TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-FIRST ANNUAL MATHEMATICS CONTEST

2017

Statistics

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Scoring formula: 4 x (Number Right) – (Number Wrong) + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work

Statistics Test:

Omit questions # 11 and # 37.

Leave the answer choice **BLANK** to avoid it being counted against you.

1. In a stemplot, if 3.1416 is represented by 1|4, whichof the following could be represented by 6|3?

   (a) 3.5632 (b) 3.5636 (c) 3.6242 (d) 3.6262 (e) 3.6351

2. Suppose that we have a random variable X where the probability associated with the value $X = k$ is ${}_{24}C_k(.3)^k(.7)^{24-k}$ for $k = 0, 1, \ldots, 24$. What is the standard deviation of X?

   (a) 0.2245 (b) 0.504 (c) 2.245 (d) 2.789 (e) 5.04

3. If a non-vertical line (L) is drawn through the point $(\bar{x}, \bar{y})$, and it is *not* the least squares regression line (LSRL) for 11 points, which of the following statements is true?

   I. The sum of the vertical deviations of the 11 actual points from the LSRL will be zero.

   II. The sum of the squared vertical deviations of the 11 actual points from the line L wil be greater than the sum of the squared vertical deviations of the 11 actual points from the LSRL.

   III. The sum of the vertical deviations of the 11 actual points from line L will not be zero.

   (a) I only (b) I and III (c) I and II (d) II and III (e) I, II, and III

4. Suppose a set of integer-valued data has the following frequency distribution:

Class	Frequency
20-27	11
28-35	25
36-43	26
44-51	22
52-59	18

   Find the lowest and highest possible values for the mean of the data set, rounded to the nearest integer.

   (a) 28 & 25 (b) 30 & 27 (c) 36 & 42 (d) 36 & 43 (e) 37 & 44

5. Which of the following statements is/are true?

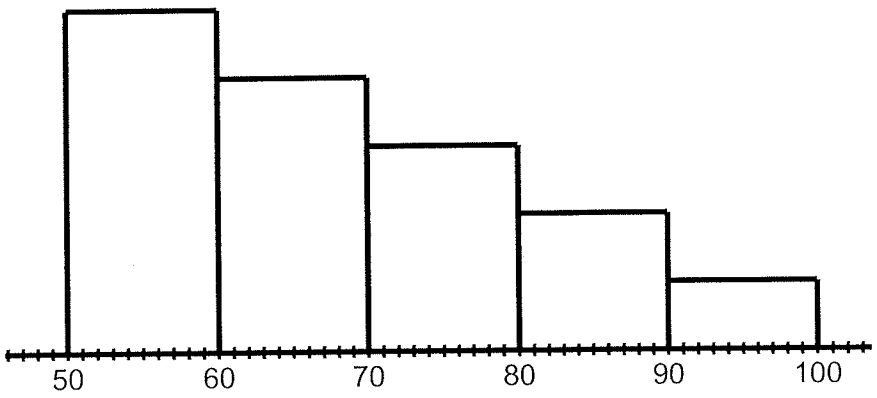
I. The standard deviation is the positive square root of the variance or it is zero.

II. The standard deviation is zero only when all of the values in the dataset are zero.

III. The range of a random sample cannot be larger than the range of the population from which it was taken.

(a) II only (b) I and III— (c) II and III (d) I and II (e) I, II, and III

6. Following is a frequency histograph of integer student test scores. There is one test score for each student. Which of the following statements must be true, if the heights are in the proportion of 5:4:3:2:1?



I. At least 6 students scored above 70.

II. At least one student scored above 80.

III. At least five students scored below 60.

(a) II only (b) I and II (c) I and III (d) II and III (e) I only

7. Consider the three points $(2, 11), (3, 17)$, and $(4, 29)$. Given any line, we can calculate the sum of the squares of the three vertical distances from these points to the line. What is the smallest possible value this sum can be?

(a) 0 (b) 6 (c) 9 (d) 29 (e) None of these

8. Which of the following is true about two different normal curves?

   I. If they have the same mean, the will have different standard deviations.

   II. If they have the same mean, and one has a standard deviation that is twice the other's, they will have exactly the same percent of values within one standard deviation of the mean.

   III. If they have the same standard deviation but different means, a value of 100 will have a smaller $z-$score for the smaller mean.

   (a) I only (b) II and III (c) I and III (d) I and II (e) I, II, and III

9. A large elementary school has 15 classrooms, with 24 children in each classroom. A sample of 30 children is chosen by the following procedure: Each of the 15 teachers selects 2 children from his/her classroom to be in the sample by numbering the students from 1 to 24 and then using a random digit table to select two different random numbers between 01 and 24. The 2 children with those numbers are in the sample.

   Did this procedure give a Simple Random Sample of 30 children from the school?

   (a) No, because not all possible groups of 30 children had the same chance of being chosen.

   (b) Yes, because each child had the same chance of being chosen.

   (c) No, because not all children had the same chance of being chosen.

   (d) Yes, because all possible groups of 30 children had the same chance of being chosen.

   (e) Yes, because the numbers were assigned randomly to the children.

10. Supose $(0.61, 0.79)$ is a 95% confidence interval for a population proportion. Which of the following is/are true statements?

    I. The population mean proportion P will be in this interval.

    II. A 90% confidence interval based on the same sample could be $(0.60, 0.80)$.

    III. The sample proportion $\hat{p}$ will be in this interval.

    (a) III only (b) I and II (c) I and II (d) II and III (e) I, II, and III

11. A simple random sample produces a 95% confidence interval for the corresponding population mean of 15 ± 3. Which of the following statements must be true?

(a) 95% of the population measurements fall between 12 and 18.

(b) 95% of the sample measurements fall between 12 and 18.

(c) If 100 samples were taken, we would expect 95 of the sample means to fall between 12 and 18.

(d) If $\mu = 19$, the probability that this interval contains μ is 0.95

(e) If $\mu = 19$, this $\bar{x}$ of 15 would be unlikely to occur.

12. If the slope of a least squares regression line is $-.00002$, and all of the data points lie on this line, what is the sample linear correlation r?

(a) -1 (b) close to -1 (c) close to 0 (d) close to +1 (e) +1

13. When rolling one balanced standard six-sided die we are interested in the number of rolls it take to get a 2. The probability that we will require at least n rolls to get the first 2 is

(a) $\left(\frac{1}{6}\right)^n$ (b) $\left(\frac{1}{6}\right)^{n-1}$ (c) $\left(\frac{5}{6}\right)^n$ (d) $\left(\frac{5}{6}\right)^{n-1}$ (e) $\left(\frac{5}{6}\right)^{n+1}$

14. A telephone executive instructs an associate to contact 52 customers using their service to obtain their opinions about the quality of service. The associate notes the number of customers whose names begin with A and uses a random number table to pick two of those names. She then proceeds to use the same procedure for each letter of the alphabet and combines the $2 \times 26 = 52$ results into a group to be contacted. Which of the following statements is/are true?

I. Each customer has an equal probability of being included in the sample.

II. Her procedure results in a Simple Random Sample.

III. Her procedure makes use of chance.

(a) I only (b) I and III (c) I and II (d) III only (e) II and III

15. Is the mean height of all adult American males between the ages of 18 and 21 now over 6 feet? If the population of all adult American males between the ages of 18 and 21 has a mean height of μ feet and a standard deviation of σ feet, to answers this question one would test which of the following null and alternative hypotheses?

(a) $H_0 : \mu \leq 6; H_a : \mu > 6$

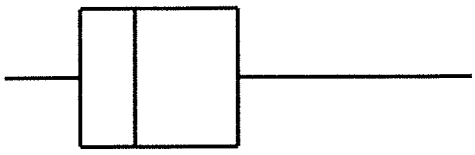
(b) $H_0 : \mu \geq 6; H_a : \mu < 6$

(c) $H_0 : \mu = 6; H_a : \mu \neq 6$

(d) $H_0 : \mu > 6; H_a : \mu \leq 6$

(e) $H_0 : \mu < 6; H_a : \mu \geq 6$

16. Suppose that some data have the following modified boxplot.



Consider the following three statements about the data:

I. The are more values above the median than below it.

II. There are no outliers.

III. The distribution is left skewed.

Which of the above statements is/are true?

(a) I only (b) II only (c) III only (d) I and II (e) II and III

17. Consider a normal distribution with mean μ and standard deviation σ. The point on the density curve where $x = \mu - \sigma$ is

(a) a minimum point

(b) a point of inflection

(c) concave up

(d) below the $x-$axis

(e) concave down

18. 200 headache sufferers were randomly divided into two groups of 100. The first group received a new drug, and the second group received an inactive sugar pill (a placebo). Of the 100 in the first group, 64 found headache relief from their treatment. Of the 100 in the second group, 36 found headache relief from the treatment. Find a 98% confidence interval for the difference in proportion of headache relief for the two treatments.

(a) $(.17, .39)$ (b) $(.15, .41)$ (c) $(.14, .42)$ (d) $(.12, .44)$ (e) $(.11, .45)$

19. If X and Y are random variables, then whcih of the following is/are true?

I. $\mu_{X+Y} = \mu_X + \mu_Y$ onlyif X and Y are independent.

II. $\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$ only if X and Y are independent.

III. $\sigma^2_{X-Y} = \sigma^2_X - \sigma^2_Y$ only if X and Y are independent.

(a) II only (b) I and II (c) I and III (d) II and III (e) I, II, and III

20. Which of the following would be the best description of the standard deviation of a sample?

(a) the average of the squared deviations from the mean

(b) an approximate average of the squared deviations from the mean

(c) the average deviation from the mean

(d) an approximate average distance from the mean

(e) an approximate average deviation from the mean

21. A binomial distribution has $n = 30$ and $p = 0.8$. Which of the following is/are true?

I. The distribution is skewed right.

II. The mean of the distribution is 24.

III. The standard deviation of the distribution is 4.8.

(a) II only (b) I and II (c) 1 and III (d) II and III (e) I, II, and III

22. Suppose that a data set of 25 elements has $\bar{x} = 22$ and variance $= 36$. If 6 is added to each element in the data set, and then each element is doubled, what are the new mean and the new standard deviation?

(a) 50, 24 (b) 50, 144 (c) 56, 12 (d) 56, 24 (e) 56, 144

Questions 23 and 24 refer to the following two-way table of 400 students' year in school and their opinion of the food service by ranking in five categories: Excellent, Above Average, Average, Below Average, and Poor.

	Opinion				
Year in School	Excellent	Above Average	Average	Below Average	Poor
Freshman	29	42	16	11	2
Sophomore	26	31	20	17	6
Junior	20	32	23	17	8
Senior	18	30	20	23	9

23. What percent of seniors rank the food service as poor?

(a) 2.25 (b) 6.25 (c) 9 (d) 25 (e) 36

24. What percent of those who rank the food service as above average or excellent are sophomores?

(a) 11.4 (b) 13.6 (c) 23 (d) 25 (e) 28

25. When we standardize the values of a normal variable, the distribution of standardized values has mean = 0 and standard deviation = 1. Suppose we measure two variables X and Y on each of several subjects. We standardize both variables and then compute the least squares regression line, finding its slope to be -0.55. We may conclude that

    (a) the linear correlation coefficient will be -0.55.

    (b) the intercept wil be -0.55.

    (c) the intercept will be -1.0.

    (d) the linear correlation coefficient will be -1.0.

    (e) the linear correlation coefficient will be 0.

26. Assume the ACT math section has a mean of 18 and a standard deviation of six points. Furthermore, assume that the SAT math section has a mean of 500 and a standard deviation of 100 points. John scored 680 and David scored 690 on the math SAT and Sam scored 29 on the math ACT. Which ordering of the students shown below indicates their relative standing on the standardized tests? The ordering should respect the lowest (left) to the highest (right) relative test score.

    (a) John, David, Sam

    (b) David, Sam, John

    (c) Sam, John, David

    (d) Sam, David, John

    (e) John, Sam, David

27. A normal distribution has a standard deviation of 15. What is the mean if 30% of the observations are greater than 60?

    (a) 52.13 (b) 56.20 (c) 60.00 (d) 63.80 (e) 67.86

28. A high school statistics class wants to conduct a survey to determine what percentage of students in the school would be willing to pay a fee for participating in after-school activities. 20 students are randomly selected from each of the freshman, sophomore, junior, and senior classes to complete the survey. Assume that each of the four classes has 100 students. This plan is an example of which type of sampling?

    (a) Systematic

    (b) Cluster

    (c) Stratified

    (d) Simple Random

    (e) Convenience

29. In a 1927 Western Electric Company study on the effect of lighting on worker productivity, productivity increased with each increase in lighting, but then also increased with every decrease in lighting. If it is assumed that the workers knew a study was in progress, this is an example of

(a) the effect of a treatment unit.

(b) voluntary response bias.

(c) the control group effect.

(d) sampling error.

(e) the placebo effect.

30. According to Chebyshev's inequality at least what proportion of the observations of any distribution will lie with two standard deviations of the mean?

(a) 0.75 (b) 0.889 (c) 0.90 (d) 0.95 (e) 0.997

31. In this question, A^c represents the complement of the event A, and $A \cup B$ represents the union of the events A and B. Suppose that $P(A^c) = 0.3$, $P(B) = 0.6$, and $P(A \cup B) = 0.88$. Consider the following four statements:

I. $P(B^c) = 0.4$

II. $P(A \cap B) = 0.18$

III. Events A and B are independent

IV. $P(A \mid B) = 0.3$

Which of the above statements are true?

(a) I only (b) I, II, and III (c) I and III (d) I, II, and IV (e) I, II, III, and IV

32. If the power of a test is 0.75, then which of the following is true?

(a) 0.75 is the probability of rejecting a false H_0

(b) 0.75 is the probability of failing to reject a false H_0

(c) 0.75 is the probability of rejecting a true H_0

(d) 0.25 is the probability of rejecting a false H_0

(e) 0.25 is the probability of rejecting a true H_0

33. A researcher has conducted a survey using a simple random sample of 50 registered voters to create a confidence interval to estimate the proportion of registered voters favoring the election of a certain candidate for mayor. Assume that the sample proportion does not change. Which of the following best describes the anticipated effect on the width of the confidence interval if the researcher were to survey a random sample of 200, rather than 50, registered voters (assuming the confidence lebel stays the same)?

(a) The width of the new interval would be about one-fourth the width of the original interval

(b) The width of the new interval would be about four times the width of the original interval

(c) The width of the new interval would be about the same width of the original interval

(d) The width of the new interval would be about twice the width of the original interval

(e) The width of the new interval would be about one-half the width of the original interval

34. A college alumni office states that 20% of the graduates eventually become lawyers, 25% doctors, and 35% corporate executives. The remaining 20% are spread out among a variety of professions. A new survey taken of 625 graduates turned up 110 lawyers, 140 doctors, 250 corporate executives, and 125 others. Is there sufficient evidence at the 1% significance level that the percentages as quoted have changed? (Use the partial χ^2 table given below)

(a) No, with $\chi^2 = 2.82$ there is not sufficient evidence.

(b) No, with $\chi^2 = 5.64$ there is not sufficient evidence.

(c) Yes, with $\chi^2 = 7.954$ there is sufficient evidence.

(d) Yes, with $\chi^2 = 2.82$ there is sufficient evidence.

(e) No, with $\chi^2 = 7.954$ there is not sufficient evidence.

Critical Values of χ^2

	Tail Probability p				
df	0.25	0.10	0.05	0.01	0.005
3	4.11	6.25	7.81	11.34	12.84
4	5.39	7.78	9.49	13.28	14.86
5	6.63	9.24	11.07	15.09	16.75

35. A shipment of 20 television sets contains 3 defective television sets. If a random sample (without replacement) of 4 television sets is taken from the shipment, what is the probability that the sample will contain at least one defective television set?

(a) 0.596 (b) 0.509 (c) 0.491 (d) 0.478 (e) 0.404

36. Which of the following is an **INCORRECT** statement?

(a) The sampling distribution of $\bar{x}$ has a mean equal to the population mean even if the population is not normally distributed.

(b) The sampling distribution of $\bar{x}$ has a standard deviation equal to $\frac{\sigma}{\sqrt{n}}$ even if the population is not normally distributed.

(c) The sampling distribution of $\bar{x}$ is normal if the population has a normal distribution.

(d) When n is large, the sampling distribution of $\bar{x}$ is approximately normal even if the population is not normally distributed.

(e) The larger the valueof the sample size n, the closer the standard deviation of teh sampling distribution of $\bar{x}$ is to the standard deviation of the population.

37. To assess the accuracy of a laboratory scale, a standard weight that is known to weigh one gram is repeatedly weighed a total of n times, and the mean $\bar{x}$ of the n weights is computed. Suppose the scale readings are normally distributed with unknown mean μ and standard deviation $\sigma = 0.01$ grams. How large should n be so that a 95% confidence interval for μ has a margin of error of ± 0.001?

(a) 19 (b) 20 (c) 284 (d) 285 (e) 38,416

38. A particular model of a Chevrolet car averages 23.5 mpg with a standard deviation of 0.8 mpg. A similar model of Ford car averages 24.2 mpg with a standard deviation of 1.1 mpg. If the mileage of both cards can be modeled with a normal distribution, what is the probability that a randomly selected Chevrolet will achieve a greater gas mileage than a randomly selected Ford (assuming independence)?

(a) 0.3050 (b) 0.5147 (c) 0.2580 (d) 0.4131 (e) 0.1950

39. Which of the following is **NOT** true regarding the variance of a sampling distribution of a sample proportion?

(a) It is always less than 1

(b) It is never greater than p

(c) It is always positive

(d) It can be 0.5

(e) For a given sample size, the maximum variance in the sampling distribution occurs when the population proportion (p) is exactly 0.5

40. If a χ^2 Test for Independence were run on the observed counts in the following table, what conclusion would be reached?

1.6	6.4
4.8	19.2

(a) Rows and Columns are independent

(b) Rows and Columns are not independent

(c) Rows and Columns are not related

(d) both (a) and (c)

(e) A χ^2 test is inappropriate

Tennessee Mathematics Teachers' Association
High School Mathematics Contest
Statistics Tables

Notation:

$P(A)$ represents the probability of event A.

The letter z always represents a quantity having a standard normal (*i.e.*, Gaussian) distribution.

Some possibly useful formulas:

$$\hat{p} \pm z^* \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$\bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

$$\bar{x}_1 - \bar{x}_2 \pm t^* \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

Only two statistical tables are needed and provided for this contest exam: the "standard" normal table of probabilities and the t-table of critical values. The form of the standard normal table provided has probabilities of the form $P(0 < z < c)$, where c is a constant ranging from 0 to 3.49. *Each problem on this contest examination has an ordinary solution not requiring any other statistical tables.*

STANDARD NORMAL PROBABILITY DISTRIBUTION

Tabulated values are P($0 < z < c$), rounded to four places

c	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998

CRITICAL VALUES OF "STUDENT'S T" DISTRIBUTION

Critical values t_p satisfy $p = P(t \geq t_p)$

d.f.	$t_{.250}$	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$t_{.0025}$	$t_{.001}$	$t_{.0005}$
1	1.000	3.078	6.314	12.706	31.821	63.657	127.32	318.31	636.62
2	0.816	1.886	2.920	4.303	6.965	9.925	14.089	22.327	31.599
3	0.765	1.638	2.353	3.182	4.541	5.841	7.453	10.215	12.924
4	0.741	1.533	2.132	2.776	3.747	4.604	5.598	7.173	8.610
5	0.727	1.476	2.015	2.571	3.365	4.032	4.773	5.893	6.869
6	0.718	1.440	1.943	2.447	3.143	3.707	4.317	5.208	5.959
7	0.711	1.415	1.895	2.365	2.998	3.499	4.029	4.785	5.408
8	0.706	1.397	1.860	2.306	2.896	3.355	3.833	4.501	5.041
9	0.703	1.383	1.833	2.262	2.821	3.250	3.690	4.297	4.781
10	0.700	1.372	1.812	2.228	2.764	3.169	3.581	4.144	4.587
11	0.697	1.363	1.796	2.201	2.718	3.106	3.497	4.025	4.437
12	0.695	1.356	1.782	2.179	2.681	3.055	3.428	3.930	4.318
13	0.694	1.350	1.771	2.160	2.650	3.012	3.372	3.852	4.221
14	0.692	1.345	1.761	2.145	2.624	2.977	3.326	3.787	4.140
15	0.691	1.341	1.753	2.131	2.602	2.947	3.286	3.733	4.073
16	0.690	1.337	1.746	2.120	2.583	2.921	3.252	3.686	4.015
17	0.689	1.333	1.740	2.110	2.567	2.898	3.222	3.646	3.965
18	0.688	1.330	1.734	2.101	2.552	2.878	3.197	3.610	3.922
19	0.688	1.328	1.729	2.093	2.539	2.861	3.174	3.579	3.883
20	0.687	1.325	1.725	2.086	2.528	2.845	3.153	3.552	3.850
21	0.686	1.323	1.721	2.080	2.518	2.831	3.135	3.527	3.819
22	0.686	1.321	1.717	2.074	2.508	2.819	3.119	3.505	3.792
23	0.685	1.319	1.714	2.069	2.500	2.807	3.104	3.485	3.768
24	0.685	1.318	1.711	2.064	2.492	2.797	3.091	3.467	3.745
25	0.684	1.316	1.708	2.060	2.485	2.787	3.078	3.450	3.725
26	0.684	1.315	1.706	2.056	2.479	2.779	3.067	3.435	3.707
27	0.684	1.314	1.703	2.052	2.473	2.771	3.057	3.421	3.690
28	0.683	1.313	1.701	2.048	2.467	2.763	3.047	3.408	3.674
29	0.683	1.311	1.699	2.045	2.462	2.756	3.038	3.396	3.659
30	0.683	1.310	1.697	2.042	2.457	2.750	3.030	3.385	3.646
40	0.681	1.303	1.684	2.021	2.423	2.704	2.971	3.307	3.551
60	0.679	1.296	1.671	2.000	2.390	2.660	2.915	3.232	3.460
∞	0.674	1.282	1.645	1.960	2.326	2.576	2.807	3.090	3.291