1$ has the same roots as the equation

(a) $x^2 - 4x - 13 = 0$

(b) $x^2 - 2x - 15 = 0$

(c) $x^2 - 2x + 17 = 0$

(d) $x^2 - 4x + 15 = 0$

(e) $x^2 - 8x + 13 = 0$

10. The sum of two numbers is 14. The first number exceeds twice the second number by 3. The first number is

(a) 10

(b) $\frac{21}{2}$

(c) $\frac{11}{3}$

(d) $\frac{31}{3}$

(e) none of these

11. The solution of the equation $a^2x + 4 = b^2 - 4x$ is

(a) $x = (b^2 - 4) / (a^2 - 4)$

(b) $x = b^2 / a^2$

(c) $x = 4 / (a^2 - b^2 + 4)$

(d) $x = 1 / (a^2 - b^2 + 4)$

(e) $x = (b^2 - 4) / (a^2 + 4)$

12. Below is a list of 5 sets, each with an algebraic operation. Which one does not have the property of closure?

(a) S = the positive real numbers; multiplication

(b) S = the positive real numbers; addition

(c) S = {-1, 1}; multiplication

(d) S = integers that are perfect squares; addition

(e) S = integers that are perfect squares; multiplication

13. Which of the following statements is true of $\frac{10}{0}$?

(a) $\frac{10}{0} = 10$

(b) $\frac{10}{0} = 0$

(c) $\frac{10}{0} = 1$

(d) $\frac{10}{0}$ is undefined

(e) none of these

14. The solution set of $x(x - 1) = 6$ is

(a) $\{6\}$

(b) $\{3\}$

(c) $\{6,7\}$

(d) $\{-2,3\}$

(e) $\{0,1\}$

15. $\sqrt[3]{54} - \sqrt[3]{16}$ simplifies to

(a) $\sqrt[3]{2}$

(b) $\sqrt[3]{38}$

(c) $\frac{38}{3}$

(d) $18 - \sqrt[3]{16}$

(e) none of these

16. Which of the following statements is not true?

(a) The product of two real numbers is a real number

(b) The product of two rational numbers is a rational number

(c) The product of two irrational numbers is an irrational number

(d) The product of an irrational number and a rational number is a real number

(e) The product of a rational number and a counting number is a rational number

17. The intersection of the graph of $y = (x - 1)^2 - 4$ and the x-axis consists of

(a) two points

(b) no points

(c) (1,0)

(d) (1,4)

(e) three points

18. $(27)^{2/3} + (9)^{-1/2}$ equals

(a) $\frac{27}{2}$

(b) 15

(c) 0

(d) 3

(e) $\frac{28}{3}$

19. The solution set of the equation $r\sqrt{\frac{1 - r^2}{75}} = 5$ is

(a) the empty set

(b) $\{-\frac{1}{2}, \frac{1}{2}\}$

(c) $\{\frac{1}{2}\}$

(d) $\{\frac{1}{4}\}$

(e) $\{\frac{3}{5}\}$

20. Consider the quadratic equation $x^2 + bx + c = 0$ with unknown coefficients b and c. The roots of this equation are integers and it is known that one is odd while the other is even. Then

(a) b and c are both even integers

(b) b and c are both odd integers

(c) b is an even integer and c is an odd integer

(d) b is an odd integer and c is an even integer

(e) neither b nor c is an integer

21. Which of the following is a factor of $12a^4 - 8a^2 - 15$?

(a) $\sqrt{2}a + \sqrt{3}$

(b) $2a + \sqrt{5}$

(c) $2a - \sqrt{5}$

(d) $2a - 3$

(e) $\sqrt{6}a + \sqrt{5}$

22. The solution set of the inequality: x is a real number and $|x - 5| / 3 \geq 2$ can be represented graphically on the real number line as

(a) a line segment of length 6

(b) a line segment of length 12

(c) a ray (half-line) with end-point $x = 11$

(d) a pair of rays (half-lines) with end-points at $x = 11$ and $x = -11$

(e) a pair of rays (half-lines) with end-points at $x = 11$ and $x = -1$

23. The simplest form of the expression $(x^3y - y^3x)/(x^2y + y^2x)$ is

(a) $xy(x + y)$

(b) $(x + y)/xy$

(c) $x - y$

(d) $x + y$

(e) $(x - y)/xy$

24. The function $f(x) = ax + b/x$ has values $f(2) = 5$ and $f(4) = 0$. Then a and b are given by:

(a) $a = 2, b = 1$

(b) $a = 2, b = 2$

(c) $a = \frac{5}{2}, b = -\frac{3}{8}$

(d) $a = -3, b = 1$

(e) $a = -\frac{5}{6}, b = \frac{40}{3}$

25. Tom, Dick, and Harry are all working at the job of putting stamps on envelopes. In a given amount of time, Tom can do 2/3 of the number done by Dick, and Dick can do 3/4 of the number done by Harry. If together the boys stamped 1800 envelopes in one hour, how many were done by Tom?

(a) 300

(b) 400

(c) 450

(d) 500

(e) 600

26. Let $i = \sqrt{-1}$. Then $\frac{1}{i} =$

(a) i

(b) 0

(c) $-i$

(d) i^2

(e) 1

27. The solutions of the equation $25x^2 - 20x + 13 = 0$ are

(a) $-\frac{1}{5}$ and 1

(b) $\frac{13}{5}$ and $\frac{1}{5}$

(c) $\frac{2}{5} - \frac{3i}{5}$ and $\frac{2}{5} + \frac{3i}{5}$

(d) $\frac{2}{5} + \frac{7i}{5}$ and $\frac{2}{5} - \frac{7i}{5}$

(e) none of these

28. Let $\cup$ and $\cap$ denote the union and intersection of sets, respectively, and if S is a set let S' denote the complement of S. Let $\emptyset$ denote the empty set. Which of the following is not a property of the operations on sets?

(a) $S \cup S = S$

(b) $S \cap S' = \emptyset$

(c) $(S \cup T)' = S' \cup T'$

(d) $S \cup (T \cap W) = (S \cup T) \cap (S \cup W)$

(e) $S \cap (T \cup W) = (S \cap T) \cup (S \cap W)$

29. Assuming that $|x^2 - 4| < 6$, then x must be

(a) $-\sqrt{2} < x < \sqrt{2}$

(b) $2 < x < 10$

(c) $-\sqrt{10} < x < \sqrt{10}$

(d) $-10 < x < 2$

(e) $-10 < x < 10$

30. The solution set of the equation $\sqrt{10 - 6x} = x - 3$ is

(a) $\{1\}$

(b) $\{-1\}$

(c) $\{0\}$

(d) $\{1, -1\}$

(e) the empty set

31. The graph of the equation $|x| + |y| = 1$ is best described by the following geometrical term:

(a) circle

(b) straight line

(c) right angle

(d) line segment

(e) square

32. In a round trip between Abilene and Dodge City, Slim is able to drive his VW Super Beetle at 50 miles per hour from Abilene to Dodge City but on the return trip with a tail wind he drives at 75 miles per hour. Then Sue drives the same round trip in the same time as Slim, but at a constant rate. How fast does Sue drive?

(a) 65 m.p.h.

(b) 64 m.p.h.

(c) 62.5 m.p.h.

(d) 60 m.p.h.

(e) none of the above

33. Let x be the length of an edge of a cube, V its volume, and S its surface area. Find V as a function of S.

(a) $V = S^{3/2} \,/\, (6\sqrt{6})$

(b) $V = 6\, S^{2/3}$

(c) $V = \left(\frac{S}{6}\right)^3$

(d) $V = S^3$

(e) $V = \sqrt{3} S^{1/2}$

34. The graph of $f(x) = 4x^3 + 4x^2 - 7x + 2$ intersects the x-axis

(a) once

(b) twice

(c) three times

(d) four times

(e) none of the above

35. What is the slope of a line perpendicular to the line whose equation is $99x + 101y = 100$?

(a) $m = \frac{101}{99}$

(b) $m = \frac{99}{101}$

(c) $m = -\frac{99}{101}$

(d) $m = -\frac{101}{99}$

(e) none of these

36. Consider the system of equations $\left\{\begin{array}{l} 3x + 8y = 7 \\ 5x + \frac{40}{3} y = k \end{array}\right\}$ with the unknown constant k. Find the set of all values of k for which the system of equations has a solution.

(a) $k = \frac{21}{5}$

(b) $k \neq \frac{21}{5}$

(c) $k \neq \frac{35}{3}$

(d) $k = \frac{35}{3}$

(e) k can be any real number

37. Suppose that $x_1, x_2, \ldots, x_{20}$ are real numbers with the k^{th} term x_k given by $x_k = \frac{1}{k} - \frac{1}{k+1}$, $k = 1,2,3,\ldots$. Then the sum $x_1 + x_2 + x_3 + \ldots + x_{20}$ equals

(a) $\frac{20}{21}$

(b) $\frac{20}{19}$

(c) $\frac{19}{20}$

(d) $\frac{21}{20}$

(e) none of these

38. If (6, 1) is the lowest point on the graph of $y = ax^2 + bx + c$, then you can be sure that

(a) $b^2 - 4ac = 0$

(b) $b^2 - 4ac < 0$

(c) $b^2 - 4ac > 0$

(d) $b^2 - 4ac = 1$

(e) $b^2 - 4ac = 6$

39. $(2\sqrt{2} - \sqrt{5}) \,/\, 3$ can be expressed as

(a) $\frac{1}{3}$

(b) $2\sqrt{3}i \,/\, 3$

(c) $1 \,/\, (2\sqrt{2} - \sqrt{5})$

(d) $-1 \,/\, 3(2\sqrt{2} + \sqrt{5})$

(e) none of these

40. Consider the expression $E = (2k - 1)^2$ in which k is an integer. Which of the following could not possibly be a value for E?

(a) 1

(b) 19

(c) 9

(d) 361

(e) 529