1998 EIGHTH GRADE MATHEMATICS COMPETITION

AUSTIN PEAY STATE UNIVERSITY
CLARKSVILLE, TENNESSEE

MIDDLE TENNESSEE STATE UNIVERSITY
MURFREESBORO, TENNESSEE

UNIVERSITY OF TENNESSEE AT MARTIN
MARTIN, TENNESSEE

Eighth Grade Test
1998
Scoring Formula: 4R - W + 40

DIRECTIONS:

This is a test of your competence in middle school mathematics. For each problem there are 5 possible answers listed. You are to work the problems, determine the correct answer, and indicate your choice on the separate answer sheet provided.

SAMPLE:

1. If $x + 1 = 2$, then x equals
   a) 0
   b) 2
   c) -1
   d) 1
   e) none of the above

The correct answer is 1, which is d); so you would answer this problem by darkening the space on the answer sheet corresponding with this choice.

If you change your mind about your answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet and begin. When you have finished one page, go on to the next. The working time for the entire test is 60 minutes.

1. Here are some examples of quadrilaterals:

   What is the maximum number of points of intersection of a quadrilateral and a triangle if no side of either polygon is on the same line?

   a. 3 b. 4 c. 6 d. 8 e. 10

2. Three apples and two pears cost $.78. But two apples and three pears cost $.82. What is the total cost of one apple and one pear?

   a. $.16 b. $.23 c. $.32 d. $.41 e. $.80

3. In your class of 30 students, 20 students indicate that they exercise at least four times a week, and 15 indicate that they average 8 hours of sleep a night. What is the greatest possible number of students who do neither?

   a. 5 b. 10 c. 12 d. 15 e. 20

4. What is the area of the shaded region given that curve BDC is a semicircle.
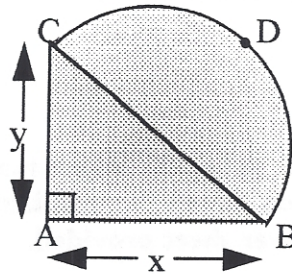
   a. $\dfrac{\pi\sqrt{x^2+y^2}+xy}{2}$

   b. $\dfrac{\pi\left(x^2+y^2\right)+4xy}{8}$

   c. $\dfrac{\pi\left(x^2+y^2\right)+8xy}{8}$

   d. $\dfrac{\pi\left(x^2+y^2\right)+2xy}{4}$

   e. $\dfrac{\pi\sqrt{x^2+y^2}+2xy}{4}$

1

5. An automobile with a speed of 50 miles per hour travels a distance of 100 miles from town A to town B. A fast-flying Texas beetle traveling 200 miles per hour leaves the windshield of the automobile as it departs from town A. The beetle flies to town B, back to the auto, back to town B, back to the auto, and so on until the automobile reaches town B. How far does the Texas beetle fly?

    a. 400 miles
    b. 200 miles
    c. 100 miles
    d. 50 miles
    e. not enough information is given

6. Fifteen pears, 25 apples, and 35 oranges are to be packed in 2 or more baskets. What is the least number of baskets needed if each basket is to have identical contents?

    a. 2   b. 3   c. 5   d. 7   e. 10

7. A bug is inside a 4 ft. x 4 ft. x 2 ft. box with closed lid. Specifically, the bug is on the ceiling of the box at a corner. On the floor of the box, in the extreme opposite corner, is a piece of candy. The bug cannot fly; it must crawl. What is the shortest distance to the candy for the bug?

    a. $4 + \sqrt{6}$ ft.   b. $2 + \sqrt{8}$ ft.   c. $4 + \sqrt{20}$ ft.   d. $2 + \sqrt{32}$ ft.   e. $2\sqrt{13}$ ft.

8. What is the 101st digit in the decimal representation of $\dfrac{26}{111}$?

    a. 1   b. 2   c. 3   d. 4   e. 5

9. If the edge of a cube is doubled, the new surface area is _____ % greater than the old surface area.

    a. 50   b. 100   c. 200   d. 300   e. 400

10. The coordinates of the center of a circle are $\left(\dfrac{1}{2}, 0\right)$. One endpoint of a diameter is $(5,-3)$. What are the coordinates of the other endpoint of that diameter?

    a. $(-4,-3)$ b. $(4,-3)$ c. $(-3,3)$ d. $(4,3)$ e. $(-4,3)$

11. Find the mean of the data illustrated in this histogram.
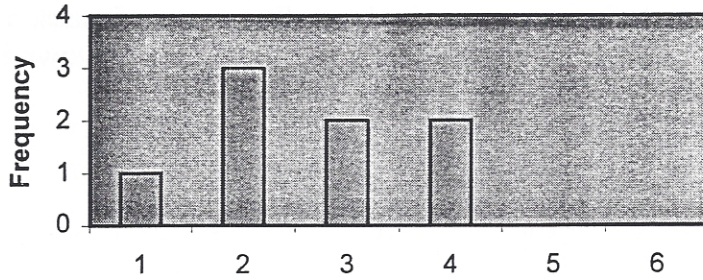
a. $\dfrac{5}{8}$ b. $\dfrac{5}{2}$ c. 2   d. $\dfrac{21}{4}$ e. $\dfrac{21}{8}$

12. In a small town, three students deliver all the newspapers. On a particular day, Abby delivered three times as many papers as Bob, and Connie delivered thirteen more than Abby. If the three students delivered a total of 496 papers, how many papers did Abby deliver?

a. 69   b. 100   c. 144   d. 207   e. 220

13. What is the circumference of a circle that has the same area as a square having a perimeter of 2π?

a. $2\sqrt{2}$ b. $\pi\sqrt{\pi}$ c. $\dfrac{\pi}{2}$ d. $\dfrac{\sqrt{2}}{\pi}$ e. 2

14. If $\dfrac{a}{b} = \dfrac{1}{4}$, where a is a positive integer, which of the following is a possible value of $\dfrac{a^2}{b}$?

I. $\dfrac{1}{4}$

II. $\dfrac{1}{2}$

III.   1

a. None   b. I only   c. I and II only   d. I and III only   e. I, II, and III

15. Consider the operation ● as defined by the following table.

●	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

For example, 3 ● 2 = 1. Find (2 ● 4) ● (1 ● 3).

a) 1   b) 2   c) 3   d) 4   e) 5

16. Suppose you are the buyer for a hat shop and decide to order hats in only one size for the coming season. To decide which size to order, you look at last year's sales figures which are itemized according to size. To determine what size to order you should find the

a. mode for the data.
b. median for the data.
c. mean for the data.
d. range for the data.
e. standard deviation for the data.

17. The Boy Scouts collected $2\frac{1}{2}$ times as many pounds of paper as the Girl Scouts and the soccer league combined. The soccer league collected 50 pounds. If all three combined collected 490 pounds of paper, how many pounds did the Girl Scouts collect?
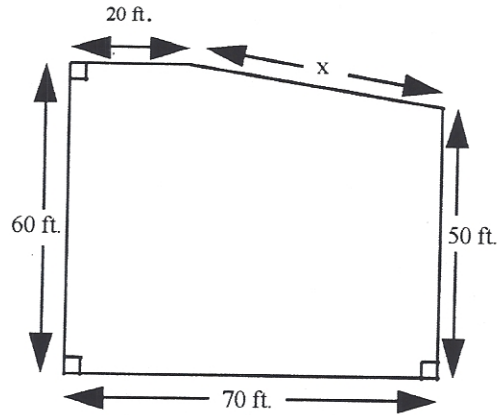
a. 11   b. 23.5   c. 41.5   d. 90   e. 147.5

4

18. The area of the shaded triangle is
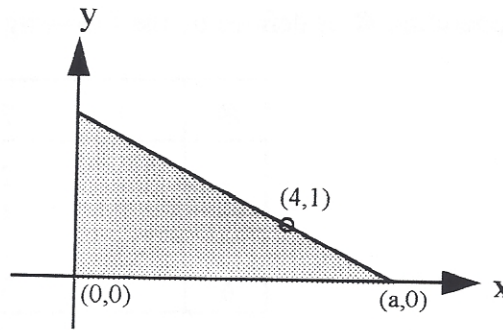
a. $\dfrac{a^2}{4}$ b. $\dfrac{a^2}{8}$ c. $\dfrac{a^2}{2a-4}$ d. $\dfrac{a^2}{2a-8}$ e. $\dfrac{a}{2a-8}$

19. The length of side x in the figure is

a. $\sqrt{2400}$ ft. b. $\sqrt{2500}$ ft. c. $\sqrt{2600}$ ft. d. $\sqrt{2800}$ ft. e. $\sqrt{2900}$ ft.

20. Which is the most likely of the events below?

a. Getting a 3 when an ordinary die is rolled.

b. Getting two heads when a penny and a dime are tossed.

c. Getting a sum of 3 when a red and green die are tossed.

d. Scoring 80% on a 5-question true-false test by guessing only.

e. Hitting your next free throw if your free-throw shooting percentage is 30%.

21. What is the probability of correctly guessing a 7-digit telephone number (without area code) if the first two digits cannot be ones or zeros?

   a. $\dfrac{1}{7^2 \cdot 9^5}$
   b. $\dfrac{1}{8^2 \cdot 9^5}$
   c. $\dfrac{1}{7^2 \cdot 10^5}$
   d. $\dfrac{1}{8^2 \cdot 10^5}$
   e. $\dfrac{1}{8^5 \cdot 10^2}$

22. Because of limited funds, two cities are to be chosen out of eight suitable ones for a study on heart disease. The eight cities are Los Angeles, New York, Miami, Boston, Seattle, Denver, Phoenix, and Detroit. If the selection of two cities is made at random, what is the probability that Miami and Phoenix will be the two cities chosen?

   a. $\dfrac{1}{56}$
   b. $\dfrac{1}{28}$
   c. $\dfrac{1}{14}$
   d. $\dfrac{1}{8}$
   e. $\dfrac{1}{4}$

23. Find the value of x in the figure below.

   3x − 10
   5x + 40
   4x − 50
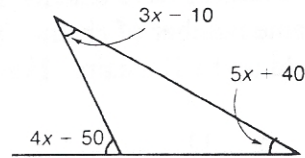
   a. 3
   b. 4
   c. 5
   d. 6
   e. 7

24. The graph below is a graph of the population of the United States (in millions of people) versus time. By what percentage did the population of the United States **increase** between the years 1850 and 1950?
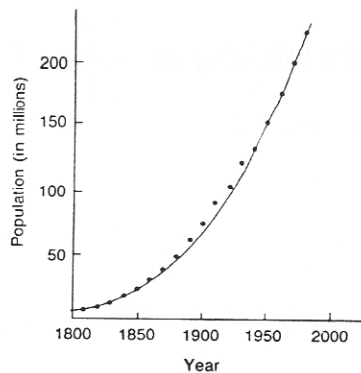
   a. 120%
   b. 200%
   c. 300%
   d. 400%
   e. 500%

6

25. What number will be printed as a result of running the following program?

```
Let A = 0
For K = 1 to 5
      Let A = A + K
Next K
Print A
End
```

    a. 0   b. 5   c. 10   d. 15   e. 20

26. When the base-ten number 628 is converted to base four, the result is

    a. 1310   b. 21304   c. 21310   d. 21311   e. 23111

27. The 135 county bus drivers are having a meeting. Jim's job is to place chairs in the auditorium. He wants to have exactly one chair for each driver; yet he also wants each row to contain the same number of chairs. The space available could hold up to 20 rows, and each row can hold up to 10 chairs. How many rows are there?

    a. 9   b. 11   c. 15   d. 17   e. not enough info given

28. John gave away all of his apples to his brothers. To Leo he gave half of his apples plus half an apple. To Ned he gave half of what he had left plus half an apple. John then had no apples left. How many apples did he have to begin with?

    a. 3   b. 6   c. 12   d. 15   e. 18

29. What is the smallest number in the following set $\left\{ x \mid x = \dfrac{a}{b} \right\}$ if a and b are natural numbers such that $a - b > 4$?
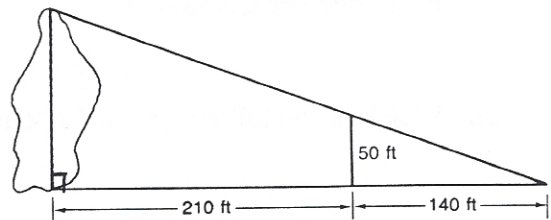
    a. -1

    b. 1

    c. $\dfrac{1}{2}$

    d. $\dfrac{1}{4}$

    e. the set does not contain a smallest element

30. John can mow the yard in $\frac{6}{7}$ of an hour. Joel can do the same job in $\frac{5}{6}$ of an hour. How long will it take them to complete the job if they work together?

a. $\frac{10}{21}$ hr.   b. $\frac{19}{42}$ hr.   c. $\frac{29}{42}$ hr.   d. $\frac{29}{71}$ hr.   e. $\frac{30}{71}$ hr.

31. What is the length of the lake pictured below?

50 ft   210 ft   140 ft

a. 75 ft.   b. 125 ft.   c. 130 ft.   d. 140 ft.   e. 150 ft

32. Before checking with the caterer, a cook cuts a cake into 35 equal pieces and an identical cake into 42 equal pieces. The caterer, however, insists that the cakes be cut exactly alike. What is the smallest number of pieces each cake can now have?

a. 77   b. 133   c. 147   d. 210   e. 735

33. The measure of each interior angle of a regular polygon is 162°. How many sides does the polygon have?

a. 10   b. 15   c. 20   d. 25   e. 30

34. Ms. Johnson wants to select a boy and a girl from Andrew, Thomas, Janet, Alice, Carl, and Mary to be hall monitors. In how many ways can this selection be made?

a. 9   b. 12   c. 15   d. 18   e. 20

8

35. If 0 degrees Centigrade is the temperature at which water freezes and 100 degrees Centigrade is the temperature at which water boils, which of the following is a reasonable temperature of a nice day to swim at the beach?

   a. 5 degrees Centigrade

   b. 10 degrees Centigrade

   c. 30 degrees Centigrade
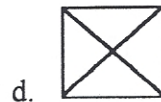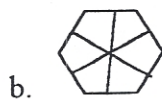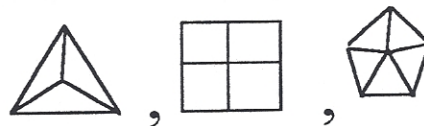
   d. 60 degrees Centigrade

   e. 80 degrees Centigrade

36. Which of the following is a reflection of about a horizontal line?
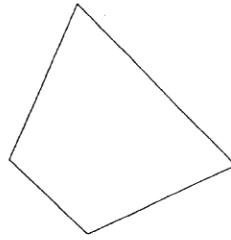
a.

b.

c.

d.

e.
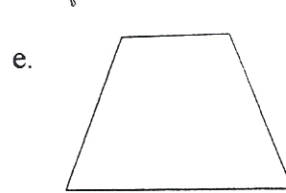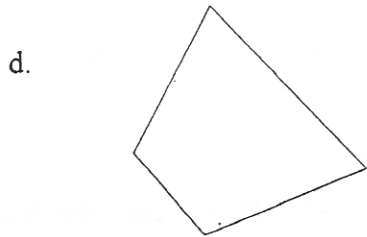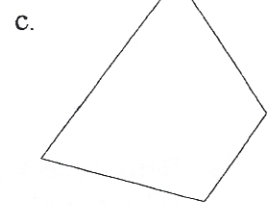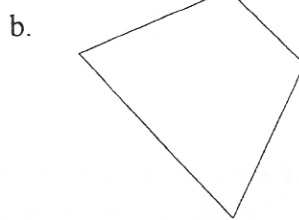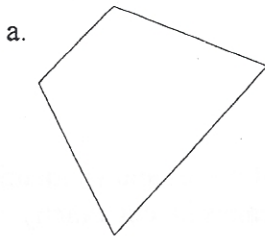
37. What is the next figure in this sequence?

a.

b.

c.

d.

e.

38. What is the next number in this sequence? −4, − 1, 4, 11, 20, 31

    a. 42   b. 43   c. 44   d. 45   e. 46

39. If a and b are integers such that $a > b$, which is the following **must be** true?

    a. $-2a > -2b$

    b. $a^2 > b^2$

    c. $\dfrac{a}{b} > 1$

    d. $|a| > |-b|$

    e. $a - b > 0$

40. Suppose $a \otimes b = a^2 b + 1$ for all real numbers a and b. Evaluate $2 \otimes (3 \otimes 1)$.

    a. 17   b. 33   c. 41   d. 170   e. 200