EIGHTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ALGEBRA I TEST

1964

Scoring Formula: 4R - W

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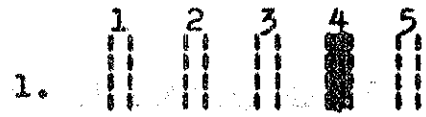
DIRECTIONS:

Do not open this booklet until you are told to do so. For each problem there are listed 5 possible answers. You are to work each problem and determine which is the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x=3$, then x equals:
   (1) 2/3; (2) 3; (3) 6;
   (4) 3/2; (5) none of these.



The correct answer for the sample question is "3/2", which is answer (4); therefore, you should answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any question. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next page. The working time for the entire test is 60 minutes.

1. The fraction $\dfrac{1 - \frac{a}{b}}{1 + \frac{a}{b}}$ may be written as

(1) $\dfrac{(1 - a)}{(1 + b)}$; (2) $\dfrac{(b - a)}{(b + a)}$; (3) $\dfrac{1 - a}{1 + a}$

(4) $\dfrac{1}{b}$; (5) none of these.

2. The sum of $\dfrac{a}{b}$ and $\dfrac{b}{a}$ is

(1) $\dfrac{ab}{ba}$; (2) $a^2 + b^2$; (3) $\dfrac{a^2 + b^2}{ab}$; (4) $\dfrac{a + b}{b + a}$;

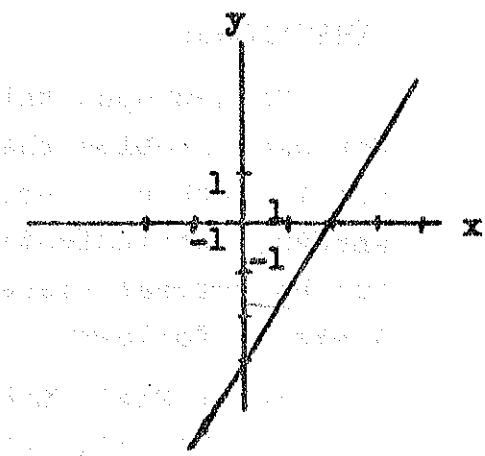
(5) none of these.

3. The line in the graph chart is the graph of the equation:

(1) $2x - 3y = 6$; (2) $3x - 2y = 6$;

(3) $2x + 3y = 6$; (4) $3x + 2y = 6$;

(5) none of these.



4. If $\dfrac{(x^2 - 5x + 6)}{(x + 2)}$ is divided by $\dfrac{(2x - 6)}{(x^2 + 4)}$ the quotient is

(1) $\dfrac{(x^2 - 4)}{2}$; (2) $\dfrac{(x^3 - 2x^2 + 4x - 8)}{(2x + 4)}$; (3) $\dfrac{-(x^2 + 4)}{2}$;

(4) $\dfrac{(x^3 + 2x^2 + 4x + 8)}{(2x + 4)}$; (5) none of these.

5. Factors of $(x^2 - 2\sqrt{2}\,x + 2)$ are $(x - \sqrt{2})$ and

(1) $(x + \sqrt{2})$; (2) $(x - \sqrt{2})$; (3) $(x - 1)$;

(4) $(x - 2)$; (5) It cannot be factored.

6. If the side, s, of a square is increased by 5 units, the area is increased by how many units of area?

(1) 25; (2) $5s + 25$; (3) $25 - 5s$; (4) $10s + s^2$;

(5) none of these.

7. If five-sixths of a number is increased by 17, the result is 62. The number is

(1) $\frac{395}{6}$; (2) $\frac{474}{5}$; (3) $\frac{225}{6}$; (4) 54; (5) none of these.

8. The coordinates of the point of intersection of the graphs of the equations $y = 4x - 3$ and $y = 2x + 3$ are

(1) $(-3, -9)$; (2) $(-9, -3)$; (3) $(9, 3)$; (4) $(3, 9)$;
(5) none of these.

9. If $A = P + Prt$, then $P =$

(1) $\frac{A}{rt}$; (2) $\frac{A}{(1 + rt)}$; (3) $\frac{A}{(1 - rt)}$; (4) $A(1 + rt)$;

(5) none of these.

10. The root of the equation $\frac{x}{2} - 2 = \frac{3x}{4} + 5$ is

(1) 28; (2) -7; (3) - 28; (4) 7; (5) none of these.

11. The sum of $\frac{3}{(x - y)}$ and $\frac{2x}{(x^2 - y^2)}$ is

(1) $\frac{(2x + 3)}{(x^2 + x - y^2 - y)}$; (2) $\frac{(2x + 3)}{(x^2 - y^2)}$;

(3) $\frac{6x}{(x - y)(x^2 - y^2)}$; (4) $\frac{(5x + 3y)}{(x^2 - y^2)}$; (5) none of these.

12. Factors of $(x^3 - y^3)$ are $(x - y)$ and

(1) $(x^2 + 2xy + y^2)$; (2) $(x^2 - 2xy + y^2)$;

(3) $(x^2 - xy + y^2)$; (4) $(x^2 + xy + y^2)$; (5) none of these.

13. Which of the following ordered pairs of numbers does not represent the coordinates of a point of the locus of the equation $y = x^2 + 4$?

(1) $(1, 5)$; (2) $(-1, 5)$; (3) $(0, 4)$; (4) $(2, 8)$;

(5) $(4, 0)$.

14. A set of straight lines is described by the equations $y = 3x + 2$, $y = 5x + 2$, and $y + x = 2$. Each of these lines has the same

(1) slope; (2) y-intercept; (3) angle of inclination; (4) x-intercept; (5) none of these.

15. If $(x^2 - 4x - 8)$ is divided by $(x - 2)$, the remainder is

(1) -4; (2) 12; (3) -12; (4) 4; (5) none of these.

16. The perimeter of a rectangle is 28. The width is denoted by W. The length is

(1) $28 - W$; (2) $14 - W$; (3) $28/W$; (4) $28 - (W/2)$; (5) none of these.

17. Two positive numbers are in the ratio 7 to 3. Their difference is 40. The larger number is

(1) 10; (2) 30; (3) 70; (4) 28; (5) none of these.

18. A board is cut into two pieces. One piece is 3/4 as long as the original board and is 13 1/2 feet long. The number of feet in the length of the original board was

(1) 18; (2) 81/8; (3) 54/7; (4) 135/8; (5) none of these.

19. Which of the following numbers has no reciprocal?

(1) 1/2; (2) 1; (3) 0; (4) 0.3; (5) none of these.

20. A store marked the selling prices of one lot of its goods so as to make a profit of 40 % of the cost. If the selling price of an article is \$39.90, what was its cost?

(1) \$23.94; (2) \$28.50; (3) \$15.96; (4) \$55.86; (5) none of these.

21. If $x < 0$, which of the following statements is true?

(1) $|x| = x$; (2) $|x| > x$; (3) $|x| < x$; (4) $|x| = 0$; (5) none of these.

22. If a, b, and c are real numbers, then $(a + b) + c = a + (b + c)$. The above equation is a statement of the
(1) associative law; (2) commutative law;
(3) distributive law; (4) reflexive law;
(5) symmetric law of addition of real numbers.

23. The number of revolutions N of a gear is inversely proportional to the radius r of the gear. This statement can be expressed as
(1) $N = kr$; (2) $N = 2\pi r$; (3) $N = 1/r$; (4) $N = k/r$;
(5) none of these.

24. The expression $|y| < 3$ is equivalent to the expression
(1) $y < 3$; (2) $y = 3$; (3) $-3 < y < 3$;
(4) $y < -3$ and $y > 3$; (5) none of these.

25. In a certain two digit number, the units' digit is 5 more than the tens' digit. If the digits were reversed, the new number would exceed twice the number by 7. The number is
(1) 38; (2) 83; (3) 49; (4) 27; (5) none of these.

26. One member of the set $f = \{(x, y) \mid y = 2x + 3\}$ is
(1) (3, 1); (2) (3, 0); (3) (3, 12); (4) (3, 9);
(5) none of these.

27. A vat contains 4 gallons of a solution that is 40 % acid and 60 % water. How much water should be added to make a mixture that is 30 % acid?
(1) 3 quarts; (2) 4/3 gallons; (3) 1.2 gallons;
(4) 6 quarts; (5) none of these.

28. The identity element with respect to the operation addition is
(1) 0; (2) 1; (3) $-a$; (4) $1/a$; (5) none of these.

29. A hiker walking in a straight line at the rate of 4 miles an hour leaves a camp at 2:00 P.M. A second hiker walking 5 miles an hour starts after him at 2:30 P.M. When will the second hiker catch up with the first one?

(1) 4:00 P.M.; (2) 4:30 P.M.; (3) 6:00 P.M.;
(4) 6:30 P.M.; (5) none of these.

30. The expression $\sqrt{(-5)^2}$ is equivalent to the expression

(1) -5; (2) 25; (3) -25; (4) $|-5|$; (5) none of these.

31. The length of the radius of circle N is three times the length of the radius of circle M. If the area of circle M is A, the area of circle N is

(1) $3A$; (2) $A/3$; (3) $9A$; (4) $\sqrt{3A}$; (5) $A/9$.

32. If a and b are real numbers, the value of x in the equation $2x + 5 + a = bx + 7$ is unique

(1) if $a \neq 2b$; (2) if $a \neq 6$; (3) if $b \neq 0$; (4) if $b \neq 2$;
(5) for all a and b.

33. The product of 3 and 6 divided by their sum is

(1) 2; (2) 1/2; (3) 18; (4) 6; (5) none of these.

34. Given that x, y, and z are non-zero real numbers, such that $x \neq z$, $y \neq z$. If $1/x + 1/y = 1/z$, then x equals

(1) $z - y$; (2) $y - z$; (3) $\frac{yz}{y - z}$; (4) $\frac{yz}{y + z}$;
(5) none of these.

35. Three times Andy's age added to Bob's age is less than 50. Four times Andy's age minus Bob's age is 34. If a represents Andy's age and b represents Bob's age then

(1) $a < 12$, $b > 14$; (2) $a < 12$, $b = 14$; (3) $a < 12$, $b < 14$;
(4) $a < 12$, but we can put no bounds on b; (5) none of these.

36. If $y = a + (b/x)$, where a and b are constants, and if $y = 1$ when $x = -1$, and $y = 9$ when $x = -5$, then $a + b$ equals

(1) -1; (2) 0; (3) 1; (4) 20; (5) 21.

37. On an examination of m questions a student answers correctly 25 of the first 30. Of the remaining questions he answers two thirds correctly. All the questions have the same credit. If the student's mark is 75%, how many different values of m can be found?

(1) 1; (2) 2; (3) 3; (4) 4;

(5) the problem can not be solved.

38. The Cartesian product of the sets A and B is defined by the following statement.

$$A \times B = \{(x, y) \mid x \varepsilon A \text{ and } y \varepsilon B\}.$$

If $A = \{r, s, t\}$ and $N = \{2, 3\}$, then $A \times N =$

(1) $\{r, 2, s, 3, t\}$; (2) $\{(r,2), (s,3)\}$;

(3) $\{(2,r), (2,s), (2,t), (3,r), (3,s), (3,t)\}$;

(4) $\{(r,2), (r,3), (s,2), (s,3), (t,2), (t,3)\}$;

(5) none of these.

39. Given $a_0 = 1$, $a_1 = 3$, and the general relation

$a_n^2 - (a_{n-1})(a_{n+1}) = (-1)^n$ for $n \geq 1$. Then a_2 equals

(1) 13/27; (2) 10; (3) 21; (4) 33; (5) - 17.

40. Assume that the following three statements are true:

I. All seniors are human.
II. All students are human.
III. Some students study.

Given the following four statements:

(a) All seniors are students.
(b) Some humans study.
(c) No seniors study.
(d) Some humans who think are not students.

Those which are logical consequences of I, II, and III are:

(1) b; (2) d; (3) b,c; (4) b,d; (5) a,b.