TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-FIRST ANNUAL MATHEMATICS CONTEST

2017

Precalculus

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Scoring formula: 4 x (Number Right) – (Number Wrong) + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work

1. Perform the indicated operation and simplify $\dfrac{i^{26}-i}{i-1}$.

A. $i+1$ B. $-i$ C. i D. $26i+1$ E. $26-i$

2. When is $\sqrt{12x^4-36x^2y^2+27y^4}=\sqrt{3}(2x^2-3y^2)$?

A. Never
B. Always
C. For positive real numbers only
D. Only if $2x^2 \geq 3y^2$
E. Only if $2x^2 \leq 3y^2$

3. Perform the indicated operation:
$\left(e^y+1\right)\left(e^y-1\right)\left(e^{2y}+1\right)\left(e^{4y}+1\right)\left(e^{8y}+1\right)$

A. $e^{8y}-1$
B. $e^{8y}+1$
C. $e^{16}-1$
D. $e^{12y}-1$
E. $e^{16y}-1$

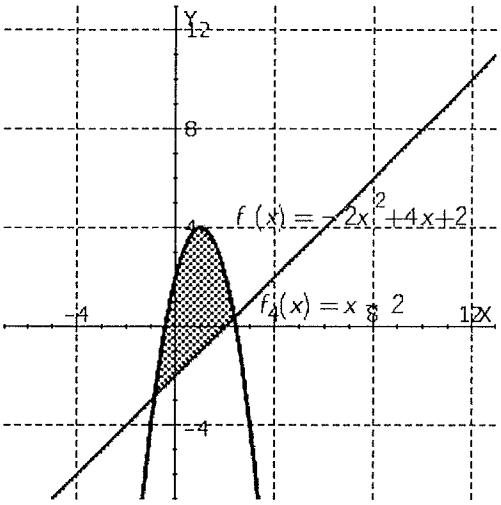
4. Consider $9(x-2)^2-16(y-3)^2=144$. Which of the following equations represents one of the two asymptotes for the graph of this conic?

A. $y-3=\frac{4}{3}(x-2)$
B. $y=\frac{-3}{4}x$
C. $y=\frac{-3}{4}(x-2)+3$
D. $y=\frac{3}{4}(x-2)-3$
E. None of the above

5. Given the graph of the equation $y^2 = 8x + 16$, find the length of its latus rectum.

A. 8
B. 4
C. 2
D. -8
E. None of the above.

6. **Find** the maximum vertical distance d between the parabola $f_1(x) = -2x^2 + 4x + 2$ and the line $f_2(x) = x - 2$ for the shaded region.

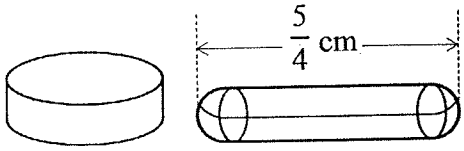


Select the correct answer.
A. $d = 8.085$
B. $d = 4.135$
C. $d = 5.125$
D. $d = 7.105$
E. none of these

7. Find the solution set to the equation $e^x(e^x - 4) = -3$.

A. {1}
B. {ln3}
C. $\{e^3, e\}$
D. {0, ln3}
E. {1, ln3}

8. An aspirin tablet is in the shape of a right circular cylinder. The manufacturer also wishes to market the aspirin in capsule form. The capsule is to be $\frac{5}{4}$ centimeters long, in the shape of a right circular cylinder with hemispheres attached at both ends (see the figure below). If r denotes the radius of a hemisphere, find a formula for the volume of the capsule.



A. $V = \pi r^2\left(\frac{5}{4} - r\right)$

B. $V = \pi r^2\left(\frac{5}{4} - \frac{2}{3}r\right)$

C. $V = \pi r^2\left(\frac{5}{4} + \frac{4}{3}r\right)$

D. $V = \pi r^3\left(\frac{2}{3}r - \frac{5}{4}\right)$

E. $V = \pi r^2\left(\frac{5}{4} + \frac{2}{3}r\right)$

9. Give a recursive definition of the sequence whose n^{th} term is $\frac{n!}{4^n}$.

A. $a_1 = 1;\ a_{n+1} = \frac{(n+1)}{4}a_n$

B. $a_1 = \frac{1}{4};\ a_{n+1} = \frac{(n+1)}{4}a_n$

C. $a_1 = 1;\ a_{n+1} = \frac{n!}{4^n}a_n$

D. $a_1 = \frac{1}{4};\ a_{n+1} = \frac{n}{4}a_n$

E. None of the above.

10. Suppose that f is a function with the following properties:
    (i) f is an even function.
    (ii) f is increasing over the interval (2, 8).
    (iii) $4 \le f(x) \le 5$ when $1 \le x \le 6$.

    Indicate which of the following statements **MUST** be true.

A. f is increasing over the interval $(-8, -2)$.

B. f is decreasing on the interval $(-8, -2)$.

C. $-5 \le f(x) \le -4$ when $-6 \le x \le -1$.

D. $g(x) = -f(x)$ is an odd function.

E. None of the above.

11. Find conditions on *w, x, y,* and *z* such that *AB* = *BA* for the following matrices.

$$A = \begin{bmatrix} w & x \\ y & z \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}.$$

A. True for all real values of w, x, y and z.

B. True only if $x = -y$ and $w = z$.

C. True only if $x = y$ and $w = -z$.

D. True only if $x = -y$ and $w = -z$.

E. *AB* never equals *BA*.

12. Find an expression for the determinant of the matrix $A = \begin{bmatrix} \lambda & 2 & 0 \\ 0 & \lambda+1 & 2 \\ 0 & 1 & \lambda \end{bmatrix}$.

A. $\lambda(\lambda+1)\lambda$

B. $\lambda[(\lambda+1)\lambda+2]$

C. $\lambda(\lambda+1)\lambda-2$

D. $\lambda^3+\lambda^2+2\lambda$

E. $\lambda^3+\lambda^2-2\lambda$

13. Four couples have purchased tickets to the school play. Their seats are next to one another in a single row. If each couple sits side by side, how many seating arrangements are possible?

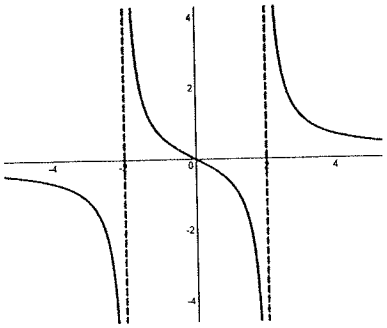
A. 384
B. 48
C. 32
D. 256
E. 40,320

14. Find a if $\log_2\left(\log_3\left(\log_4(a)\right)\right)=0$

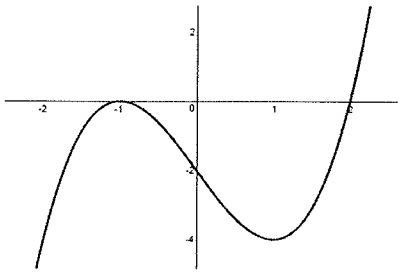
A. 64
B. e^{12}
C. 16
D. 12
E. 24

15. Let f be the function represented by the graph below. Choose the best answer below.

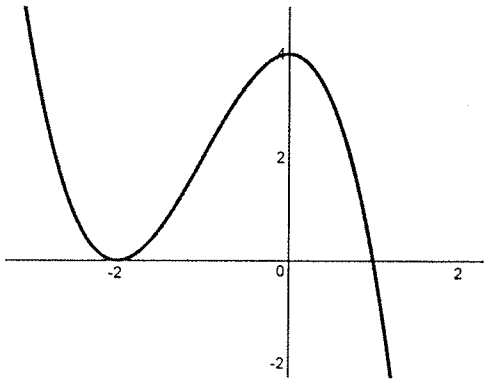
A. f is an odd function.
B. f is an even function.
C. f is neither even or odd.
D. f is both even and odd.
E. None of these are true.



16. Let g be the function represented by the graph below



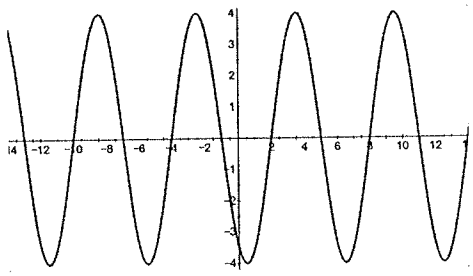
Find an expression for the function below in terms of g.



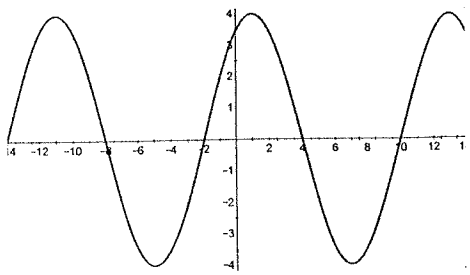
A. $y = -g(x-1)$
B. $y = g(1-x)$
C. $y = g(x+1)$
D. $y = -g(x+1)$
E. $y = g(x+1)+4$

17. Which of the following graphs represents a sine function having period 12, amplitude 4 and phase shift of 2?

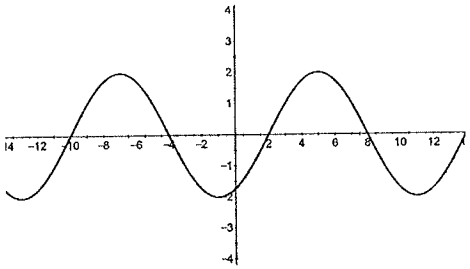
A.



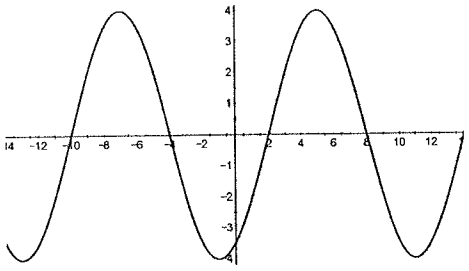
B.



C.



D.



E. None of these.

18. Write an equation for the line that goes through the center of the circle $x^2 + y^2 - 2x + 4y = 4$ and is perpendicular to the line $3x + y = 7$.

A. y = 3x +5

B. $y - 2 = \frac{1}{3}(x+1)$

C. y = -3x + 1

D. $y + 2 = \frac{1}{3}(x-1)$

E. $y = -3x + 2$

19. If f(1) = 5, what can you say about g(x) = -f(x + 2) -3?

A. g(-1) = -2

B. g(4) = -2

C. g(-1) = -8

D. g(4) = -2

E. Not enough information.

20. Find the 5th term of the expansion of $\left(x - y^2\right)^6$ if the terms are listed with the powers of x in decreasing order.

A. $-20x^3y^6$

B. $15x^2y^3$

C. $-15x^4y^4$

D. $6x^2y^{10}$

E. None of these.

21. A rocket is fired at sea level and climbs at a constant angle of 78° through a distance of 10,500 feet. Approximate its altitude to the nearest foot.

A. 10,270 ft

B. 49,399 ft

C. 10,271 ft

D. 2,183 ft

E. 5,397 ft

22. The second and sixth terms of a geometric progression are $\frac{3}{10}$ and $\frac{243}{160}$ respectively. What is the first term of this sequence?

A. 3/10

B. 1/10

C. 3/5

D. 3/5

E. 1/5

23. Find the angle between the two vectors $\mathbf{v} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{w} = -\mathbf{i} + 4\mathbf{j}$.

A. 42.3°
B. 73.8°
C. 137.7°
D. 87°
E. 145°

24. A force of 6 pounds acts in the direction of 40° to the horizontal. The force moves an object along a straight line from the point (5, 9) to the point (8, 20), with the distance measured in feet. Find the work done by the force.

A. 56 ft-lb
B. 62 ft-lb
C. 37 ft-lb
D. 50 ft-lb
E. None of these.

25. A piece of commercial real estate is priced at $3.50 per square foot. Find the cost, to the nearest dollar, of a triangular lot measuring 240 feet by 300 feet by 420 feet.

A. $126,175
B. $ 220,512
C. $63,225
D. $96,450
E. $123,454

26. Which equation below represents the line that is perpendicular to the line $3x - 5y = -10$ and has the same x-intercept as the line $2x + y = 8$.

A. $y = \frac{-5}{3}x + 8$

B. $y = \frac{-5}{3}(x - 4)$

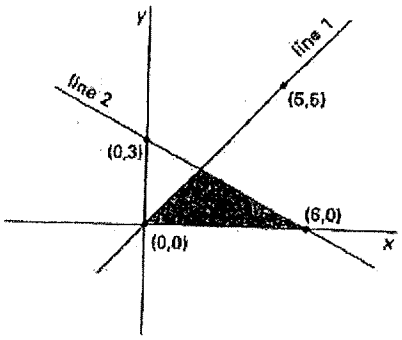
C. $y = \frac{3}{5}(x - 8)$

D. $y = -\frac{3}{5}(x - 4)$

E. $y = \frac{5}{3}x + 4$

27. Find the area in square units of the shaded triangle given. Figure is not to scale.

A. 4.5
B. 5
C. 5.5
D. 6
E. 6.5



28. The points of intersection of the graphs of $xy = 20$ and $x^2 + y^2 = 41$ are joined to form a rectangle. Find the area of the rectangle.

A. 80 sq units

B. 20 sq units

C. 18 sq units

D. 10 sq units

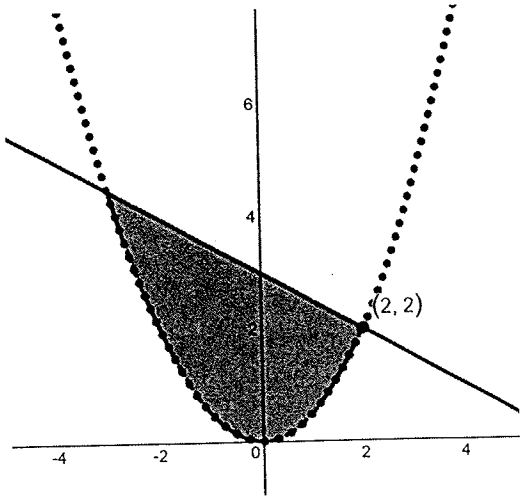
E. 22 sq units

29. Find the values of x, y and z that satisfy the following system of equation. What is their sum?

$$3x+y-z=9$$
$$x+2y+5z=-13$$
$$4x-3y+2z=2$$

A. -1
B. 5
C. 1
D. 0
E. -5

30. Which system of inequalities describes the set of ordered pairs graphed below.



A. $x^2 < 2y$
$x > 6-2y$

B. $x^2 > 2y$
$x \leq 6-2y$

C. $x^2 < 2y$
$x \leq 6-2y$

D. $x^2 < 2y$
$x \geq 6-2y$

E. None of these.

31. A circular gear in a motor rotates at the rate of 100 rpm (revolutions per minute). What is the linear speed of a point on the gear 4 cm from the center?

A. 800π cm/min

B. 1600π cm/min

C. 400π cm/min

D. 600π cm/min

E. 1000π cm/min

32. In April 1803, US representatives in Paris agreed to pay Napoleon $15 million for about 828,000 square miles of land called the Louisiana Purchase. If Napoleon had invested that amount at 4% annual interest compounded continuously, how much would it be worth today?

A. $694.8 million

B. $78,280.2 million

C. $3,771,111,026 million

D. Less than any of these.

E. More than any of these.

33. An air traffic controller spots a plane at coordinates $\left(-10,-10\sqrt{3}\right)$ in relation to the air traffic control tower if the tower is at (0, 0) on the Cartesian coordinate plane. What are the coordinates for the plane when the air traffic controller switches to polar coordinates?

A. $\left(20,\frac{\pi}{3}\right)$

B. $\left(20,\frac{7\pi}{6}\right)$

C. $\left(-20,\frac{2\pi}{3}\right)$

D. $\left(-20,\frac{\pi}{3}\right)$

E. $\left(10,\frac{4\pi}{3}\right)$

34. Find the solution set for the inequality

$$\frac{(4-x^2)(2x-3)}{(x-5)\sqrt{x+1}} \leq 0$$

A. $\left(-1, \frac{3}{2}\right] \cup [2,5)$

B. $\left[\frac{3}{2}, 2\right] \cup (5, \infty)$

C. $\left(\frac{3}{2}, 2\right) \cup (5, \infty)$

D. $(-\infty, -1) \cup \left(-1, \frac{3}{2}\right] \cup [2,5)$

E. $(-1, 2]$

35. If $\log(x^3 y) = a$ and $\log(y^2) = b$, then $\log x = \ldots$

A. $\frac{a}{3} - \frac{b}{2}$

B. $\frac{b}{6} - \frac{a}{3}$

C. $\frac{2a-3b}{6}$

D. $\frac{2a-b}{-6}$

E. $\frac{2a-b}{6}$

36. If θ is a first quadrant angle with $\sin(\theta) = \frac{a}{b}$, then $\sin(2\theta) = \ldots$ (You may assume that a and b are both positive.)

A. $\frac{2a(b-a)}{b^2}$

B. $\frac{2ab}{\sqrt{a^2+b^2}}$

C. $\frac{a^2-2b^2}{a^2}$

D. $\frac{2b\sqrt{b^2-a^2}}{a^2}$

E. $\frac{2a\sqrt{b^2-a^2}}{b^2}$

37. Twice the width of a rectangle is 1 inch more than the length. The sum of the width and length is 5 inches. What is the area of the rectangle?

A. $\frac{44}{9}\ \text{in}^2$

B. $5\ \text{in}^2$

C. $4\ \text{in}^2$

D. $6\ \text{in}^2$

E. $\frac{50}{9}\ \text{in}^2$

38. A bit string is a string of 0's and 1's. For example a three digit bit string could be 010 or 110 or even 000. How many 5 digit bit strings start with a 0 or end with a 1?

A. 24

B. 32

C. 10

D. 16

E. 8

39. An airplane's velocity with no wind is 500 km/h with a bearing of N60°E. The wind at the altitude of the plane has a velocity of 60 km/h and is blowing SE (which means S45°E.) What is the true bearing of the plane?

A. N23.6°E

B. N66.4°E

C. S23.6°E

D. S66.4°E

E. N15°E

40. Bob and Sue invest $2,000 in a mutual fund earning 4% annual interest compounded quarterly to save money for their newborn, Candy's college tuition. If no money is added to the fund other than interest, how old will Candy be when there is $6,000 in the fund?

A. 40

B. 36

C. 28

D. 25

E. 30