Tenth Annual Mathematics Contest

Algebra I 1966

1) Let $A = \{x \mid x$ is a counting number less $53\}$.
Which of the following is false?
(1) $3/2 \in A$; (2) $17 \in A$; (3) $61 \notin A$; (4) $-18 \notin A$; (5) none

2) A can do a piece of work in 10 hours and B can do the work in 12 hours.
How long will it take them to do it together?
(1) 5.5; (2) 5; (3) $5\ 5/11$; (4) 6; (5) none of these.

3) The denominator of a fraction is 7 more than the numerator. If 2 be added to the numerator, the value of the fraction becomes $1/2$. What is the fraction?
(1) $5/12$; (2) $9/16$; (3) $1/8$; (4) $3/10$; (5) none

4) The solution of the equation $1 = \frac{x}{x-2} + \frac{4}{x+1}$ is:

(1) 2; (2) $1\ 1/2$; (3) 1; (4) -1; (5) none of these

5) The solution set for the system of equations
$\begin{cases} 4x - 3y = -3 \\ 7x + 2y = 2 \end{cases}$ is (1) $\{(1,0)\}$; (2) $\{(0,1)\}$; (3) $\{(-3,-3)\}$;
(4) $\{(0,-1)\}$; (5) none of these

6) Twice the square of a certain number exceeds five times the number by 7.
One such number is:
(1) -1; (2) $-7/2$; (3) $2/7$; (4) 1; (5) none of these

7.) If $a = \frac{nt-b}{n}$, then b equals
(1) $nt-a$; (2) $n(t+a)$; (3) $-n+na$ (4) nt;
(5) none of these.

8.) The expression $\left(\frac{x}{y}-\frac{y}{x}\right) \div \left(\frac{x}{y}+\frac{y}{x}\right)$ is equal to (1) $\frac{x^2-y^2}{x^2+y^2}$; (2) $\frac{x-y}{x+y}$; (3) 0; (4) -1; (5) none

9.) An equivalent expression for $8x^2-6xy-9x^2$ is (1) $(4x-3y)(2x+3y)$; (2) $(2\sqrt{2x}-3y)^2$; (3) $2x(4x-3y+1)$; (4) $(4x+3y)(2x-3y)$; (5) none

10.) One automobile takes 2 hours less time than another for a trip of 180 miles. If the rate of the first automobile is $3/2$ that of the second, what is the rate of each?
(1) (45 mph and 30 mph) (2) (30 mph and 20 mph)
(3) (54 mph and 36 mph) (4) (36 mph and 18 mph)
(5) none of these

11.) The expression $15\sqrt{18} \div 3\sqrt{3}$ simplifies to
(1) $-5\sqrt{6}$; (2) 30; (3) $5\sqrt{6}$; (4) $\frac{\sqrt{6}}{30}$; (5) none

12.) Suppose that x and y vary inversely, and that $y=5$ when $x=6$. What is y when x is 15?
(1) $1/2$; (2) 2; (3) $25/2$; (4) -2; (5) none

13.) If $S \subset T$, then $S \cup T =$
(1) T; (2) S; (3) $S \cap T$; (4) null set (5) none

14.) The solution set for the inequality
$-3w + 6 \neq -8$ is
(1) $w \neq 14/3$; (2) $w \geq 14/3$ (3) $w > 14/3$;
(4) $w < 14/3$; (5) none of these

15.) The slope of the equation $y/2 + x/3 = 0$ is
(1) $2/3$; (2) 1.5; (3) $-1/6$; (4) $-3/2$; (5) none

16.) The discriminant of the equation
$4y^2 + 5y + 1 = 0$ is
(1) 41; (2) 3; (3) 9; (4) -3; (5) none

17.) Suppose $F = \{(x, y) \mid y$ is the additive inverse of $x^3\}$, then an ordered pair in F is
(1) $(2, 1/2)$; (2) $(2, -2)$; (3) $(1/2, 2)$; (4) $(2, 4)$; (5) $(4, 2)$

18.) In a chemistry lab there are two jars containing a mixture of acid and water, one jar containing a 20 per cent acid solution and the other a 40 per cent acid solution. The instructor wants to get 4 ounces of a 25 per cent solution. How many ounces should he take from each jar?

(con't on next page)

(Problem 18 cont'd)

(1) 3 ounces of 20 per cent solution and one ounce of 40 per cent solution.
(2.) 2 ounces of 20 per cent solution and two ounces of 40 per cent solution.
(3.) 3 ounces of 20 per cent solution and two ounces 40 per cent solution,
(4.) 1 ounce of 20 per cent and one ounce of 40 per cent solution.
(5) none of these

19.) The expression $5\{3[12-(81-78)]\}$ is equivalent to (1) 72; (2) 225; (3) -60; (4) 135; (5) none of these

20.) For any real numbers a, b, and c, the distributive law allows us to state that (1) $a+(b+c)=(a+b)+c$
(2) $abc=cba$ (3) $a(b+c)=ab+ac$
(4) $a+b=c$ (5) none of these

21.) The product of $(4x+3)$ and $(2x-3)$ is
(1) $8x^2-9$; (2) $8x^2-6x-9$; (3) $6x$;
(4) $8x^2+6x-9$; (5) none of these.

22.) The difference of y^2-2y+5 and $2y^2+3y-6$ is
(1) $-y^2-5y+11$; (2) $y^2-5y-11$; (3) y^2-5y-1;
(4) $y^2+5y+11$; (5) none of these

23.) The fraction: $\dfrac{\frac{x}{5} - \frac{y}{2}}{\frac{2x^2}{5} - \frac{5y^2}{2}}$

may be simplified and expressed as

(1) $2x+5y$; (2) $\frac{1}{2x-5y}$; (3) $\frac{10}{2x+5y}$ (4) $\frac{1}{2(x-5y)}$

(5) $\frac{1}{2x+5y}$

24.) If $\dfrac{3}{\sqrt{3n+1}} = 2$, then n is

(1) 1/3; (2) 5/12; (3) 1/6; (4) 12/5 (5) none

25.) Find a two digit integer such that the tens digit is one less than twice the ones digit, and the number itself is 36 more than the number obtained by reversing the digits of the original integer

(1) 59; (2) 36; (3) 95; (4) 72; (5) no such no

26.) If the polynomial $x^2 + 3x^2 - 9x + 10$ is divided by $x-1$, the remainder is

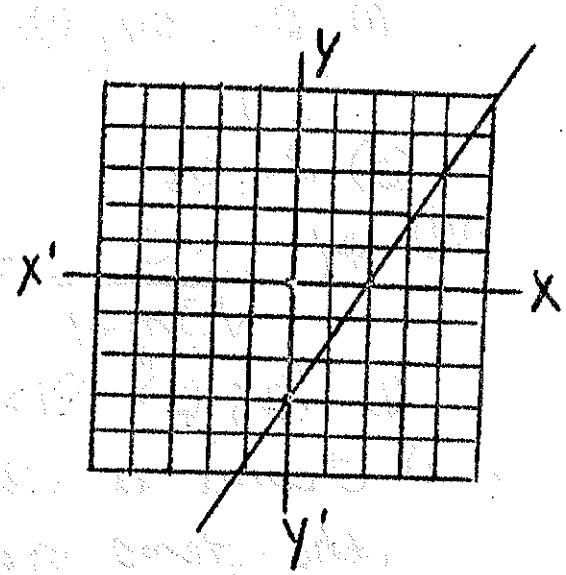
(1) -5; (2) 23; (3) 5; (4) 17; (5) 21

27.) The root of the equation $\dfrac{x+3}{x-4} = \dfrac{x+3}{x+2}$ is

(1) 4; (2) -2; (3) 3; (4) -3; (5) none of these

28.) The expression $13\sqrt{6} + 5\sqrt{150}$ simplifies to
(1) $3\sqrt{2}$; (2) $-12\sqrt{2}$; (3) $180\sqrt{3}$; (4) $36\sqrt{13}$;
(5) none of these

29.) The slanted line in the figure represents the graph of



(1) $3x + 2y = 6$
(2) $2x - 3y = 9$
(3) $2x + 3y = 9$
(4) $2x + 3y = 6$
(5) $3x - 2y = 6$

30.) Four years ago Jake was 8 years older than Joe is now. The sum of their present age is 44. What are their ages?
(1) (28 & 16); (2) (26 & 18) (3) (20 & 24); (4) (36 & 8); (5) none

31.) The expression $(16a^8)^{1/4}$ is equivalent to
(1) $2a^4$; (2) $2a^2$; (3) $16^4 a^{32}$; (4) $4a^4$ (5) none

32.) To complete the square of $2x^2 + 3x$ you would add (1) $9/4$; (2) 9; (3) $9/8$; (4) $9/16$;
(5) none of these

33.) The solution set of the equation $2x^2 + 9 = 9x$ is (1) $\{-3/2, 3\}$; (2) $\{1.5, -3\}$;
(3) $(-3/2, -3)$; (4) $\{3/2, 3)\}$; (5) $\{9/2, 1\}$.

34) A box contains $3.30 in nickles, dimes, and quarters. There are 8 more dimes than quarters and 2 more nickels than dimes. How many nickels are in the box
(1) 5; (2) 13; (3) 15; (4) 10; (5) 12

35) The perimeter of a farm is 600 rods. The length of the farm is 60 rods more than the width. What are the 2 dimensions?
(1) (30 by 20); (2) (150 by 150) (3) (60 by 90);
(4) (40 by 15); (5) (120 by 180)

36) If $A = \{(x,y) \mid y = x + 4\}$ and
$B = \{(x,y) \mid 2y = x + 14\}$ then $A \cap B$ is
(1) $\{-6, -2\}$; (2) $\{10, 14\}$; (3) $\{2, 6\}$; (4) $\{6, 10\}$ (5) none

37) Let $C = \{s \mid -5 \leq s \leq 2\}$ and $D = \{x \mid -8 \leq x \leq -2\}$. If the domain of all variables is the set of real numbers, then $C \cup D$ is
(1) $-8 \leq x \leq 2$; (2) $x \leq 2$; (3) $-8 \leq x < 2$;
(4) $-5 \leq x \leq -2$; (5) none of these

38) If $(a,b) * (c,d) = (ad + bc, ac + bd)$ and
$(a,b) = (c,d)$ means $a = c$ and $b = d$, then the solution set for $(1,2) * (x,y) = (3,6)$ is
(1) $\{6, 15\}$; (2) $\{3, 0\}$; (3) $\{5, 2\}$; (4) $\{(0,3)\}$;
(5) $\{0, -3\}$.

39) If $f(x) = [x]$ where $[x]$ is the largest integer smaller than x, $f(-2.5)$ is

(1) -2; (2) -3; (3) 2.5; (4) 2; (5) none of these.

40) Which of the following is not a lower bound of $y = |x| + 1$?

(1) 0; (2) $-\frac{1}{2}$; (3) $\frac{1}{2}$; (4) -2; (5) none of these