

SIXTY-FIRST ANNUAL MATHEMATICS CONTEST 2017

Algebra I

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Scoring formula: 4 x (Number Right) - (Number Wrong) + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the <u>best</u> answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

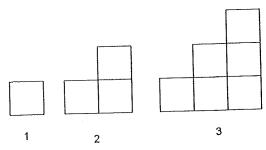
This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

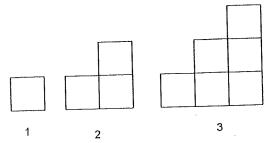
When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

- 1) Multiply: (2x + 3)(x + 1)
 - a) $2x^2 + 3$
 - b) $2x^2 + 4x + 3$
 - c) $2x^2 + 5x + 1$
 - d) $2x^2 + 5x + 3$
 - e) $2x^2 + 6x + 3$
- 2) Simplify: $\sqrt[3]{\frac{x^3y^{-6}}{z^3}}$
 - a) $\frac{xy^2}{z}$
 - b) $\frac{x}{yz}$
 - c) $\frac{x^2}{y^3z^2}$
 - d) $\frac{1}{xy^2z}$
 - e) $\frac{x}{y^2z}$
 - f)
- 3) Which of the following is a solution to $x^2 y < 3$?
 - a) (0, -3)
 - b) (1, -1)
 - c) (1, -4)
 - d) (0, -4)
 - e) (-1, -4)
- 4) Which expression is equivalent to 2x 3?
 - a) $x = \frac{3}{2}$
 - b) 4x 6
 - c) $\frac{12x \ 18}{6}$
 - d) 0.2x 0.3
 - e) 2x 3 + 3

- 5) What set of numbers is defined by $\{2k-1 \mid k \text{ is an integer}\}$?
 - a) Negative integers
 - b) Non-positive integers
 - c) Even negative integers
 - d) Odd negative integers
 - e) Odd integers
- 6) Toothpicks are used to design the following figures. Figure Number 1 has a perimeter of 4 units. What is the perimeter of Figure Number 126?



- a) 32 units
- b) 64 units
- c) 126 units
- d) 252 units
- e) 504 units
- 7) Toothpicks are used to design the following figures. Figure Number 1 has an area of 1 square unit. What is the area of Figure Number 10?

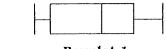


- a) 10 square units
- b) 45 square units
- c) 50 square units
- d) 55 square units
- e) 100 square units

- 8) How many lines of symmetry does the following figure have?
 - a) 0
 - b) 1
 - c) 2
 - d) 3
 - e) 4



- 9) What is the sum of the x-intercepts for function $f(x) = 8x^2 14x 15$?
 - a) 0
 - b) $\frac{7}{4}$
 - c) $-\frac{7}{4}$
 - d) $\frac{13}{4}$
 - e) $-\frac{13}{4}$
- 10) Which of the following statements must be true about the data sets corresponding to Boxplots 1 and 2 below, assuming they are on the same scale?



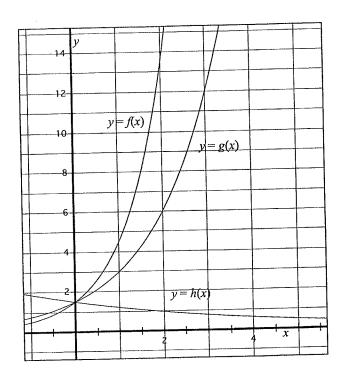
Boxplot 1



Boxplot 2

- (i) The data set corresponding to Boxplot 2 has a greater median.
- (ii) The data set corresponding to Boxplot 1 has a smaller range.
- a) (i) and (ii) are both true.
- b) Only (i) is true.
- c) Only (ii) is true.
- d) Neither (i) or (ii) is true.
- e) There is not enough information provided to answer the question.

- 11) What is the equation of the line perpendicular to 2x+3y=1 and passing through the point (0, k)?
 - a) 3x 2y = -2k
 - b) 3x + 2y = -2k
 - c) 3x 2y = 2k
 - d) 2x + 3y = 3k
 - e) 2x 3y = 3k
- 12) Below are graphs of three exponential functions. Let $f(x) = a \times b^x$. Let $g(x) = m \times n^x$. Let $h(x) = r \times s^x$. Which statement is true about the parameters a, b, m, n, r, and s?



- a) a = m = r and b < n < s.
- b) a = m = r and b > n > s.
- c) a > m > r and b = n = s.
- d) a < m < r and b = n = s.
- e) a > m > r and b > n > s.

13) Which of the following is a solution to $(x k)^2 < 4$?

- a) k-2
- b) k + 2
- c) k-3
- d) k+3
- e) *k*

14) When I enter 0.058 into my calculator, the display reads 3.90625E⁻¹¹. Which statement below describes what that result means?

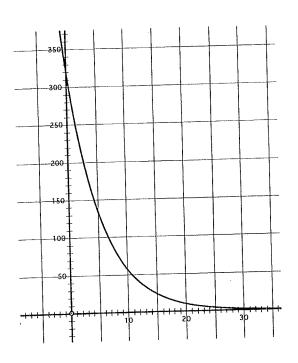
- a) 0.05^8 is exactly equal to 3.90625^{-11} .
- b) 0.058 is approximately 0.00 000 000 000 390 625.
- c) 0.058 is exactly equal to 0.00 000 000 000 390 625.
- d) 0.058 is exactly equal to 0.00 000 000 003 906 25.
- e) 0.058 is approximately 0.00 000 000 003 906 25.

15) Let the operation \blacksquare be defined in this way: $a \blacksquare b = (a + b)^2$. Let the operation \therefore be defined in this way: $a \therefore b = (a - b)^2$.

What is the value of $\frac{1}{9}$:: $\left(\frac{2}{98} \blacksquare \frac{48}{49}\right)$?

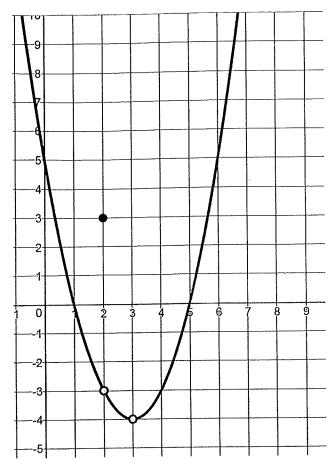
- a) 1
- b) $\frac{64}{81}$
- c) $\frac{8}{9}$
- d) $\frac{8}{9}$
- e) $-\frac{64}{81}$

16) The graph below represents one of the situations described below. Which situation is represented?



- a) \$320 is invested in an account that pays 4% interest compounded annually. The balance in the account is a function of the number of years invested.
- b) 320 grams of a radioactive substance has a half-life of 4 days. The amount in grams of the substance is a function of the number of days.
- c) 320 dice are rolled. After every roll the odd numbers are removed. The number of dice remaining is a function of the number of rolls.
- d) 320 pounds of potatoes are put into a slicing machine. The machine slices 20 pounds of potatoes per minute. The weight of the UNSLICED potatoes is a function of the number of minutes elapsed.
- e) 320 tires are put into a shredding machine. Every minute 40 pounds of tires are shredded. The weight of the UNSHREDDED tires is a function of the number of minutes elapsed.

17) The graph of a function, y = f(x), is shown below. According to the graph below, which of the following statements is FALSE?



- a) f(2)=3
- b) f(3) = 4
- c) f(1)=0
- d) f(5) = 0
- e) f(0) = 5

18) The table below shows several groups' concert tickets for the Mathemusician and the total sales.

| Premium | General Admission | Balcony | Total Sales |
|---------|----------------------|---------|-------------|
| 1 | 1 | | \$90 |
| | 1 | 1 | \$55 |
| 1 | 1 | 1 | \$110 |
| 2 | 1 | | \$145 |

All Premium tickets are the same price. All General Admission tickets are the same price and they are cheaper than the Premium tickets. All Balcony tickets are the same price, but their price is different from either of the other two types of tickets. What are the differences in the prices of the tickets?

- a) Premium are \$20 more than General. General are \$15 more than Balcony.
- b) Premium are \$35 more than General. General are \$20 more than Balcony.
- c) Premium are \$30 more than General. General are \$15 more than Balcony.
- d) Premium are \$25 more than General. General are \$10 more than Balcony.
- e) Premium are \$15 more than General. General are \$10 more than Balcony.

19) Below are two functions.

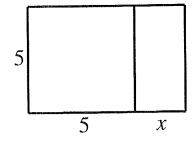
$$y = f(x) = 2x - 7$$
 $y = g(x) = \frac{2x^2 - 5x - 7}{x+1}$

Which statement is true?

- a) The graph of f(x) is identical to the graph of g(x).
- b) The domain of both functions is the set of all real numbers.
- c) The range of both functions is the set of all real numbers.
- d) (0, -7) is an element of both functions.
- e) (-1, -9) is an element of both functions.

- 20) Jim and Kim can paint a fence in 144 minutes. (This assumes that both work the whole time and that each works at a constant pace.) Jim working alone could paint the whole fence in two hours less than it would take Kim working alone to paint the whole fence. How long would it take Kim working alone to paint the whole fence?
 - a) It would take Kim 3 hours to paint the fence alone.
 - b) It would take Kim 4 hours to paint the fence alone.
 - c) It would take Kim 4.2 hours to paint the fence alone.
 - d) It would take Kim 4.5 hours to paint the fence alone.
 - e) It would take Kim 6 hours to paint the fence alone.
- 21) A formula for the surface area of a right circular cone is r(r+l), if r is the radius of the base of the cone and l is the slant height. A formula for the volume of the right circular cone is $r^2\sqrt{l^2-r^2}$. Suppose that a right circular cone has a volume of 300π cubic units. The radius of the base is 5 units. What is the surface area of the cone?
 - a) The surface area is 90π square units.
 - b) The surface area is 144π square units.
 - c) The surface area is 167π square units.
 - d) The surface area is 324π square units.
 - e) The surface area is 450π square units.
- 22) An old coin was valued at \$24 ten years ago. This year, the same coin was valued at \$84. Which of the following statements is false?
 - a) The value of the coin 10 years ago is 3.5% of its current value.
 - b) The coin is worth 350% as much this year as it was 10 years ago.
 - c) The coin is worth 250% more this year than it was 10 years ago.
 - d) The value of the coin 10 years ago is $\frac{2}{7}$ of its current value.
 - e) The value of the coin 10 years ago is $\frac{5}{7}$ less than its current value.

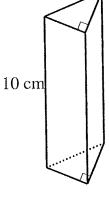
- 23) What is the coefficient x^4y the binomial expansion of $(3x 2y)^5$?
 - a) 5
 - b) 405
 - c) -81
 - d) 405
 - e) 810
- 24) Three children all start their savings account at the same time. Alice put in \$50 the first month, and then put in \$10 each month thereafter. Marco put in \$1 to begin Month 1, and then put in double the previous amount each month. So in Month 2 he put in \$2; in Month 3 he put in \$4. Kali put in 1¢ the first month, and tripled the amount she put in each month after. In Month 2 she put in 3¢, and in Month 3 she put in 9¢. Which of the following shows the children in order, from most saved in 10 years to least saved in 10 years?
 - a) Alice, Marco, Kali
 - b) Kali, Marco, Alice
 - c) Marco, Alice, Kali
 - d) Alice, Kali, Marco
 - e) Kali, Alice, Marco
- 25) Ms. Vogel was surprised when her students wrote many different expressions to represent the area of the figure below. She wanted to make sure that she did not mark as incorrect any that were actually right. Which of the following expressions correctly represents the area of the figure?



- i) $5^2 + x$
- ii) $5^2 + 5x$
- iii) 5(5+x)
- iv) $(5 + x)^2$

- a) i) only
- b) ii) only
- c) ii) and iii) only
- d) iii) and iv) only
- e) iii) only

- 26) A triangular base prism has a volume of 16.2 cubic centimeters. The base is an isosceles right triangle. The height of the prism is 10 centimeters. What is the approximate length of the hypotenuse of the triangular base of the prism?
 - a) 1.8 cm
 - b) 2.5 cm
 - c) 3.24 cm
 - d) 4.6 cm
 - e) 6.48 cm.



- 27) A price change from \$14 to \$21 is an increase of what percent?
 - a) 25%
 - b) $33\frac{1}{3}\%$
 - c) 50%
 - d) 75%
 - e) 100%
- 28) $3 \text{ m}^3 = \underline{}?\underline{}\text{ dm}^3$
 - a) 30
 - b) 270
 - c) 300
 - d) 900
 - e) 3000
- 29) What is the 99th letter in the following pattern?

ABBCCCDDDD...

- a) L
- b) M
- c) N
- d) O
- e) P

- 30) Simplify the expression: $\frac{x}{x^2-16} \frac{7}{x^2+5x+4}$
 - a) $\frac{x^2-6x+28}{(x-4)(x+4)(x+1)}$
 - b) $\frac{x^2-6x+28}{(x-4)(x+4)}$
 - c) $\frac{x^2+6x+28}{(x-4)(x+4)(x+1)}$
 - d) $\frac{x^2+6x+28}{(x-4)(x+4)(x-1)}$
 - e) $\frac{x^2+6x-28}{(x-4)(x+4)(x-1)}$
- 31) Simplify the fraction: $\frac{9 + \frac{3}{x}}{\frac{x}{4} + \frac{1}{12}}$
 - a) $\frac{x}{12}$
 - b) $\frac{4}{x+1}$
 - c) $\frac{1}{x}$
 - d) $\frac{36}{x}$
 - e) 1
- 32) What is the remainder when the following polynomials are divided?

$$\frac{x^3 - 2x^2 + 4x - 8}{x + 2}$$

- a) 32
- b) 16
- c) 0
- d) 8
- e) 16
- 33) Simplify the complex number: (2 + 3i)(5 2i)
 - a) 4 + 11i
 - b) 10 6i
 - c) 7+i
 - d) 16 + 11i
 - e) 17i

- 34) There are 18 yards of ribbon left on the spool. It requires $2\frac{2}{3}$ yards of ribbon to trim a band uniform. After you trim the greatest number of uniforms that can be trimmed with the ribbon, how many yards of ribbon will be leftover?
 - a) $\frac{3}{4}$
 - b) $\frac{3}{8}$
 - c) 2
 - d) 1
 - e) $\frac{1}{2}$
- 35) If 45 + 45 = 134, what base are you using?
 - a) 3
 - b) 4
 - c) 6
 - d) 8
 - e) 10
- 36) Jen drove 40 miles per hour from home to work. Going home, there was traffic and she could only drive 30 miles per hour from work to home. What was her average rate for the entire trip (to the nearest tenth)?
 - a) 17.1 mph
 - b) 33.9 mph
 - c) 34.3 mph
 - d) 35.0 mph
 - e) 35.4 mph
- 37) For the function $f(x) = 2x^2 3x + 5$, find the difference quotient $\frac{f(x+h)-f(x)}{h}$.
 - a) 4x 3
 - b) 2x + h + 3
 - c) 4x + 2h
 - d) 4x + 2h 3
 - e) 4x + h

- 38) The height of an object dropped from a tall building is given by the table, where t is the elapsed time in seconds and h is the height in feet.
 - t 0 1 2 3 4 5 h 600 584 536 456 344 200

If the height is modeled by $h(t) = h_0 - 16t^2$, find h_0 .

- a) 300 ft
- b) 400 ft
- c) 600 ft
- d) 900 ft
- e) 1200 ft
- 39) Which of the following statements concerning the linear correlation coefficient are true?
 - A: If the linear correlation coefficient for two variables is zero, then there is no relationship between the variables.
 - B: If the slope of the regression line is negative, then the linear correlation coefficient is negative.
 - C: The value of the linear correlation coefficient always lies between -1 and 1.
 - D: A linear correlation coefficient of 0.62 suggests a stronger linear relationship than a linear correlation coefficient of -0.82.
 - a) A and B
 - b) B and C
 - c) C and D
 - d) A and C
 - e) A and D
- 40) State the domain of the function $f(x) = \sqrt{4-x}$.
 - a) $\{x \mid x < 4\}$
 - b) $\{x \mid x \ge 4\}$
 - c) $\{x \mid x \le 4\}$
 - d) $\{x \mid x \geq 0\}$
 - e) $\{x \mid x \le 0\}$