2003 MIDDLE SCHOOL MATHEMATICS COMPETITION

AUSTIN PEAY STATE UNIVERSITY
CLARKSVILLE, TENNESSEE

MIDDLE TENNESSEE STATE UNIVERSITY
MURFREESBORO, TENNESSEE

Eighth Grade
Scoring Formula: 4R - W + 30

Tests were developed with support from the Tennessee Center for the Advancement of Mathematics, Science and Technology Education at Middle Tennessee State University, Ray Phillips, Director

DIRECTIONS:

For each problem there are 5 possible answers listed. You are to work the problems, determine the correct answer, and indicate your choice on the separate answer sheet provided.

SAMPLE:

1. If $x + 1 = 2$, then x equals
    a) 0
    b) 2
    c) -1
    d) 1
    e) none of the above

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      A B C D E
1  ① ② ③ ● ⑤
      A B C D E
2  ① ② ③ ④ ⑤
      A B C D E
3  ① ② ③ ④ ⑤
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The correct answer is 1, which is d); so you would answer this problem by darkening the space on the answer sheet corresponding with this choice.

If you change your mind about your answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin. When you have finished one page, go on to the next. There are 30 questions in all. The working time for the entire test is 60 minutes.

1. A penny is made of only zinc and copper. A penny weighs 2.5 grams, and 2.5% of each penny is copper. What is the weight of the zinc in each penny?

    a. 0.625 grams of zinc
    b. 0.0625 grams of zinc
    c. 0.1825 grams of zinc
    d. 2.4375 grams of zinc
    e. 2.45 grams of zinc

2. The population of the United States is about 272,000,000. The national debt is about $6,000,000,000,000. About how much would each person's share be if the national debt were divided evenly among everyone living in the United States?

    a. $22,000
    b. $22,000,000
    c. $1,600,000,000
    d. $16,000
    e. $160,000

3. When I use a calculator to calculate 2^{50}, my calculator says 1.125899907E15. Which statement is true?

    a. 2^{50} is exactly equal to $1.125899907 \times 10^{15}$
    b. 2^{50} is approximately equal to $1.125899907 \times 10^{15}$
    c. 2^{50} is exactly equal to 11,258,999,070,000,000
    d. 2^{50} is exactly equal to 1125899907^{15}.
    e. 2^{50} is exactly equal to 1.125899907^{15}

4. The set of Integers is <u>closed under addition</u> because the sum of every pair of integers is an integer. The set of Integers is <u>not closed under division</u> because the quotient of some pairs of integers is not an integer. Which statement is true about the set of Even Whole Numbers (0, 2, 4, 6, ...)?

    a. The set of Even Whole Numbers is closed under addition and multiplication.
    b. The set of Even Whole Numbers is closed under addition only.
    c. The set of Even Whole Numbers is closed under addition and subtraction.
    d. The set of Even Whole Numbers is closed under multiplication only.
    e. There is no operation under which the set of Even Whole Numbers is closed.

5. Which statement about parallelograms is false?

    a. The diagonals of any parallelogram bisect each other.
    b. Some parallelograms have four right angles.
    c. Some parallelograms are equilateral.
    d. Opposite angles of any parallelogram are supplementary.
    e. Opposite angles of any parallelogram are congruent.

6. Which expression is NOT equivalent to $12x - 6$?

    a. $12(x - \frac{1}{2})$
    b. $6(2x - 1)$
    c. $6x - 6(x + 1)$
    d. $6x - 6(1 - x)$
    e. $3(4x - 2)$

7. Three fifths of the eighth-grade students are girls. Half the eighth-grade boys get to school by bus. One sixth of the eighth-grade girls get to school by bus. There are 147 eighth-grade students who do not ride the bus. How many students are in the eighth grade?

    a. There are 441 students in eighth grade.
    b. There are 882 students in eighth grade.
    c. There are 210 students in eighth grade.
    d. There are 192 students in eighth grade.
    e. There are 368 students in eighth grade.

8. Which statement is true about $\left(\frac{2}{3}\right)^{20}$?

    a. $\left(\frac{2}{3}\right)^{20}$ is greater than $\left(\frac{2}{3}\right)^{10}$.

    b. $\left(\frac{2}{3}\right)^{20}$ is twice as much as $\left(\frac{1}{3}\right)^{20}$

    c. $\left(\frac{2}{3}\right)^{20}$ is less than $\left(\frac{2}{3}\right)^{40}$.

    d. $\left(\frac{2}{3}\right)^{20}$ is less than $\frac{2}{3}$.

    e. $\left(\frac{2}{3}\right)^{20}$ is equal to $\left(\frac{1}{3}\right)^{40}$.

9. The letters A, B, C, D, and E represent the numbers 0, 1, 2, 4, and 8 (not necessarily in that order). Each letter represents exactly one number. If the following equations are true, which letter represents 4?

$$C^D = E$$
$$A^A = C$$
$$\sqrt{B} = A \cdot \sqrt{A}$$

    a. A represents 4.
    b. B represents 4.
    c. C represents 4.
    d. D represents 4.
    e. E represents 4.

10. The greatest common factor of two numbers is 18. The least common multiple of the two numbers is 1386. Neither number is 18. What is the <u>sum</u> of the two numbers?
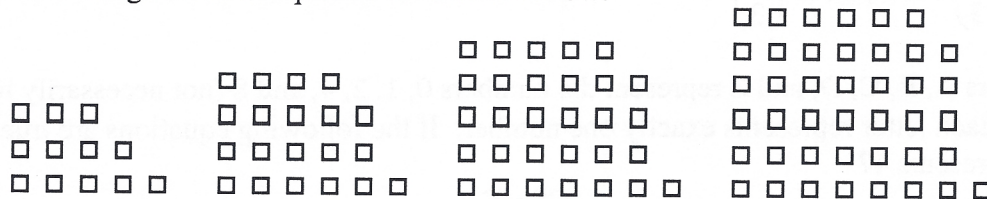
    a. The sum of the numbers is 324.
    b. The sum of the numbers is 77.
    c. The sum of the numbers is 154.
    d. The sum of the numbers is 95.
    e. The sum of the numbers is 36.

11. The graph shown here can be represented by which equation.
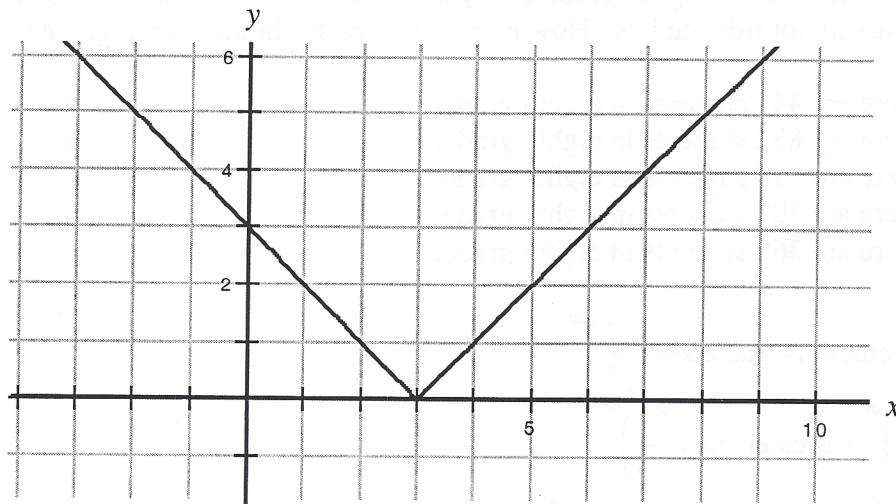
    a. $y = |x + 3|$
    b. $y = |x - 3|$
    c. $y = |x| + 3$
    d. $y = |x| - 3$
    e. $y = 3 - |x|$

12. The first four figures in a sequence are shown below.

How many squares would be in the 50th figure in the sequence?

    a. 2500 squares
    b. 2550 squares
   c. 2756 squares
    d. 3175 squares
    e. 3250 squares

13. A CD club has a one-time membership fee of $25. Members can then order CD's for $5 each. The total cost for joining and buying CD's can be represented by a linear graph with the number of CD's ordered represented on the *x*-axis and the total cost represented on the *y*-axis. Which statement is true?

   a. The y-intercept corresponds to the membership fee.
   b. The slope corresponds to the membership fee.
   c. The y-intercept corresponds to the cost per CD.
   d. The slope would increase if the price per CD decreased.
   e. The y-intercept would change if the price per CD changed.

14. Consider the first 40 numbers in this sequence: 1, 1, 3, 1, 3, 5, 7, 1, 3, 5, 7, 9, 11, 13, 15, 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 1, 3, 5, 7, 9, 11, 13, 15, 17, ...
Notice that the first number is 1. The third number is 3. The seventh number is 7. When will be the next time (after the 40th number) a number in the sequence is the same as its position?

   a. The 63rd number is 63.
   b. The 61st number is 61.
   c. The 47th number is 47.
   d. The 53rd number is 53.
   e. The 64th number is 64.

15. A troll and an ogre take jobs at the monster factory. The troll is very greedy and asks for $100 for his first day's work, $200 for his second day's work, $300 for his third day's work, and so on. (Each day he makes $100 more than he did the previous day.) The ogre says he does not care much for money. He will accept 1¢ on the first day, 2¢ the second day, 4¢ the third day, 8¢ on the fourth day, and so one. (Each day he will make twice as much as he did the previous day.) If they start to work on the same day and work the same days, what is the first day that the ogre's paycheck will be greater than the troll's? (You may assume they get a paycheck at the end of each day.)

   a. The ogre's paycheck will be greater than the troll's on the 12th day.
   b. The ogre's paycheck will be greater than the troll's on the 15th day.
   c. The ogre's paycheck will be greater than the troll's on the 19th day.
   d. The ogre's paycheck will be greater than the troll's on the 21st day.
   e. The ogre's paycheck will never be greater than the troll's.

16. A trapezoid has bases of 7 inches and 3 inches. The area of the trapezoid is 20 square inches. What is the altitude of the trapezoid?

   a. 1 inch
   b. 2 inches
   c. 3 inches
   d. 4 inches
   e. 5 inches

17. In an isosceles trapezoid the two sides that are not parallel are congruent. An isosceles trapezoid has one angle whose measure is 48°. What are the measures of the other three angles?

    a. All angles would have measure of 48°.
    b. There would be one more 48° angle but the measures of the other two angles are impossible to determine.
    c. There would be one more 48° angle and two 90° angles.
    d. There would be one more 48° angle and two 42° angles.
    e. There would be one more 48° angle and two 132° angles.

18. A prism has bases that are octagons and lateral faces that are rectangles. How many vertices, faces and edges does the prism have?

    a. 16 vertices, 24 edges, and 10 faces.
    b. 16 vertices, 16 edges, and 16 faces.
    c. 16 vertices, 16 edges, and 10 faces.
    d. 24 vertices, 24 edges, and 10 faces.
    e. 24 vertices, 16 edges, and 10 faces.

19. What is the relationship between a square centimeter and a square meter?

    a. It takes 1000 square centimeters to equal 1 square meter.
    b. It takes 10,000 square centimeters to equal 1 square meter.
    c. It takes 100 square centimeters to equal 1 square meter.
    d. It takes 100,000 square centimeters to equal 1 square meter.
    e. It takes 1,000,000 square centimeters to equal 1 square meter.

20. A giant tortoise's maximum land speed is 0.17 miles per hour. At that speed, how many inches can the tortoise travel in 30 seconds? (5280 feet =1 mile. Round to the nearest inch.)

    a. 7 inches
    b. 90 inches
    c. 180 inches
    d. 18 inches
    e. 35 inches

21. Larry bought 2 Biggie Burgers, 1 Lemon Squirt, and 1 Sweet Pie. Before tax, Larry's total was $7.56. Liz got 1 Biggie Burger and 1 Lemon Squirt. Liz's total, before tax, was $3.98. Lynn bought 2 Sweet Pies and paid $1.98 before taxes. How much does a Biggie Burger cost?
    a. A Biggie Burger costs $1.60.
    b. A Biggie Burger costs $1.89.
    c. A Biggie Burger costs $1.99
    d. A Biggie Burger costs $2.49
    e. A Biggie Burger costs $2.59

22. In the figure below line m is parallel to line n and angle B is a right angle. $m\angle CAB = 68°$. What is the measure of the angle X?
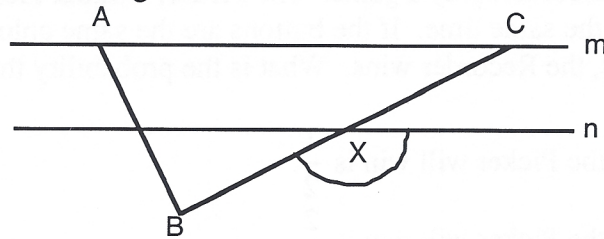
    a. 112°
    b. 102°
    c. 156°
    d. 158°
    e. 162°

23. On the grid shown below the lower left corner has coordinates (0, 0). The upper right corner has coordinates (7, 4). A rhombus is drawn on the grid with vertices at (1, 2), (4, 3), (7, 2), and (4, 1). If the distance from (0, 0) to (1, 0) is 1 centimeter and the distance from (0, 0) to (0, 1) is 1 centimeter, what is the perimeter of the rhombus?
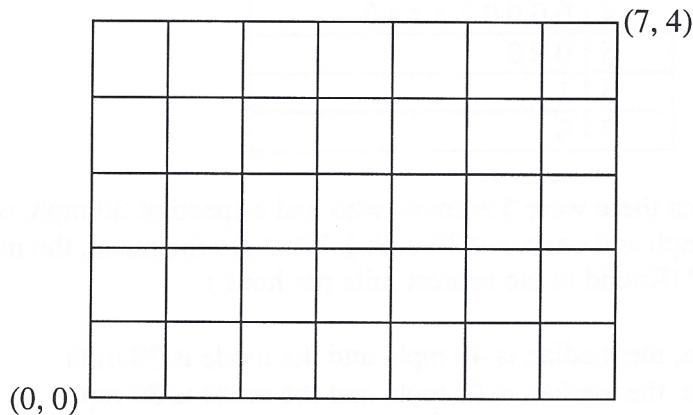
    a. 12 centimeters
    b. $4\sqrt{10}$ centimeters
    c. $\sqrt{40}$ centimeters
    d. 8 centimeters
    e. 10 square centimeters

24. The area of the base of a cylindrical oatmeal box is 50 square inches. Its height is 8 inches. The formula for finding the volume of a cylinder is $V = \pi r^2 h$, where r is the radius of the base and h is the height. What is the volume of the box?

    a. There is not enough information given to determine the volume.
    b. The volume is 400 cubic inches.
    c. The volume is 400π cubic inches.
    d. The volume is 200π cubic inches.
    e. The volume is 625π cubic inches.

25. A bag has 5 buttons that are the same size and shape; three are green and 2 are red. Two people, a Picker and a Recorder, play a game. The Picker, without looking, draws two buttons from the bag at the same time. If the buttons are the same color, the Picker wins. If they are different colors, the Recorder wins. What is the probability that the Picker will win?

   a. The probability the Picker will win is $\frac{1}{2}$.

   b. The probability the Picker will win is $\frac{2}{5}$.

   c. The probability the Picker will win is $\frac{3}{5}$.

   d. The probability the Picker will win is $\frac{7}{10}$.

   e. The probability the Picker will win is $\frac{3}{10}$.

26. The maximum speeds (in miles per hour) of 25 animals are given in the stem and leaf chart below.

3	0 0 0 0 0 2 5 5 5 5 9
4	0 0 0 0 2 3 4 4 7
5	0 0 0
6	1
7	0

(The first row means that there were 5 animals who had a speed of 30 mph, one could go 32 mph, four went 35 mph and one went 39 mph.) What are the mean, the median, and the mode of these data? (Round to the nearest mile per hour.)

   a. The mean is 44 mph, the median is 40 mph, and the mode is 30 mph.
   b. The mean is 41 mph, the median is 40 mph, and the mode is 30 mph.
   c. The mean is 40 mph, the median is 40 mph, and the mode is 30 mph.
   d. The mean is 40 mph, the median is 41 mph, and the mode is 30 mph.
   e. The mean is 40 mph, the median is 30 mph, and the mode is 30 mph.

27. Becky bought a horse on Monday for $200. She sold the horse on Tuesday for $300. On Thursday she bought the horse back again for $400. On Friday she sold the horse for $500. What was Becky's total gain or loss on all the horse deals?

   a. She gained $200.
   b. She lost $200.
   c. She broke even.
   d. She gained $100.
   e. She lost $100.

28. Consider the bar graph of the heights of the eighth grade members of the middle school basketball team. Which statement is false?

Heights of the Seven Eighth Grade Players on the Middle School Basketball Team
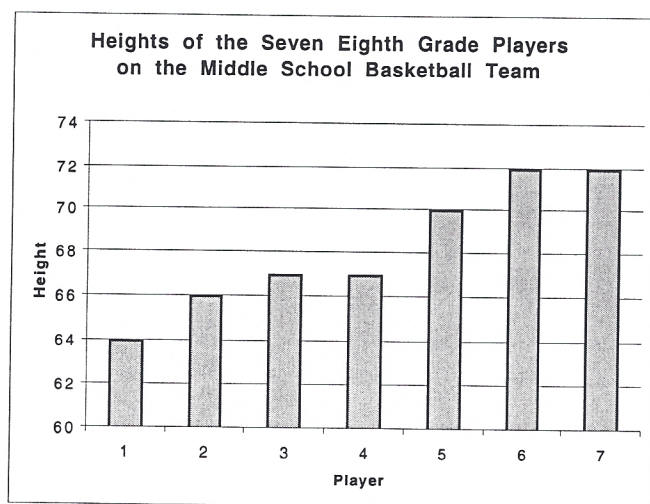
   a. The graph indicates that player 6 is twice as tall as player 2.
   b. The graph indicates that player 6 is 2 inches taller than player 5.
   c. The graph indicates that player 1 is 2 inches shorter than player 2.
   d. The graph indicates that players 3 and 4 are the same height.
   e. The graph indicates that player 5 is 70 inches tall.

29. A baby's weight at birth was 7 pounds and 5 ounces. When the baby went for her 3-month check-up, the baby's weight had increased by 110%. What was the baby's weight at the three-month check-up? (Round to the nearest ounce.)

   a. 15 pounds and 4 ounces
   b. 15 pounds and 6 ounces
   c. 129 ounces
   d. 15 pounds and 8 ounces
   e. 15 pounds and 10 ounces

30. One hundred people at the mall were asked to tell what kinds of soft drinks they had drunk in the last two days. Fifty six people said they had drunk Cola A. Sixty one people said they had drunk Cola B. What <u>must</u> be true about the group of people?

   a. Exactly 17 people drank both Cola A and Cola B.
   b. At least 17 people drank both Cola A and Cola B.
   c. Everyone drank Cola A or Cola B or both.
   d. There were no people who drank neither Cola A nor Cola B.
   e. Exactly five people drank both Cola A and Cola B.