EIGHTEENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHER'S ASSOCIATION

ALGEBRA I TEST

1974

Scoring Formula: 4R - W

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This test was prepared from a list of Algebra I questions submitted by Middle Tennessee State University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school algebra. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (a). 2/3.   (b). 3.   (c). 6.

   (d). 3/2.   (e). none of these

The correct answer for the sample problem is 3/2, which is answer (D); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If x is a real number, what are <u>all</u> the values of x for which
    $x^4 + 16$ is a positive number?

    (a). All x greater than -2
    (b). All x greater than zero
    (c). All x greater than 2
    (d). All x between -2 and 2
    (e). All values of x

2. For what values of x is $\frac{x}{6} = \frac{1}{2} (x - 3) - \frac{x}{3}$ a true statement?

    (a). 0 only
    (b). 3 only
    (c). 0 and 3 only
    (d). All values
    (e). No value

3. Solve the formula $E = \frac{ar}{a + r}$ for r.

    (a). r = (aE)/(a - E)
    (b). r = (aE)/(a + E)
    (c). r = aE - a + E
    (d). r = aE - a - E
    (e). r = a - (E)/(a - E)

4. If x is greater than 3, which of the following is the smallest?

    (a). 3/x
    (b). 3/(x + 1)
    (c). 3/(x - 1)
    (d). x/3
    (e). (x + 1)/3

5. Evaluate $i^2 + i^4 + i^6$, where $i = \sqrt{-1}$.

   (a). -3
   (b). -1
   (c). 0
   (d). 1
   (e). 3

6. $-16^{-1/2} + 8^{2/3}$ is equal to

   (a). -8/3.
   (b). 0.
   (c). 3 3/4.
   (d). 4 1/4.
   (e). 8.

7. What are all the values of c for which $x^2 + 2x + c = 0$ has imaginary roots?

   (a). c < -1
   (b). c < 1
   (c). c > -1
   (d). c > 0
   (e). c > 1

8. If $\log_b 10 = p$ and $\log_b 2 = q$, then $\log_b 20$ is equal to

   (a). pq.

   (b). p + q.

   (c). p^q.

   (d). q^p.

   (e). 2(p + q).

9. If $x - 7$ is a factor of $x^2 - 3x + p$, what is the value of p?

    (a). -28

    (b). -21

    (c). -10

    (d). 21

    (e). 28

10. If a, b, and c are real numbers, which of the following is always true:

    (a). $a + (b \cdot c) = (a + b) \cdot c$

    (b). $(a \cdot b) - (a \cdot c) = (b - c) \cdot a$

    (c). $a(b - c) = (a \cdot b) - c$

    (d). $(b + c) \div a = b + (c \div a)$

    (e). $(b + c)a = b + ca.$

11. When x is a real number, $\sqrt{x^2}$ is always equal to

    (a). x.

    (b). - x.

    (c). 1.

    (d). $|x|$.

    (e). $2\sqrt{x}$.

12. If x is a real number and $|x| = -x$, then which of the following is true?

    (a). $x \neq 0$

    (b). $x > 0$

    (c). $x \leq 0$

    (d). $x = - x$

    (e). All of these are true.

13. The distance between points P(-3, -1) and Q(9, 4) is equal to

    (a). $\sqrt{61}$.
    (b). $\sqrt{153}$.
    (c). 13.
    (d). 17.
    (e). $3\sqrt{5}$.

14. The solution for the equation $\frac{m+1}{6} - \frac{m-2}{5} + \frac{m+3}{10} = 4$, is

    (a). m = 17.
    (b). m = 63.
    (c). m = 28.
    (d). m = 52.
    (e). m = 47.

15. The area of a rectangle is $(y^2 + 4y - 8)$ square inches. A square of equal area has a side with measure y inches. The area of the rectangle is equal to

    (a). 2 square inches.
    (b). 6 square inches.
    (c). 1 square inch.
    (d). 12 square inches.
    (e). 4 square inches.

16. The 11th term of the geometric progression 1/2, 1, 2, ... is

    (a). 512.
    (b). 10.
    (c). 11/2.
    (d). 1024.
    (e). 256.

17. $\sqrt{12}$ + $\sqrt{27}$ simplifies to

    (a). $5\sqrt{3}$.

    (b). $\sqrt{39}$.

    (c). $\sqrt{324}$.

    (d). 39.

    (e). $3\sqrt{5}$.

18. The product of $(2x^{2/5}y)$ and $(1/2)(x^{3/5}y^{-1})$ can be expressed as

    (a). y.

    (b). xy^{-2}.

    (c). x.

    (d). $x^{1/5}y^{-2}$.

    (e). $x^{6/5}y^{-1}$.

19. Which one of the following sets has the dense order property?

    (a). The integers

    (b). The counting numbers

    (c). The rational numbers

    (d). The even numbers

    (e). The complex numbers

20. Which one of the following sets is not closed under addition?

    (a). The real numbers

    (b). The integers

    (c). The even integers

    (d). The odd integers

    (e). The positive rational numbers

21. Given the linear equation $2x + 3y - 5 = 0$, whose slope is m and whose y intercept is b. Which of the following is true?

   (a). $m = 5/3$, $b = -2/3$

   (b). $m = -3/2$, $b = 5/2$

   (c). $m = 2/3$, $b = -5/2$

   (d). $m = -2/3$, $b = -5/3$

   (e). $m = -2/3$, $b = 5/3$

22. A certain number exceeds its reciprocal by 15/4. Which one of the following is a solution?

   (a). -4

   (b). 1/4

   (c). -3/4

   (d). 3/4

   (e). -1/4

23. A 550 mile long trip could be traveled in 1 hour less if the present speed were to be increased by 5 m.p.h. What is the present speed?

   (a). 30 m.p.h.

   (b). 40 m.p.h.

   (c). 45 m.p.h.

   (d). 50 m.p.h.

   (e). 55 m.p.h.

24. Which of the following statements correctly describes the graph of $2x - y > 1$?

   (a). The set of points above the line $y = 2x - 1$.

   (b). The set of points below the line $y = 2x - 1$.

   (c). The set of points on the line $y = 2x - 1$.

   (d). The graph contains the point $(2, 5)$.

   (e). None of these describes the given graph.

25. The formula for simple interest is a = P(1 + rt). Find the value for t when a = $295.30, P = $250.25, and r = 4½%.

    (a). 5 years

    (b). 2½ years

    (c). 3½ years

    (d). 4 years

    (e). 6 years

26. Find the value of the expression $[x^2 - 3x + 4x^0 + x^{-1} - 5x^{-2} + x^{-3}]$ when x = 1/2.

    (a). -7 1/4

    (b). 8 1/2

    (c). -10 1/4

    (d). 11 3/4

    (e). - 9 1/2

27. What is the remainder when $x^5 - 1$ is divided by x - 2?

    (a). 63

    (b). 31

    (c). -31

    (d). -33

    (e). 33

28. The diagonal of a cube with an edge 6 inches long is:

    (a). $6\sqrt{2}$ inches

    (b). 18 inches

    (c). 12 inches

    (d). $6\sqrt{3}$ inches

    (e). 10.7 inches

29. The quadratic equation whose roots are 3 and -2 is:

    (a). $x^2 - x - 5 = 0$

    (b). $x^2 + x - 6 = 0$

    (c). $x^2 - 5x - 6 = 0$

    (d). $x^2 - x - 6 = 0$

    (e). $x^2 - 6 = 0$

30. The equation of the line through the points (3, 2) and (0, -7) is

    (a). $3x - y - 7 = 0$.
    (b). $3x + 2y - 7 = 0$.
    (c). $5x + 3y + 21 = 0$.
    (d). $3x - y - 3 = 0$.
    (e). $5x - 3y - 9 = 0$.

31. In the solution set for the system of equations $\begin{cases} x + y = 5 \\ x^2 + y^2 = 17 \end{cases}$, the larger value for x is:

    (a). 5
    (b). 2
    (c). 3
    (d). $3\frac{1}{2}$
    (e). none of these

32. One of the roots of $x^4 - 12x^2 + 32 = 0$ is

    (a). 8.
    (b). $2\sqrt{2}$.
    (c). $-\sqrt{2}$.
    (d). -4.
    (e). 4.

33. The roots of $x^2 - 2x - 1 = 0$ are:

    (a). 1 and -1

    (b). 1 and 1

    (c). $1 \pm 2\sqrt{2}$

    (d). $2 \pm \sqrt{2}$

    (e). $1 \pm \sqrt{2}$

34. Find the solution set for t, the number of seconds required for a ball thrown upward with initial velocity, v = 48 feet per second, to be at a height, h, of 32 feet when $h = vt - 16t^2$.

    (a). $\{1\}$

    (b). $\{2\}$

    (c). $\{1, 2\}$

    (d). $\{(3 \pm \sqrt{17})/2\}$

    (e). $\{2/3\}$

35. If S is the sum of a non-zero real number and its reciprocal then which of the following is true about S?

    (a). $S \geq 1$

    (b). $S \geq 2$

    (c). $|S| \geq 2$

    (d). $|S| \leq 2$

    (e). S is positive

36. If $(2 + i)(x + yi) = 5 + 5i$ then the values for x and y are:

    (a). $x = 10/3, y = 5/3$

    (b). $x = 5/3, y = 10/3$

    (c). $x = 1, y = 3$

    (d). $x = 5/3, y = 5/3$

    (e). $x = 3, y = 1$

37. Two numbers have a ratio of 1 to 3. If the difference between the squares of the numbers is 1800, what is the larger of the two numbers?

    (a). 45

    (b). 54

    (c). 30

    (d). 15

    (e). 48

38. The solution set $\{x, y\}$ for the system $\left\{\begin{array}{l} ax + by = b \\ x - y = 2 \end{array}\right\}$ is:

    (a). $\{(3b)/(a + b), (b - 2a)/(a + b)\}$, $a \neq -b$

    (b). $\{(b - 2a)/(a + b), (3b)/(a + b)\}$, $a \neq -b$

    (c). $\{(2a - b)/(a + b), (3b)/(a + b)\}$, $a \neq -b$

    (d). $\{(3b)/(a + b), (2a - b)/(a + b)\}$, $a \neq -b$

    (e). $\{(b - 2a)/(a - b), (3b)/(a - b)\}$, $a \neq b$

39. A solution for $\dfrac{2x + 1}{2x - 1} - \dfrac{10}{4x^2 - 1} = \dfrac{2x - 1}{2x + 1}$ is:

    (a). $x = 1$

    (b). $x = 1/2$

    (c). $x = 4/5$

    (d). $x = 5/4$

    (e). $x = 2$

40. $\{(x,y) \mid P(x,y)\}$ is a <u>function</u> when $P(x,y)$ is which one of the following?

    (a). $x^2 + y^2 = 9$

    (b). $x - y^2 = 1$

    (c). $y - x^2 = 1$

    (d). $|y| = |2x|$

    (e). $x^2 - y^2 = 9$