

TWELFTH ANNUAL MATHEMATICS CONTEST

Sponsored by

THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

1968

Scoring Formula: $4R - W$

Prepared by:

Herbert Willcox, Chairman
Evelyn Brown
Roger Lessman
Jerry Powell

- Tennessee Technological
University

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in Algebra I. For each of the 40 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) $\frac{2}{3}$ (2) 3 (3) 6

(4) $\frac{3}{2}$ (5) none of these

1.

1	2	3	4	5
			■	

The correct answer for the sample is $\frac{3}{2}$, which is answer (4); therefore, you should answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any question. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next page. The working time for the entire test is 80 minutes.

1. The solution set of the inequality $|x - 3| \leq 2$ is the set of all real numbers satisfying

- (1) $0 \leq x \leq 5$. (3) $-1 \leq x \leq 5$. (5) $-2 \leq x \leq 2$.
 (2) $1 \leq x \leq 5$. (4) $x \leq -5$.

2. In the equation $3x^2 + x = 0$,

- (1) both roots are positive.
 (2) one root is positive and the other is zero.
 (3) one root is negative and the other is zero.
 (4) both roots are negative.
 (5) one root is positive and the other negative.

3. The solution set of the equation

$$\frac{2}{(x-1)} + \frac{1}{(x+1)} = \frac{-2}{x^2-1} \text{ is}$$

- (1) $\{1\}$. (2) $\{-1\}$. (3) $\{1, -1\}$. (4) \emptyset .
 (5) the set of all real numbers.

4. Consider the statement "If it rains, then the wind blows." The contrapositive of this statement is

- (1) "If the wind blows, then it rains."
 (2) "If the wind does not blow, then it does not rain."
 (3) "If the wind blows, then it does not rain."
 (4) "If it does not rain, then the wind does not blow."
 (5) "If it rains, then the wind does not blow."

5. A boat can travel 38 miles downstream in the same time that it takes to go 26 miles upstream. If its rate in still water is 16 miles an hour, what is the rate of the current?

- (1) 2 miles an hour (3) 3 miles an hour (5) 4 miles an hour
 (2) $2\frac{1}{2}$ miles an hour (4) $3\frac{1}{2}$ miles an hour

6. Let $U = \{1, 2, 3, 4, 5\}$ be the universal set. Let $A = \{2, 3\}$ and $B = \{1, 3, 4\}$. If B' is the notation used for the complement of B , then $A \cup B'$ is
- (1) $\{2\}$. (3) $\{2, 5\}$. (5) $\{2, 3, 5\}$.
 (2) $\{3\}$. (4) $\{1, 2, 3, 4\}$.
7. Let p and q be statements. In symbolic language the statement, "p implies q", is written in the form " $p \rightarrow q$ " and the negation of p is indicated by " $\sim p$ ". Given that $p \rightarrow q$ is a valid implication, which of the following is also a valid implication?
- (1) $q \rightarrow p$ (3) $q \rightarrow \sim p$ (5) $\sim q \rightarrow \sim p$
 (2) $\sim p \rightarrow q$ (4) $\sim p \rightarrow \sim q$
8. Consider the equation $ax^2 + bx + c = 0$ where a , b , and c are real and $b^2 - 4ac$ is positive. The roots are always
- (1) imaginary.
 (2) rational.
 (3) natural numbers
 (4) real.
 (5) negative numbers.
9. If x is a real number, $|x|$ is
- (1) $-x$ if $x < 0$. (3) 0 if $x > 0$. (5) always positive.
 (2) $-x$ if $x > 0$. (4) x if $x < 0$.
10. $\left(x \div \frac{1}{2}\right) + x = 3\sqrt{N}$, where N is a counting number. Which one of the following satisfies this equation?
- (1) $2\sqrt{N}$ (3) \sqrt{N} (5) none of these
 (2) $\frac{\sqrt{N}}{2}$ (4) $3\sqrt{N}$

11. If a car travels at a rate of 45 m.p.h., how many miles does it travel in 45 minutes?
- (1) 60 (2) 35 (3) 45 (4) 50 (5) none of these
12. If x is a non-zero real number and $m = 2n$, where m and n are positive integers, then $\frac{x^m}{x^{2n}}$ is which of the following?
- (1) x^{m-n} (2) x^{4n} (3) 1 (4) 0 (5) none of these
13. $7\frac{1}{2}\%$ of what number is 75?
- (1) 5.625 (2) 56.25 (3) 10 (4) 100 (5) 1000
14. Simplify $\frac{\pi r^2}{\pi R^2 - \pi r^2} \div \frac{2\pi r}{R + r}$.
- (1) $\frac{r}{2\pi(R - r)}$
- (2) $\frac{r}{2(R - r)}$
- (3) $\frac{1}{2(R - 1)}$
- (4) $\frac{R + 1}{2\pi R^2 - 1}$
- (5) $\frac{1}{2\pi(R - 1)}$
15. There are 100 people in a room. If each person shakes hands with every other person exactly one time, how many hand shakes occur?
- (1) 10,000 (3) 999 (5) none of these
- (2) 4,950 (4) 4,450
16. The inequality $\frac{p}{q} > \frac{r}{s}$, where $p, q, r,$ and s are positive integers, implies which of the following?
- (1) $p > r$ (3) $ps < qr$ (5) none of these
- (2) $q > s$ (4) $ps > qr$

17. The simplest form of $\sqrt{(-16)^2} - (\sqrt{16})^2$ is
 (1) 32. (2) -32. (3) 8. (4) 0. (5) none of these.
18. The solution set of $\frac{1}{x} > 1$ is
 (1) $\{x \mid x > 1\}$.
 (2) $\{x \mid 0 < x < 1\}$.
 (3) $\{x \mid x < 1\}$.
 (4) $\{x \mid x > 0\}$.
 (5) $\{x \mid x < 0\}$.
19. The domain of the relation $\{(x,y) \mid y = x^2 + 2\}$ is the set $\{-1,0,1,2\}$.
 The range of the same relation is
 (1) $\{(0,2), (1,3), (2,6)\}$.
 (2) $\{0,2,3,6\}$.
 (3) $\{(-1,3), (0,2), (1,3), (2,6)\}$.
 (4) $\{2,3,6\}$.
 (5) none of these.
20. A(0,3) and B(1,5) are two points in the coordinate plane. The slope of the line that contains A and B is
 (1) $\frac{1}{2}$. (2) 8. (3) 2. (4) $\frac{1}{8}$. (5) none of these.
21. An equation of a circle with the center at (1,2) and a radius of 3 is
 (1) $x^2 + y^2 = 9$.
 (2) $(x-1)^2 + (y-2)^2 = 3$.
 (3) $(x+1)^2 + (y+2)^2 = 9$.
 (4) $(x-1)^2 + (y-2)^2 = 9$.
 (5) none of these.





34. A factorization of $x^3 - 8$ is
- (1) $(x - 2)(x^2 - 2x + 4)$.
 - (2) $(x + 2)(x^2 - 2x + 4)$.
 - (3) $(x - 2)(x^2 + 2x + 4)$.
 - (4) $(x - 2)(x^2 + 4x + 4)$.
 - (5) none of these.
35. The equation $x^3 + x^2 + 2x + 1 = 0$ has a real root between
- (1) 0 and $\frac{1}{2}$.
 - (2) $\frac{1}{2}$ and 1.
 - (3) 2 and 3.
 - (4) 3 and 4.
 - (5) -1 and 0.
36. If the area of the interior of an equilateral triangle is $\frac{8}{\sqrt{3}}$ square units, what is the height of the triangle?
- (1) $\sqrt{8}$ units
 - (2) $\frac{\sqrt{8}}{2}$ units
 - (3) $\frac{\sqrt{8}}{3}$ units
 - (4) $\frac{\sqrt{8}}{6}$ units
 - (5) none of these
37. Solve $x + 2\sqrt{x+2} - 1 = 0$ for x .
- (1) -1
 - (2) +1
 - (3) 7
 - (4) -7
 - (5) No solution is possible if x is a real number.
38. Find the solution for the given system of simultaneous equations.
- $$\begin{cases} x - y = 4 \\ x = y \end{cases}$$
- (1) $x = 2, y = 2$
 - (2) $x = -2, y = -2$
 - (3) both the solutions in (1) and (2)
 - (4) $x = 0, y = 0$
 - (5) These equations have no solution.

39. Solve $x + \frac{4}{x} = \frac{3}{x} - 7$ for x .

(1) $\frac{-7 \pm \sqrt{53}}{2}$

(2) 0

(3) $\frac{-7 \pm \sqrt{45}}{2}$

(4) 1

(5) none of these

40. The curves $y = x^2$ and $x^2 + (y - 1)^2 = 1$ intersect in how many distinct points?

(1) 0 (2) 1 (3) 2 (4) 3 (5) 4





