2022 Grade 8 Mathematics Contest

1. Arrange the following lengths in order from shortest to longest.

$$10^5 \text{ km}, \ 10^{11} \text{ cm}, \ 10^6 \text{ m}$$

A. 10^{11} cm, 10^6 m, 10^5 km

B. 10^6 m, 10^5 km, 10^{11} cm

C. 10^5 km, 10^6 m, 10^{11} cm

D. 10^5 km, 10^{11} cm, 10^6 m

E.   All three lengths are the same.

2. In three years, Sidda will be twice as old as her brother Jace was two years ago. If Sidda is currently 15 years old, how old is Jace?

A.   8 years old

B.   9 years old

C.   10 years old

D.   11 years old

E.   12 years old

3. The picture depicts the shape and lengths of sides of a polygonal region. The picture is not to scale. The area of the region is 68 square feet. To the nearest tenth of a foot, find the length of the side labeled m.

A.   15.4 ft

B.   8.0 ft

C.   7.4 ft

D.   6.0 ft

E.   5.3 ft
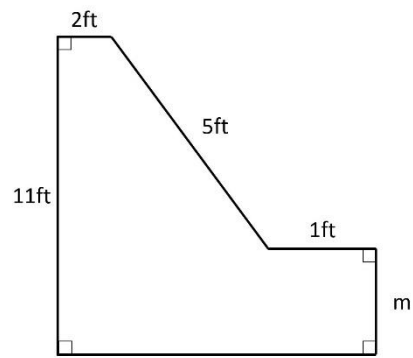
4. The angle ∠DEF is the image of ∠ABC under a reflection about the line m. Which of the following statements is necessarily true?

A.   The measure of ∠DEF depends on how far ∠ABC is from the line m.

B.   The measure of ∠DEF depends on the angle formed by line $\overleftrightarrow{AB}$ and line m.

C.   The measure of ∠DEF is twice the measure of ∠ABC.

D.   The angles ∠DEF and ∠ABC are congruent.

E.   Angle ∠DEF is a straight angle.

5. The Richter scale is a base 10 logarithmic scale used to describe the magnitude of earthquakes. An earthquake with magnitude m on the Richter scale has a measured amplitude 10 times greater than that of an earthquake with magnitude $m - 1$. The earthquake that created Reelfoot Lake in northwest Tennessee was estimated to be magnitude 8.1. An earthquake of magnitude 6.2 occurred on October 23, 2021, in Taiwan. How many times larger was the measured amplitude of the earthquake creating Reelfoot Lake than that of the earthquake in Taiwan?

A.   Approximately 79.4 times larger

B.   Approximately 0.0126 times larger

C.   Approximately 1,585,000 times larger

D.   Approximately 50.22 times larger

E.   Approximately 1.31 times larger

6. A contractor makes observations of a roof using a theodolite and tape measure. As indicated in the picture, the lowest point of the roof closest to the theodolite has a horizontal distance of 20 feet from the theodolite and is 5 feet above the theodolite. The peak of the roof has a horizontal distance of 40 feet from the theodolite and it is 11 feet above the theodolite. As indicated in the picture, a piece of siding in the shape of a right triangle is 8 feet long and tangent to the roof. The picture is not to scale. To the nearest tenth of a square foot, what is the area of the right-triangular piece of siding?

A.   9.6 square feet

B.   8.8 square feet

C.   16.0 square feet

D.   32.0 square feet

E.   106.7 square feet
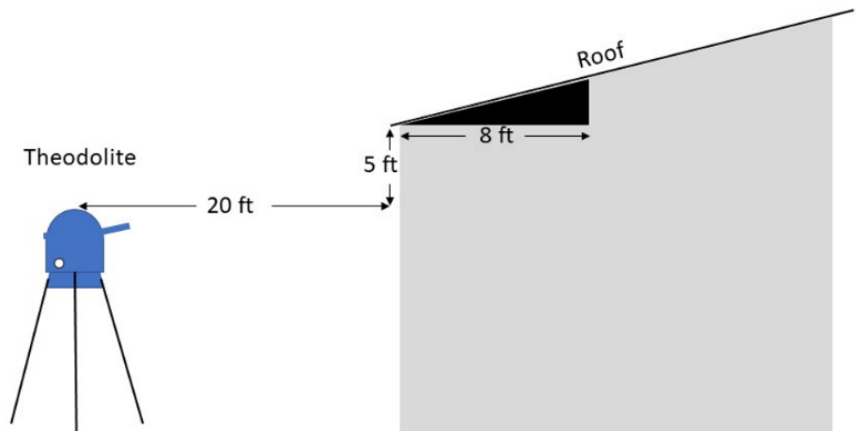
7. Which of the following equations describes y as a linear function of x?

A. $y = x^2 + 3$

B. $y = \sqrt{x^2 + 7}$

C. $y = \frac{-x}{4} + \sqrt[2]{7}$

D. $y = \frac{-1}{4x} + 7$

E. In none of the above is y a linear function of x.

8. Solve the following equation for x. $\qquad -2(x + 7) = \frac{-1}{4}x + 3$

A. $x = \frac{68}{7}$

B. $x = \frac{-7}{8}$

C. $x = \frac{-8}{5}$

D. $x = \frac{16}{7}$

E. $x = \frac{-68}{7}$

9. Emily runs a snow cone stand. Her ice machine can produce 6000 cm³ of ice every 30 minutes. A complete snow cone consists of ice filling a conical paper cup with a hemisphere of ice on top. Her snow cone cups are 15 centimeters deep and have a circular top with a diameter of 6 cm. How many complete snow cones can she make in an hour?

A. 60 snow cones
B. 61 snow cones
C. 8 snow cones
D. 30 snow cones
E. 25 snow cones

10. Ashley wants to build a sandbox for her little sister's birthday. Her parents allow her to build one in the corner of their back yard. After measuring the area her parents want her to use, she decides to make a triangular sandbox. The sides of the triangle will be 25 in, 32 in, and 40 in. Is the triangle she plans to build a right triangle? Why or why not?

A.   Yes, because the perimeter is 97 in.
B.   Yes, because $25^2 + 32^2 = 40^2$
C.   No, because $25^2 + 32^2 \neq 40^2$
D.   No, because the perimeter is 97 in.
E.   There is not enough information to answer this question.

11. The picture shows two points, A and B, in the xy-plane. Find the distance between the points.

A.   5
B.   12
C. $\sqrt{6}$
D. $\sqrt{20}$
E. $\sqrt{8}$
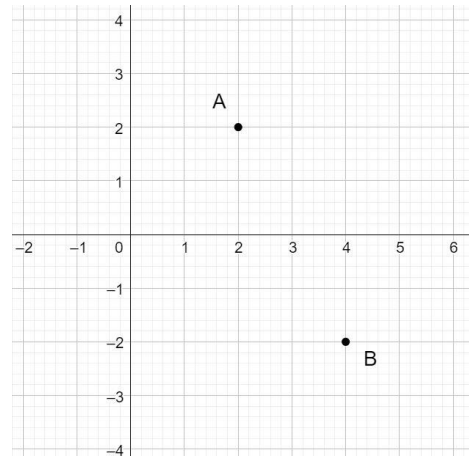

12. Points A, C and D are collinear with point C being between points A and D. Point B is not on the line containing points A, C, and D. $m(\angle BAC)$ denotes the measure of angle BAC. Which of the following statements must be true?
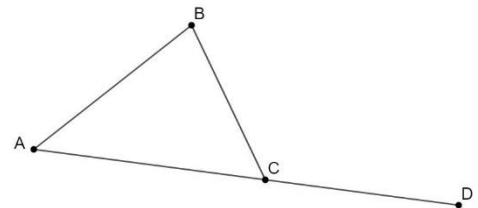
A. $m(\angle BAC) = m(\angle BCA)$

B. $m(\angle ABC) + m(\angle BCA) = m(\angle BCD)$

C. $m(\angle BCD)$ is less than 90°

D. $m(\angle CAB) + m(\angle ABC) = m(\angle BCD)$

E.   Each of the above statements must be true.

13. As indicated in the picture $m(\angle QTU) = 110°$, and $m(\angle SQT) = 65°$. Lines $\overleftrightarrow{ST}$ and $\overleftrightarrow{PR}$ are not necessarily parallel. What is the measure of $\angle QPR$?

A. 45°

B. 65°

C. 70°

D. 35°

E. There is not enough information to determine the measure of $\angle QPR$.
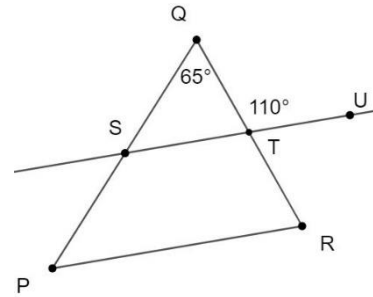
14. If possible, find a value of d so that the system of equations has infinitely many solutions.

$$\begin{cases} 3s + 4t = 21 \\ 5s + dt = 35 \end{cases}$$

A. 5/4

B. 4/5

C. 20/3

D. 0

E. No matter what number we choose for d, the system of equations will always have exactly one solution.

15. Which of the following lists contains only rational numbers?

A. $\frac{2}{3}, \sqrt{2}, \sqrt{7}, \pi$

B. $\frac{2}{3}, 1.4142, (\sqrt{7})^2, 3.1415$

C. $\frac{2}{3}, (\sqrt{2})^3, (\sqrt{7})^3, \pi^3$

D. All numbers in lists B and C are rational.

E. Each of these lists contains at least one number which is not rational.

16. A number cube is a cube with sides labeled with the numbers 1 through 6. A *fair* number cube is one for which each side is equally likely to show after the cube is rolled. Consider a game in which a player rolls two fair number cubes. The player wins with a 1 or doubles. That is, the player wins if the result is "Both cubes show the same number" or "At least one cube shows a 1." Notice that if both cubes show a 1, then that is a winning roll. What is the expected number of winning rolls if the game is played 360 times?

A.   180
B.   160
C.   170
D.   16
E.   300

17. Hannah is trying to raise money for her school's basketball team trip. She decides to do a fundraiser at her school's track field. The more laps she runs the more money she will make for the trip. Hannah created the graph below to record her progress. It shows the relationship between the number of laps (x) she runs and the money (y) she earns in dollars.

y

x
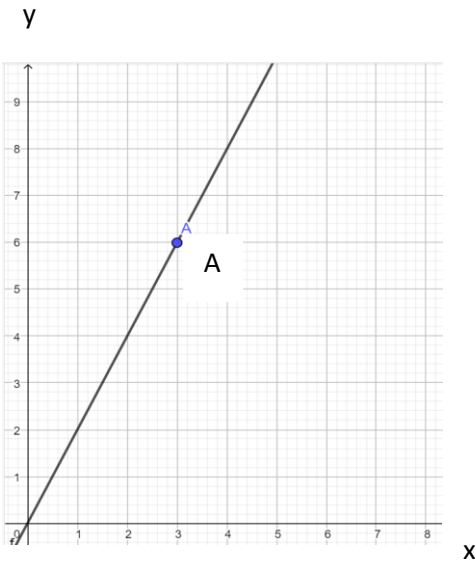
What does point A on the graph represent?

A.   Hannah raises 3 dollars for running 6 laps.

B.   On average Hannah earns 6 dollars for each lap she runs.

C.   On average Hannah earns 3 dollars for each lap she runs.

D.   Hannah raises 6 dollars for running 3 laps.

E.   The point represents how much money she raised overall.

18. Data for the midterm English and math grades of 10 students are provided in the table below. Based on the data, which of the following statements is true?

Student	English	Math
1	85	84
2	96	61
3	86	71
4	88	74
5	78	80
6	80	85
7	68	90
8	64	93
9	60	96
10	59	100

A. A student who makes a low English midterm grade will typically make the same Math midterm grade.

B. A student who makes a high English midterm grade will typically make a high Math midterm grade.

C. A student who makes a low English midterm grade will typically make a low Math midterm grade.

D. A student who makes a high English midterm grade will typically make a low Math midterm grade.

E. No information about the relationship between English midterm grades and Math midterm grades is available.

19. A box with a volume of 36 cubic inches has length of 6 inches and width of 3 inches. What is the distance from the top, left, front corner to the bottom, right, back corner?

A. 7 inches

B. 2 inches

C. 8 inches

D. 12 inches

E. 20 inches

20. The following data were collected by Company A about their employees. Each dot represents a single employee. Determine the average years of experience and the average salary (in thousands of dollars).


Years of Experience vs Salary

A. The average years of experience is 15 years, and the average salary is 35 thousand dollars.
B. The average years of experience is 10 years, and the average salary is 25 thousand dollars
C. The average years of experience is 20 years, and the average salary is 43 thousand dollars.
D. The average years of experience is 25 years, and the average salary is 51 thousand dollars.
E. The average years of experience is 5 years, and the average salary is 21 thousand dollars.

21. Mr. Mike has an ice cream business, and for 10 months he recorded the average temperature and the profit he made in that month, recording the data on the graph below. Which of the linear models presented in the choices represents the line of best fit for Mr. Mike's data?
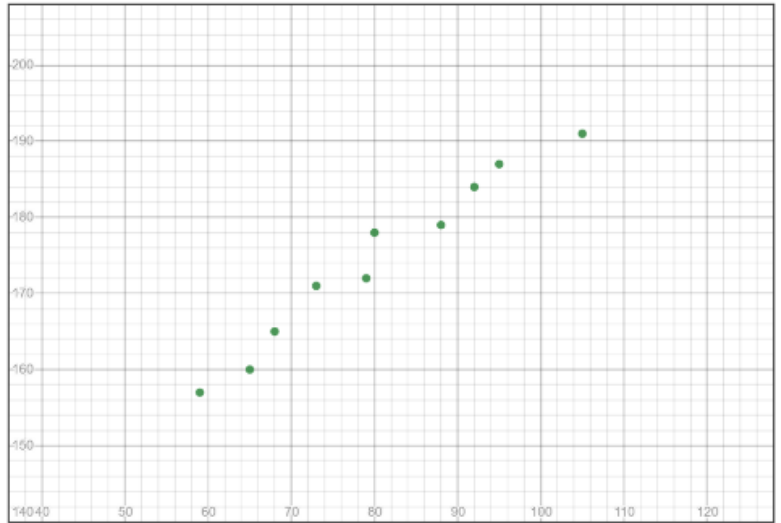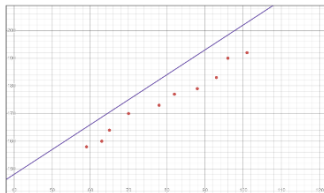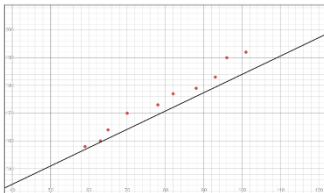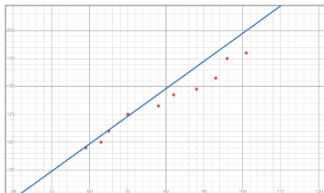

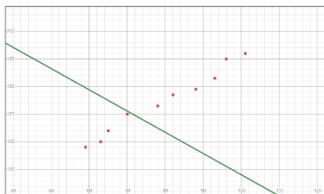*Mr. Mike's Data*

A.
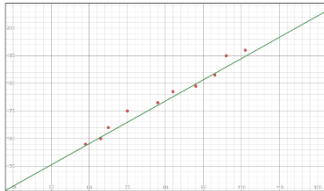
B.

C.

D.

E.

22. When cumulousene decomposes, two by-products are goloshene and umbrelline. Data was collected to analyze the relationship between the quantities of galoshene (G) and umbrelline (U) produced from decomposition of different cumulousene samples. Both quantities are measured in milligrams. The following linear model describes the relationship between the quantities.

$$G = 3.2 + 10.5U$$

In this linear model, what does the slope imply?

A.	If the amount of umbrelline in a sample is 0, then we predict that the amount of galoshene present in the by-products is 10.5 milligrams.

B.	If the amount of umbrelline in a sample is 0, then we predict that the amount of galoshene present in the by-products is 3.2 milligrams.

C.	If the amount of umbrelline increases by 1 milligram, then we assume that the amount of ' galoshene present in the by-products will increase by 10.5 milligrams.

D.	If the amount of galoshene increases by 1 milligram, then we assume that the amount of umbrelline present in the by-products will increase by 10.5 milligrams.

E.	If the amount of umbrelline increases by 10.5 milligram, then we assume that the amount of galoshene present in the by-products will increase by 3.2 milligrams.

23. Two nomads were resting at an oasis when a third one, Nasrudin, showed up. Nasrudin asked them for some food since he had run out of it during his journey. The first two nomads shared some bread with Nasrudin so they all had the same amount to eat. The first nomad had 3 pieces of bread in his pouch and the ratio of bread given to Nasrudin by him to the amount given to Nasrudin by the second nomad was 1:7. How many pieces of bread did the second nomad have in his pouch to begin with?

A.	1

B.	5

C.	7

D.	21

E.	None of the above are possible.

24. Five students were asked to approximate $2 \times 10^{20} + 4 \times 10^{10}$. Their approximations appear in the choices below. Which approximation is closest to the true value?

A. 2×10^{20}
B. 6×10^{30}
C. 8×10^{30}
D. 2.4×10^{20}
E. 4×10^{10}

25. The graph and table below represent two different linear functions. How do their rates of change compare?
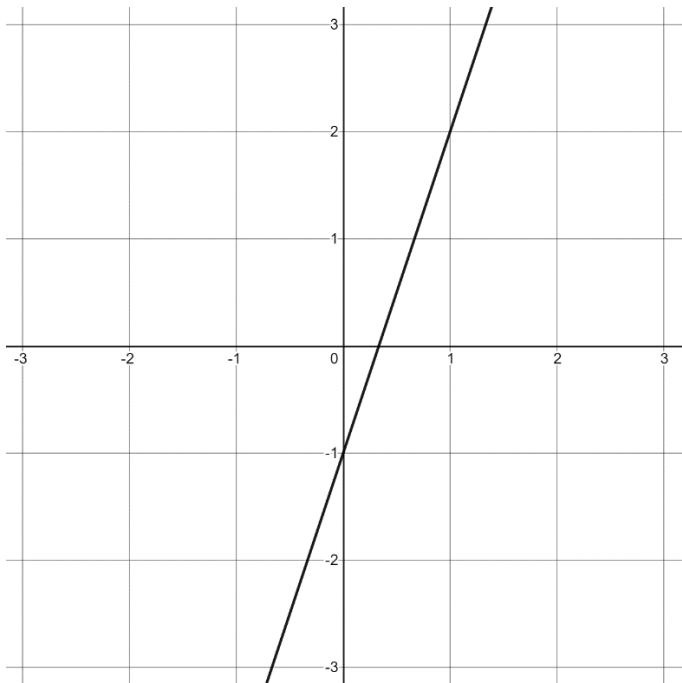
x	y
-2	-8
-1	-3
0	2
1	7
2	12

A.   The rate of change of the function on the right is the same as the rate of change of the function on the left.

B.   The rate of change of the function on the right is 2 ft/s more than the rate of change of the function on the left.

C.   The rate of change of the function on the right is 2ft/s less than the rate of change of the function on the left.

D.   The rate of change of the function on the right is 5 times the rate of change of the function on the left.

E.   The rate of change of the function on the right is -5 times the rate of change of the function on the left.

26. A cone is called awesome if its height is equal to its radius. Suppose Sasha has an awesome cone and a sphere and that the sphere has an opening in the top that she can pour water into the sphere. She does not know the radius of either, but she knows the two radii are equal. How many times can she completely fill the cone with water and then pour it into the sphere?

A. The answer depends on the radius of the awesome cone and sphere.

B. 2

C. 3

D. 4

E. 5

27. Suppose two rays emanate from a common point, making an obtuse angle. What effect does a dilation have on the angle between these rays?

A. It becomes even more obtuse.

B. It will be obtuse, but closer to a right angle than it started.

C. It becomes less obtuse, possibly even acute depending on the magnitude of the dilation.

D. There is not enough information to answer the question.

E. The angle is unchanged.

28. Millicent Millman, magical milliner, makes marvelous moleskin hats. A new customer, Geordie Goodman, a garden gnome, requested a conical hat with a volume, to the nearest ten-thousandth, of 1809.5574 cubic centimeters. At a glance, Millicent knew that the diameter of Geordie's head is no greater than 20cm. After measuring his head, Millicent made a perfectly fitting hat from a piece of moleskin in the shape of a circular sector as shown in the picture, creating a cone by putting the edges marked with dots together with no overlap and gluing the edges. If the area of the sector in the picture is, to the nearest ten-thousandth, 707.7444 square centimeters, what is the diameter, to the nearest centimeter, of Geordie Goodman's round head? Note that the "base" of the conical hat is the hole where Geordie's head goes. There is no moleskin there to contribute to the surface area. The picture is not necessarily to scale.

A. 16 cm

B. 8 cm

C. 12 cm

D. 16π cm

E. 8π cm
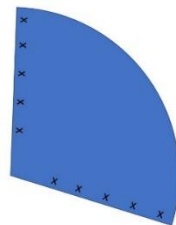
29. In the Dart Game, a player throws a dart at a board containing 15 regions numbered from 5 to 19. Each region is equally likely to be hit. A player wins by hitting a region with an even number in it, and loses by hitting a region with a prime number in it. If a player hits an odd composite number, he or she throws again. Monique will only play a game if she is more likely to win than lose. Will she play the Dart Game?

A.   Yes, because the probability of winning is 7/15.

B.   No, because the probability of winning is 7/15.

C.   Yes, because the probability of winning is 7/13.

D.   No, because the probability of winning is 7/13.

E.   No, because she is equally likely to win or lose.

30. Suppose x denotes the number 0.333... with the 3s repeating forever. Which of the following is an accurate description of 3x?

A.   3x is to the left of 1 on the number line, and there is no number between 3x and 1.

B.   3x is to the left of 1 on the number line, but there are numbers between 3x and 1.

C.   3x is to the right of 1 on the number line, and there is no number between 3x and 1.

D.   3x is to the right of 1 on the number line, but there are numbers between 3x and 1.

E.   3x = 1.