FOURTEENTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

1970

Scoring Formula: 4R - W.

Prepared by:

Glenn F. Webb, Chairman
Michael R. Vaughan-Lee
Stephen D. Comer
Robert H. Bowman

- Vanderbilt Univ.

DIRECTIONS:

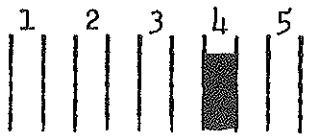
Do not open this booklet until you are told to do so.

This is a test of your competence in First-year Algebra. For each of the 40 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) $\frac{2}{3}$ (2) 3 (3) 6

(4) $\frac{3}{2}$ (5) none of these.

1. 

The correct answer for the sample is " $\frac{3}{2}$ ", which is answer (4); therefore, you should answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any question. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next page. The working time for the entire test is 80 minutes.

1. The expression $\sqrt{2x} + \sqrt{18x}$ is equal to which of the following:

   (1) $\sqrt{20x}$

   (2) $\sqrt{20}\ \sqrt{x}$

   (3) $x\sqrt{2}\ \sqrt{18}$

   (4) $4\sqrt{2x}$

   (5) $4x\sqrt{2}$

2. Simplify the expression $\frac{2x^2y}{6xz^3} \cdot \frac{3xz^4}{y^2}$.

   (1) $\frac{z^2}{y}$

   (2) $\frac{x^3z}{y}$

   (3) $\frac{zx^2}{y}$

   (4) $\frac{xz}{y}$

   (5) none of these

3. If $A + B = C + D$ and $A + C = 2C$ then

   (1) $A = B$

   (2) $A = C$

   (3) $B = C$

   (4) $B = D$

   (5) $C = D$

4. What is the unique number a such that $ax = x$ and $xa = x$ for all real numbers x?

   (1) $a = 0$

   (2) $a = x$

   (3) $a = -x$

   (4) $a = 1$

   (5) no such number exists

5. Find the intersection of the sets $\{10,20,30\}$ and $\{15,25,35\}$.

   (1) $\{10,20,30,15,25,35\}$

   (2) $\{12\ 1/2, 22\ 1/2, 32\ 1/3\}$

   (3) $\{25,45,65\}$

   (4) $\{5\}$

   (5) ϕ

6. What is the solution of the equation $x = 1$?

   (1) i

   (2) 2

   (3) -1

   (4) there is no solution

   (5) none of these

7. The roots of $x^2 + 7x + 3$ are

   (1) real and distinct

   (2) real and not distinct

   (3) imaginary and distinct

   (4) imaginary and not distinct

   (5) real and imaginary

8. Which of the following numbers is less than each of the others?

   (1) $-\frac{99}{100}$

   (2) $-\frac{101}{100}$

   (3) $-\frac{999}{1000}$

   (4) $-\frac{1,001}{1,000}$

   (5) $-\frac{9,999}{10,000}$

9. What are the numbers x such that $\frac{x^2-1}{x^2-1} = 0$?

(1) 1

(2) -1

(3) 1 and -1

(4) i

(5) none exist

10. A perfect number is a positive integer which is equal to the sum of its positive divisors other than itself. For example 6 is a perfect number since $6 = 1 + 2 + 3$. Which of the following is a perfect number?

(1) 30
(2) 12
(3) 57
(4) 28
(5) 143

11. The expression $(x^3+1)(x^2+1)(x+1)$ is equal to

(1) $x^7 + x^6 + x^4 + x^3 + x^2 + x + 1$

(2) $x^7 + x^6 + x^4 + 2x^3 + x^2 + x + 1$

(3) $x^9 + x^6 + x^4 + 2x^3 + x^2 + x + 1$

(4) $x^6 + x^5 + x^4 + 2x^3 + x^2 + x + 1$

(5) $x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

12. Which of the following numbers is the best approximation for $\sqrt{2}$?

(1) .9

(2) 1.1

(3) 1.9

(4) 2.0

(5) 2.1

13. The solutions of equation $\sqrt{x+7} = 1 + x$ are

(1) -3 and 2

(2) 3 and 2

(3) 1 and -2

(4) -1 and -3

(5) none of these

14. The statement that $ab = 0$ implies that

(1) $a = 0$

(2) $b = 0$

(3) $a = 0$ and $b = 0$

(4) $a = 0$ and/or $b = 0$

(5) $a \neq 0$ and $b \neq 0$

15. A factorization of $x^4 - 4$ is

(1) $(x^2-2)^2(x+2)$

(2) $(x-\sqrt{2})^2(x+\sqrt{2})^2$

(3) $(x-\sqrt{2})(x+\sqrt{2})(x^2+2)$

(4) $(x^2-2)(x+2)^2$

(5) $(x^2-2)(x+\sqrt{2})$

16. The time in town T is one hour ahead of the time in town R. The time in town S is 2 hours ahead of the time in town T. The time in town S is one hour ahead of the time in town U. The time in town U is 2 hours ahead of the time in town V. Which two towns have the same time?

(1) U and T

(2) R and V

(3) T and V

(4) R and U

(5) R and V

17. What is $\sqrt[5]{-x^2} \cdot \sqrt[5]{x^3}$?

(1) ix

(2) $|x|$

(3) $-x$

(4) $-x^{6/5}$

(5) $-x^5$

18. Let U be the set $\{1,2,3,4,5\}$, let A be the set $\{2,3\}$, and let B be the set $\{3,4,5\}$. What are the elements of U that are not in A or not in B?

(1) $\{1,3,5\}$

(2) $\{1,2,4,5\}$

(3) $\{1,2\}$

(4) $\{3\}$

(5) none of these

19. What are the chances of tossing a head on each of three successive tosses of a coin?

(1) 1 in 2

(2) 2 in 3

(3) 1 in 8

(4) 1 in 6

(5) none of these

20. Two runners A and B are running on a 1 mile circular track. Suppose that runner A runs at 6 miles per hour and runner B at 8 miles per hour. If runner A starts at 1:00 p.m. and runner B starts at 1:30 p.m. at what time will runner B overtake runner A?

(1) 2:00 p.m.
(2) 2:30 p.m.
(3) 3:00 p.m.
(4) 3:30 p.m.
(5) 4:00 p.m.

21. Assume that the statement "A Democrat is a nice person" is true. Which of the following does it imply?

(1) "A Republican is not a nice person."

(2) "A nice person is a Democrat."

(3) "A nice person is not a Democrat."

(4) "A person who is not nice is not a Democrat."

(5) "A person who is not a Democrat is not a nice person."

22. Consider the number-sequence 2,6,15,60,180. Which number in the sequence should be eliminated in order for each number to divide all the numbers which follow it?

(1) 2

(2) 6

(3) 15

(4) 60

(5) 180

23. Simplify the expression

$$\cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1+x}}}$$

(1) $\frac{1}{1+x}$

(2) $\frac{3+2x}{1+x}$

(3) $\frac{2+x}{3+2x}$

(4) $\frac{2}{3+x}$

(5) $1+x$

24. For what values of a does $x^2 + 2ax + 1 = 0$ have a real solution?

(1) $\{a: a \geq 1 \text{ or } a \leq -1\}$

(2) $\{a: -1 \leq a \leq 1\}$

(3) $\{1,-1\}$

(4) $\{a: a \geq 1\}$

(5) $\{a: a \leq -1\}$

25. The range of the set of ordered-pairs (x,y) such that x is a real number and $y = x^2$ is the set of

(1) all real numbers

(2) all negative real numbers

(3) all nonnegative real numbers

(4) all positive real numbers

(5) all nonpositive real numbers

26. What is $\sqrt{|-x|^2}$?

(1) $-x$

(2) x

(3) $-|x|$

(4) $|x|$

(5) $-|-x|$

27. A man drives at a speed of 45 miles per hour for the first 10 miles of a 20 mile trip. How fast must he drive the remaining 10 miles if he is to average 60 miles per hour for the entire trip?

(1) 75
(2) 80
(3) 90
(4) 120
(5) none of these

28. The expression $(x^2y^2z^2w^2)^3$ is not equal to

(1) $(xyzw)^6$

(2) $(xy)^2(zw)^2(xy)^3(zw)^3(xy)(zw)$

(3) $x^6y^6z^6w^6$

(4) $x^5y^5z^5w^5$

(5) $(x^3y^3z^3w^3)^2$

29. The solution set of the inequality $|x-5| \leq \frac{3}{2}$ is

(1) $\{x: \ x > \frac{13}{2} \text{ or } x < \frac{7}{2}\}$

(2) $\{x: \ 0 \leq x \leq \frac{3}{2}\}$

(3) $\{x: \ \frac{7}{2} \leq x \leq \frac{13}{2}\}$

(4) $\{x: \ \frac{9}{2} \leq x \leq \frac{15}{2}\}$

(5) $\{x: \ x \geq \frac{13}{2} \text{ or } x \leq \frac{7}{2}\}$

30. The curves $y = x^3$ and $y = x$ intersect in how many distinct points?

(1) 0

(2) 1

(3) 2

(4) 3

(5) 4

31. The two simultaneous equations $2x + 5y = 2$
$-4x - 10y = 0$

have which of the following as their solution:

(1) $x = 6$ and $y = -2$

(2) $x = 5$ and $y = -2$

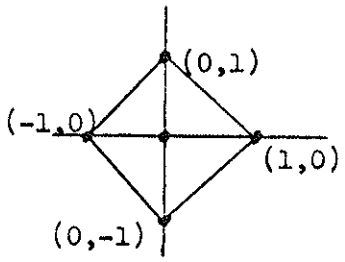
(3) $x = 1$ and $y = 1$

(4) $x = 0$ and $y = 0$

(5) no solutions exist

32. The set of points $\{(x,y) \mid |x| \leq 1 \text{ and } |y| \leq 1\}$ represented in the plane is

(1)



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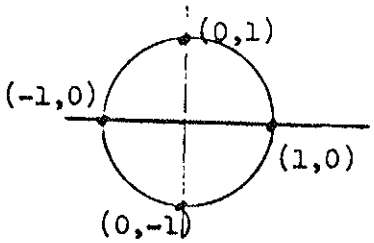

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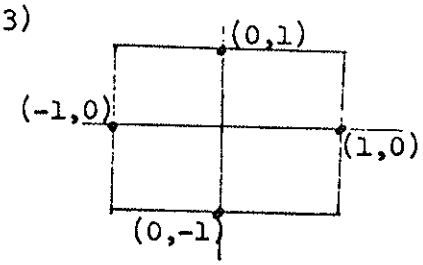

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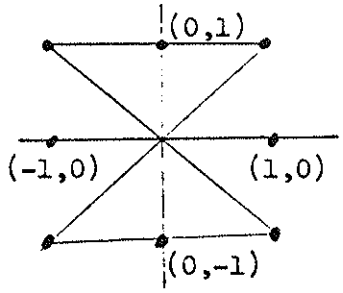

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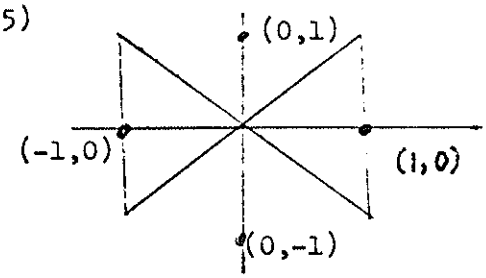

33. Which of the following regions has the largest area?

(1) a square with side 1

(2) a equilateral triangel with side 1

(3) a square with diagonal 1

(4) a circle with diameter 1

(5) a semi-circle with diameter 1

34. The line through the point $(1,5)$ with slope 2 intersects the y-axis at the point

(1) $(0,0)$

(2) $(3,0)$

(3) $(-5,0)$

(4) $(0,3)$

(5) $(1,2)$

35. If $0 < \varepsilon < 1$ then which of the following numbers is the greatest?

(1) 1

(2) $1 + \varepsilon$

(3) $(1+\varepsilon)^2$

(4) $1 + \varepsilon^2$

(5) $(1 + \varepsilon^2)^2$

36. If A, B, C, and D are real numbers and $A > B$ and $C > D$, then

(1) $AB > CD$

(2) $BC > D^2$

(3) $A^2 > BC$

(4) $AC > BD$

(5) none of these

37. Let S be the sum of all odd numbers from 1 to 100 inclusively and let T be the sum of all even numbers from 1 to 100 inclusively. What is $T - S$?

(1) 0

(2) 1

(3) 50

(4) 100

(5) none of these

38. In how many different ways can 4 students be seated in 4 chairs?

(1) 1

(2) 4

(3) 12

(4) 16

(5) 24

39. Observe that

(1) + (1) = 2

(1+2) + (1+2) = 3 + 3

(1+2+3) + (1+2+3) = 4 + 4 + 4

(1+2+3+4) + (1+2+3+4) = 5 + 5 + 5 + 5

What is the sum of all the positive integers between 1 and 1000inclusively?

(1) 500,500

(2) 1,001,100

(3) 2,002,000

(4) 250,250

(5) 499,500

40. A radiator holds 10 quarts which is 20% antifreeze and 80% water. Each 1% of the anti-freeze in the total mixture reduces the freezing point of the mixture one degree. How much of the mixture should be drained off so that if it is replaced with pure anti-freeze the new mixture will be protected to 0°? (The freezing point of water is 32°).

(1) 1.2 quarts
(2) 1.5 quarts
(3) .8 quarts
(4) 1.8 quarts
(5) none of these