ELEVENTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

1967

Scoring Formula: 4R - W.

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DIRECTIONS:

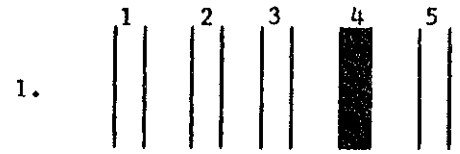
Do not open this booklet until you are told to do so.

This is a test of your competence in First-year Algebra. For each of the 40 problems there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) $\frac{2}{3}$ (2) 3 (3) 6

(4) $\frac{3}{2}$ (5) none of these.



The correct answer for the sample is " $\frac{3}{2}$ ", which is answer (4); therefore, you should answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any question. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next page. The working time for the entire test is 80 minutes.

1. In the formula $A = S^2$, if S is doubled, then A is:

   (1) doubled

   (2) quadrupled

   (3) halved

   (3) constant

   (5) none of these.

2. Indicate which one of the following is the largest:

   (1) $\frac{2}{3}$ (2) $\frac{9}{10}$ (3) $\frac{7}{8}$ (4) $\frac{47}{64}$ (5) $\frac{8}{9}$.

3. Factor $x^4 - 16y^8$ into prime factors.

   (1) $(x - 2y^2)(x + 2y^2)(x^2 + 4y^4)$

   (2) $(x - 2y)(x + 2y)(x - 2y^2)\ x + 2y^2)$

   (3) $(x - 2y)(x + 2y)(x - 4y^2)(x + y^2)$

   (4) $(x - 4y^2)(x + 4y^2)(x^2 - y^4)$

   (5) none of these.

4. Simplify: $\frac{2x^2 - xy + 6x - 3y}{2x - y}$

   (1) $7x + 3$

   (2) $x + 3$

   (3) $2x^2 - xy + 3$

   (4) $x - 3$

   (5) none of these.

5. The statement that am = an implies m = n is:

   (1) always true

   (2) true except when m = 0

   (3) true except when n = 0

   (4) true except when a = 0

   (5) never true.

6. Find the greatest common divisor of the following: $x^5 - x$, $x^5 - x^2$, $x^5 - x^3$.

   (1) $x^3(x^2 - 1)$

   (2) $x^3(x^4 - 1)(x^2 + x + 1)$

   (3) $x^3(x - 1)$

   (4) $x(x + 1)$

   (5) $x(x - 1)$.

7. If $S = \{(x,y) \mid 0 \leq x \leq 1 \text{ and } 0 \leq y \leq 1\}$, the set of points in the xy-plane described by S is:

   (1) a circular region

   (2) a square region

   (3) a one-dimensional region

   (4) a triangular region

   (5) none of these.

8. If $(x^3 - 3x^2 - x + 1)$ is divided by $(x^2 - 2)$ the remainder is:

   (1) $x - 5$

   (2) $x - 7$

   (3) $-3x - 5$

   (4) $-3x + 7$

   (5) none of these.

9. What is the value of "a" such that the line with equation $3x - 2y + 1 = 0$ contains the point (a, 2)?

   (1) $a = 0$

   (2) $a = 1$

   (3) $a = -1$

   (4) $a = 2$

   (5) none of these.

10. If pencils can be bought for x cents per dozen, the number of pencils that can be bought for \$2 is:

    (1) $\frac{2}{x}$

    (2) $\frac{24}{x}$

    (3) $\frac{240}{x}$

    (4) $\frac{2400}{x}$

    (5) $\frac{200}{x}$.

11. If $x = 1$, then $1 + \dfrac{1}{1 + \dfrac{1}{1 + \frac{1}{x}}} =$

    (1) 1 (2) $1\frac{1}{3}$ (3) $\frac{1}{4}$ (4) $1\frac{1}{5}$ (5) none of these.

12. Find the values of x for which this inequality holds: $x^2 - 3x + 2 \leq 0$:

    (1) $x \geq 2$ (2) $x \leq 1$ (3) $x \geq 2$ or $x \leq 1$ (4) $1 \leq x \leq 2$ (5) $x \geq 0$.

13. In the decimal numeration system (base 10), the number 123 means $1 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$. How would the decimal number 13 be expressed in the base two numeration system?

    (1) 1111 (2) 1011 (3) 1101 (4) 1110 (5) none of these.

14. Brian can do a piece of work in 7 days, while Joe requires 9 days. How many days will it take Brian and Joe working together?

    (1) 8 (2) $4\frac{15}{16}$ (3) $3\frac{5}{8}$ (4) $3\frac{15}{16}$ (5) 16.

15. Which property of the real numbers justifies the equation $2(4 + 3) = 2 \cdot 4 + 2 \cdot$

    (1) commutative property for multiplication

    (2) associative property for multiplication

    (3) associative property for addition

    (4) distributive property

    (5) none of these.

16. Find all solutions of $x^4 - 16 = 0$.

(1) 2, 2, -2, -2

(2) $-\sqrt{2}, \sqrt{2}, \quad 2\sqrt{2}, \quad -2\sqrt{2}$

(3) 2, -2, 2i, -2i

(4) $i\sqrt{2}, \quad -i\sqrt{2}, \quad 2\sqrt{2}, \quad -2\sqrt{2}$

(5) 2, -2, 4, -4.

17. The solution set for the inequality $|x| \leq 2$ is the set of all real x satisfying:

(1) $0 \leq x \leq 2$

(2) $x \leq -2$

(3) $-2 \leq x \leq 0$

(4) $x \geq 2$

(5) $-2 \leq x \leq 2$

18. A man wants to receive \$1000 immediately and pay it back in one year. The bank charges a discount of 6% payable at once. How much must be borrowed?

(1) \$1060 (2) \$1166.67 (3) \$1094 (4) \$940 (5) \$1063.83.

19. The solution set for the inequality $x^2 + 6x + 10 > 0$ is:

(1) the set of all real x satisfying $-3 < x < 3$

(2) the set of all real x satisfying $-3 \leq x \leq 3$

(3) the set of all real x

(4) the set of all real x satisfying $-3 < x$

(5) the set of all real x except $x = 0$.

20. Simplify: $(a^x \cdot a^{2x}) \div a^{-3x}$.

(1) 1

(2) $\frac{1}{a^{6x}}$

(3) a^{6x}

(4) a^{9x}

(5) none of these.

21. In the table shown, the formula relating x and y is:

x	1	2	3	4	5
y	3	7	13	21	31

(1) $y = 4x - 1$

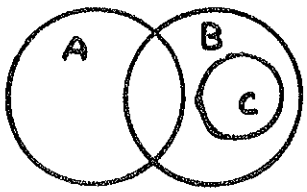
(2) $y = x^3 - x^2 + x + 2$

(3) $y = x^2 + x + 1$

(4) $y = (x^2 + x + 1)(x - 1)$

(5) none of these.

22. Which figure represents sets A, B, and C if $C \subset B$ and $C \cap A = \emptyset$?



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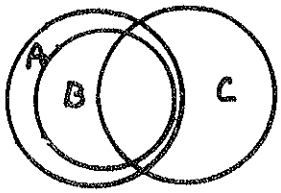

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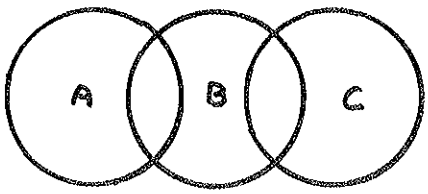

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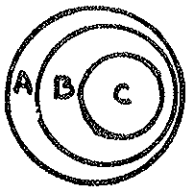

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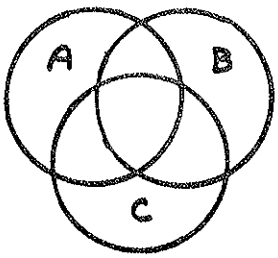

(5)

23. The statement $|x - 1| < 3$ implies that:

(1) $x = 4$

(2) $x < 4$

(3) $x > -2$

(4) $-2 < x < 4$

(5) $x < -2$ or $x > 4$.

24. A real number a such that $-a$ is a root of the equation $2y + 7 = y + 5$ is:

(1) -2 (2) $+2$ (3) -4 (4) $+4$ (5) none of these.

25. If $[x]$ represents the largest integer that is less than or equal to x, then $[-1.68] + [.01]$ is:

(1) -2 (2) -1 (3) -3 (4) -4 (5) $+1$.

26. The largest value taken by the function $f(x) = 2x + 4$ when x is restricted to the interval $-1 \leq x \leq +1$ is:

(1) 6 (2) 1 (3) 2 (4) -1 (5) none of these.

27. A plane leaves Knoxville for New York at 3:00 P.M. and a plane leaves New York for Knoxville at 4:00 P.M. and both planes travel at a speed of 300 m.p.h. Assuming that New York is 600 miles from Knoxville, at what time will the planes meet?

(1) 4:15 P.M. (2) 4:30 P.M. (3) 4:45 P.M. (4) 4:20 P.M. (5) none of these.

28. Which one of the following sets of points does the graph represent?

(1) $-2 \leq x < 3$

(2) $x \leq -2$ and $x > 3$

(3) $-2 \geq x > 3$

(4) $x \geq -2$ or $x < 3$

(5) $x \leq -2$ or $x > 3$.

29. Given that $3^{x+y} = 81$ and $3^{x-y} = 3$, then:

(1) $x = \frac{5}{2}$, $y = \frac{3}{2}$

(2) $x = 5$, $y = 4$

(3) $x = \frac{2}{5}$, $y = \frac{2}{3}$

(4) $x = 2$, $y = 2$

(5) none of these.

30. Which of the following values of y is an upper bound for $y = -|x| + 1$?

(1) 0 (2) -1 (3) -2 (4) $\frac{1}{2}$ (5) none of these.

31. Mary's and John's ages add up to equal Dave's age. Mary is 14 years older than John who is $\frac{1}{3}$ as old as Dave. How old is Dave?

(1) 46 (2) 36 (3) 26 (4) 16 (5) 42.

32. If $A = \{x \mid x \leq 2\}$ and $B = \{x \mid x < -3\}$ then $A \cap B =$

(1) $\{x \mid x \leq -3\}$

(2) $\{x \mid x \leq 2\}$

(3) $\{x \mid x < -3\}$

(4) $\{x \mid x < 2\}$

(5) $\{x \mid -3 < x \leq 2\}$.

33. If the area of the interior region of an equilateral triangle is $4\sqrt{3}$ sq. units, what is the length of a side?

(1) $2\sqrt{3}$ units

(2) $\frac{4}{3}\sqrt{3}$ units

(3) $\sqrt{3}$ units

(4) 4 units

(5) none of these.

34. The diagonal of a square is $b + c$. The perimeter of a second square which has four times the area of the first one is:

(1) $4\sqrt{2}(b + c)$

(2) $4(b + c)$

(3) $\sqrt{2}(b + c)$

(4) $4\sqrt{2}(b + c)^2$

(5) none of these.

35. The solution set for the system of equations $\begin{cases} 2x - 3y = 3 \\ 4x + y = 6 \end{cases}$ is:

(1) $\{(0,6)\}$

(2) $\left\{\left(\frac{9}{10}, -\frac{12}{5}\right)\right\}$

(3) $\{(1,2)\}$

(4) $\left\{\left(\frac{3}{2}, 0\right)\right\}$

(5) $\left\{\left(-\frac{3}{2}, -2\right)\right\}$.

36. Suppose that a collection of coins consisting of nickels, dimes and quarters, is worth \$6.25 and contains 35 coins. If there are twice as many dimes as nickels, then how many quarters are there?

(1) 5 (2) 10 (3) 15 (4) 20 (5) 25.

37. The roots of the equation $4x^2 + 12x + 9 = 0$ are:

(1) equal (2) unequal (3) integral (4) irrational (5) imaginary.

38. Which one of the following is a solution ox $x^3 - 8x^2 + 19x - 12 = 0$?

(1) 2 (2) 1 (3) -1 (4) -3 (5) none of these.

39. The rational roots of $x^4 - x^3 - x^2 - x - 2 = 0$ are:

(1) 1,2 (2) 1,-2 (3) -1,2 (4) -1,-2 (5) 0,1.

40. The equation $\begin{vmatrix} x & 1 & 0 \\ 2 & x & -1 \\ 1 & -1 & 1 \end{vmatrix} = 0$

(1) has no solutions

(2) has two complex conjugate solutions

(3) has one real and one imaginary solution

(4) has two distinct real solutions

(5) has a double real solution.